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A simple criterion for the global exponential stability of uncertain T-S fuzzy singularly perturbed discrete-time systems*

by

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Abstract: In this paper, the global exponential stability for a class of uncertain Takagi-Sugeno (T-S) fuzzy singularly perturbed systems is investigated. Based on time-domain approach with difference inequality technique, a simple criterion is derived to guarantee the global exponential stability of such systems. The upper bound of the singular perturbation parameter is also provided by estimating the unique positive zero of specific function. Finally, two numerical examples are given to demonstrate the feasibility and effectiveness of the obtained result.

Keywords: Takagi-Sugeno fuzzy system, singularly perturbed systems, stability analysis, uncertain systems.

1. Introduction

Recently, much effort has been devoted to the stability analysis, control design, and industrial applications of singularly perturbed systems; see, for instance, Alwan and Liu (2009), Chen et al. (2002a, 2002b), Meng and Jing (2009), Prljaca and Gajic (2008), Sun et al. (1996), Sun (2009), Xu and Feng (2009), and the references therein. This is due not only to theoretical interests, but also to the relevance of this topic for various engineering applications. Typical singularly perturbed systems include flexible mechanical systems, armature-controlled DC motors, magnetic-ball suspension systems, direct-drive robots, flexible joint robots, flexible space structures, high-gain control systems, tunnel diode circuits, Josephson junction circuits, nonlinear time-invariant RLC networks, control system of an inverted pendulum, control system of an airplane, and IEEE type 2 voltage regulators (Sun et al., 1996). Objectively speaking, multiple time-scale phenomena are almost unavoidable in real systems and the

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singular perturbation methodology has proven to be an effective tool for system analysis and control design in this context. Due to the very small singular perturbation parameter , the stability analysis of singularly perturbed systems consists in decomposition of the original system into two sub-problems, for the slow and fast dynamics. The two respective stability criteria are then combined to give a criterion for the full systems.

During the past decades, on the other hands, T-S fuzzy systems, with or without uncertainties, have received a great deal of interest; see, for example, Chen (2009), Chung and Wu (2009), Farzaneh and Tootoonchi (2009), Lagrat et al. (2008), Lien (2006), Rhee and Won (2006), Tanaka and Sugeno (1992), Yu (2009), Zhang et al. (2009). In particular, T-S fuzzy system models can be viewed as complex nonlinear systems and these models frequently appear in several engineering systems. Recently, the stability analysis for T-S fuzzy system has been extensively explored; see, for instance, Chen (2009), Lien (2006), and the references cited therein. Over the past years, various methodologies in robust stability analysis for T-S fuzzy system have been offered, such as LMI approach, Lyapunov-based methodology, time-domain approach, and frequency-domain approach. In this paper, the robust stability for a class of uncertain T-S fuzzy singularly perturbed systems will be studied. Based on time-domain approach with difference inequality technique, a simple criterion will be derived to guarantee the global exponential stability of such systems. Besides, the upper bound of the singular perturbation parameter is given by estimating the unique positive zero of a specific function. Finally, a simulation example is provided to demonstrate the feasibility and effectiveness of the main result.

This paper is organized as follows. The problem formulation and the main result are presented in Section 2. Two numerical examples are given in Section 3 to illustrate the main result. Finally, conclusions are drawn in Section 4.

2. Problem formulation and the main result

Notations

Z^+	the set of all non-negative integers
$\Re^{m\times n}$	the set of all real m by n matrices
$\|\lambda\|$	the modulus of the complex number λ
$\|\|x\|\|$	the Euclidean norm of the vector $x \in \Re^{p\times 1}$
$\lambda_{\max}(A)$	the maximum eigenvalue of the matrix A with real eigenvalues
A^*	the conjugate transpose of the matrix A
$\|\|A\|\|$	the induced Euclidean norm of the matrix A; $\|\|A\|\| = [\lambda_{\max}(A^*A)]^{\frac{1}{2}}$
$\underline{m}$	$\{1, 2, \ldots, m\}$
$\overline{m}$	$\{0, 1, \ldots, m\}$.

In this paper, we consider the following uncertain fuzzy singularly perturbed systems, which is represented by a T-S fuzzy model (Lagrat, Ouakka, Boumhidi,

2008) and composed by a set of fuzzy implications. Each implication is expressed by the uncertain singularly perturbed time-varying systems and the ith rule of the T-S model is written in the following form.

Rule j: If $h_1(t)$ is about $S_{1,j}$, $h_2(t)$ is about $S_{2,j}$, …, and $h_r(t)$ is about $S_{r,j}$, then

$$x(k+n) = \sum_{i=0}^{n-1} \Delta A_{i,j}(k)x(k+i) + \Delta B_{i,j}(k)z(k+i), \quad \forall k \geq 0, \tag{1a}$$

$$z(k+n) = \sum_{i=0}^{n-1} \varepsilon \Delta C_{i,j}(k)x(k+i) + \varepsilon \Delta D_{i,j}(k)z(k+i), \quad \forall k \geq 0 \tag{1b}$$

$$\begin{bmatrix} x(k) \\ x(k) \end{bmatrix} = \phi_j(k), \quad 0 \leq \leq n-1, \tag{1c}$$

where $h_1(t)$, $h_2(t)$, …, $h_r(t)$, are premise variables, $S_{i,j}$, $\forall i \in \underline{r}, j \in \underline{m}$, are fuzzy sets, m is the number of If-then rules, $x \in \Re^{p\times 1}$ is the slow state, $z \in \Re^{q\times 1}$ is the fast state, $\Delta A_{i,j}$, $\Delta B_{i,j}$, $\Delta C_{i,j}$, and $\Delta D_{i,j}$ are uncertain matrices of appropriate dimensions, the singular perturbation parameter ε is a positive and sufficiently small scalar, and $\phi_j(k)$ is the initial vector-valued function.

Before presenting the main result, we make an assumption as follows.

(A1) There exist nonnegative constants $a_{i,j}$, $b_{i,j}$, $c_{i,j}$, and $d_{i,j}$ such that

$$\begin{aligned} &\|\Delta A_{i,j}(k)\| \leq a_{i,j}, \quad \|\Delta B_{i,j}(k)\| \leq b_{i,j}, \\ &\|\Delta C_{i,j}(k)\| \leq c_{i,j}, \quad \|\Delta D_{i,j}(k)\| \leq d_{i,j}, \quad \forall i \in \overline{n-1}, j \in \underline{m},\ k \in Z^+. \end{aligned}$$

For brevity, let us define

$$p_i := \max_{1\leq j\leq m} (a_{i,j} + b_{i,j}), \quad q_i := \max_{1\leq j\leq m} (c_{i,j} + d_{i,j}), \quad \forall i \in \overline{n-1}, \tag{2}$$

$$g(x) := 1 - \sum_{i=0}^{n-1} \sqrt{p_i^2 + q_i^2 x^2}, \quad \text{with } x \geq 0, \tag{3}$$

$$y(k) := \begin{bmatrix} x(k) \\ z(k) \end{bmatrix}, \ \Delta E_{i,j}(k) := \begin{bmatrix} \Delta A_{i,j}(k) & \Delta B_{i,j}(k) \\ \varepsilon \Delta C_{i,j}(k) & \varepsilon \Delta D_{i,j}(k) \end{bmatrix}, \ \forall i \in \overline{n-1},\ j \in \underline{m}.$$

If we use the standard fuzzy inference method (Takagi, Sugeno, 1985), i.e. minimum fuzzy inference, singleton fuzzifier, and central-average defuzzifier, system (1) is inferred as follows.

$$\begin{aligned} y(k+n) &= \frac{1}{\sum_{i=1}^m u_i(h(t))} \cdot \sum_{i=1}^m u_i(h(t)) \cdot \{\Delta E_{n-1,i}(k)y(k+n-1) \\ &+\Delta E_{n-2,i}(k)y(k+n-2) + \cdots + \Delta E_{1,i}(k)y(k+1) + \Delta E_{0,i}(k)y(k)\}, \quad \forall k \geq 0 \end{aligned} \tag{4a}$$

$$y(k) = \frac{1}{\sum_{i=1}^m u_i(h(t))} \cdot \sum_{i=1}^m [u_i(h(t)) \cdot \phi_i(k)], \quad 0 \leq k \leq n-1, \tag{4b}$$

where $u_i(h(t)) = \prod_{j=1}^{r} \Phi_{ij}(h_j(t))$ and $\Phi_{ij}(h_j(t))$ is the grade of membership of $h_j(t)$ in fuzzy set S_{ij}. Define $\lambda_i(h(t)) = \frac{u_i(h(t))}{\sum_{i=1}^{m} u_i(h(t))}$, and we assume, in this paper, $u_i(h(t)) \geq 0$ for each $i \in \underline{m}$ and $\sum_{i=1}^{m} u_i(h(t)) \geq 0$. Thus, the system (4) can be represented as

$$y(k+n) = \sum_{i=1}^{m} \lambda_i(h(t)) \cdot \{\Delta E_{n-1,i}(k) y(k+n-1) + \Delta E_{n-2,i}(k) y(k+n-2) + \cdots + \Delta E_{1,i}(k) y(k+1) + \Delta E_{0,i}(k) y(k)\}, \quad \forall k \geq 0, \tag{5a}$$

$$y(k+n) = \sum_{i=1}^{m} [\lambda_i(h(t)) \cdot \phi_i(k)], \quad 0 \leq k \leq n-1, \tag{5b}$$

with $0 \leq \lambda_i(h(t)) \leq 1$, $\forall i \in \underline{m}$ and $\sum_{i=1}^{m} \lambda_i(h(t)) = 1$.

Now, we are in a position to present the main result.

THEOREM 1 *The uncertain T-S fuzzy singularly perturbed system (1) with (A1) and* $0 < \varepsilon < \bar{\varepsilon}$ *is globally exponentially stable provided that*

$$\sum_{i=0}^{n-1} p_i < 1, \tag{6}$$

where

$$\bar{\varepsilon} := \begin{cases} \delta, & \text{if } \sum_{i=0}^{n-1} q_i^2 \neq 0 \\ \infty, & \text{if } \sum_{i=0}^{n-1} q_i^2 = 0 \end{cases}$$

and δ *is the unique positive zero of the function* $g(x)$.

Proof. Define $r_{i,j} := \sqrt{(a_{i,j} + b_{i,j})^2 + \varepsilon^2 (c_{i,j} + d_{i,j})^2}$ and $b_i := \sqrt{p_i^2 + \varepsilon^2 q_i^2}$. By (2), it can be easily obtained that

$$b_i \geq r_{i,j}, \quad \forall i \in \overline{n-1},\ j \in \underline{m}. \tag{7}$$

In addition, the function of $g(x)$ is a decreasing function with

$$g(0) > 0, \quad g(\varepsilon) > 0,\ \forall \varepsilon \in (0, \delta), \quad g(\delta) = 0, \text{ if } \sum_{i=0}^{n-1} q_i^2 \neq 0;$$

$$g(\varepsilon) > 0,\ \forall \varepsilon \geq 0, \text{ if } \sum_{i=0}^{n-1} q_i^2 = 0,$$

in view of (6). This implies that

$$g(\varepsilon) > 0, \quad \forall\, \varepsilon \in (0,\ \bar{\varepsilon}). \tag{8}$$

From (A1), one has

$$\|\Delta E_{i,j}(k)\| = \left\| \begin{bmatrix} \Delta A_{i,j}(k) & \Delta B_{i,j}(k) \\ \varepsilon\Delta C_{i,j}(k) & \varepsilon\Delta D_{i,j}(k) \end{bmatrix} \right\|$$
$$= \sup_{\left\|\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right\|=1} \left\| \begin{bmatrix} \Delta A_{i,j}x_1 + \Delta B_{i,j}x_2 \\ \varepsilon\Delta C_{i,j}x_1 + \varepsilon\Delta D_{i,j}x_2 \end{bmatrix} \right\|$$
$$\leq \sup_{\left\|\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right\|=1} \sqrt{(\|\Delta A_{i,j}\|\cdot\|x_1\|+\|\Delta B_{i,j}\|\cdot\|x_2\|)^2+(\|\varepsilon\Delta C_{i,j}\|\cdot\|x_1\|+\|\varepsilon\Delta D_{i,j}\|\cdot\|x_2\|)^2}$$
$$\leq \sqrt{(\|\Delta A_{i,j}\|+\|\Delta B_{i,j}\|)^2+(\|\varepsilon\Delta C_{i,j}\|+\|\varepsilon\Delta D_{i,j}\|)^2}$$
$$\leq \sqrt{(a_{i,j}+b_{i,j})^2+\varepsilon^2(c_{i,j}+d_{i,j})^2}$$
$$= r_{i,j}, \quad \forall i \in \overline{n-1}, j \in \underline{m}. \tag{9}$$

From (4), (7)-(9), with (A1), it is easy to see that

$$\|y(k+n)\| = \Big\| \sum_{i=1}^{m} \lambda_i(h(t)) \cdot \{\Delta E_{n-1,i}(k)y(k+n-1)$$
$$+\Delta E_{n-2,i}(k)y(k+n-2) + \cdots + \Delta E_{1,i}(k)y(k+1) + \Delta E_{0,i}(k)y(k)\} \Big\|$$
$$\leq \sum_{i=1}^{m} \lambda_i(h(t)) \cdot \big\{\|\Delta E_{n-1,i}(k)\| \cdot \|y(k+n-1)\|$$
$$+\|\Delta E_{n-2,i}(k)\| \cdot \|y(k+n-2)\| + \cdots$$
$$+\|\Delta E_{1,i}(k)\| \cdot \|y(k+1)\| + \|\Delta E_{0,i}(k)\| \cdot \|y(k)\|\big\}$$
$$\leq \sum_{i=1}^{m} \lambda_i(h(t)) \cdot \big\{r_{n-1,i} \cdot \|y(k+n-1)\| + r_{n-2,i} \cdot \|y(k+n-2)\| + \cdots$$
$$+r_{1,i} \cdot \|y(k+1)\| + r_{0,i} \cdot \|y(k)\|\big\}$$
$$\leq \Big(\sum_{i=1}^{m} \lambda_i(h(t))\Big) \cdot b_{n-1} \cdot \|y(k+n-1)\|$$
$$+\Big(\sum_{i=1}^{m} \lambda_i(h(t))\Big) \cdot b_{n-2} \cdot \|y(k+n-2)\| + \cdots$$
$$+\Big(\sum_{i=1}^{m} \lambda_i(h(t))\Big) \cdot b_1 \cdot \|y(k+1)\| + \Big(\sum_{i=1}^{m} \lambda_i(h(t))\Big) \cdot b_0 \cdot \|y(k)\|$$

$$= b_{n-1} \cdot \|y(k+n-1)\| + b_{n-2} \cdot \|y(k+n-2)\| + \cdots$$
$$+ b_1 \cdot \|y(k+1)\| + b_0 \cdot \|y(k)\|, \quad \forall k \geq 0. \tag{10}$$

Now we define a scalar difference system

$$w(k+n) = \sum_{i=0}^{n-1} b_i w(k+i), \quad \forall k \geq 0, \tag{11a}$$

$$w(i) = \|y(i)\|, \quad \forall i \in \overline{n-1}. \tag{11b}$$

From (10) and (11) with $e(k) := \|y(k)\| - w(k)$, $\forall k \geq 0$, it is easy to see that

$$e(n) \leq \sum_{i=0}^{n-1} b_i e(i) = 0;$$

$$e(n+1) \leq \sum_{i=0}^{n-1} b_i e(i+1) \leq b_{n-1} e(n) \leq 0;$$

$$e(n+2) \leq \sum_{i=0}^{n-1} b_i e(i+2) \leq b_{n-1} e(n+1) \leq 0;$$

$$e(n+3) \leq \sum_{i=0}^{n-1} b_i e(i+3) \leq b_{n-1} e(n+2) \leq 0;$$

$$\vdots$$

$$\vdots$$

Consequently, we conclude that $e(k) = \|y(k)\| - w(k) \leq 0$, $\forall k \geq 0$, which implies $\|y(k)\| \leq w(k)$, $\forall\ k \geq 0$. The characteristic polynomial of system (11) is given by

$$G(z) = z^n - \sum_{i=0}^{n-1} b_i z^i. \tag{12}$$

By Descartes' rule of signs (Ledermann, Vajda, 1980) in (12), it is obvious that the polynomial equation $G(x) = 0$ has a unique positive root, denoted α^*. Moreover, it is easy to see that

$$G(x) < 0,\ \forall x \in (0, \alpha^*),\ G(\alpha^*) = 0,\ G(x) > 0,\ \forall x \in (\alpha^*, \infty)\ . \tag{13}$$

By (8), we have $G(1) = g(\varepsilon) > 0$. This implies $0 < \alpha^* < 1$. Now, let λ be any root of the equation $G(z) = 0$. Then, we have

$$|\lambda|^n = \left| \sum_{i=0}^{n-1} b_i \lambda^i \right| \leq \sum_{i=0}^{n-1} b_i |\lambda|^i,$$

from which we obtain $G(|\lambda|) \le 0$. Hence, from (13), one has $|\lambda| \le \alpha^*$. Consequently, we conclude that there exist $\beta \ge 1$ and $0 < \mu < 1 - \alpha^*$ such that $\|y(k)\| \le w(k) \le \beta(\alpha^* + \mu)^k$, $\forall\, k \ge 0$. This completes the proof. ■

Remark 1 *In case of $\sum_{i=0}^{n-1} q_i^2 \neq 0$, it is easy to see that $g\left(\frac{1}{a}\right) \le 0$ with $a := \max_{i \in \overline{n-1}} q_i$. It follows that the value of δ can be directly evaluated using the Newton's method in $g(x) = 0$ with the starting value $x_1 = \frac{1}{a}$.*

Remark 2 *Consider the non-fuzzy discrete-time system described as in Sun (2009):*

$$\begin{bmatrix} x(k+n) \\ z(k+n) \end{bmatrix} = \sum_{i=0}^{n-1} \begin{bmatrix} \Delta A_i(k) & \Delta B_i(k) \\ \varepsilon \Delta C_i(k) & \varepsilon \Delta D_i(k) \end{bmatrix} \begin{bmatrix} x(k+i) \\ z(k+i) \end{bmatrix}.$$

This is the special case of (1) with $m = 1$. Then the result of Theorem 1 is exactly the same as Corollary 1 in Sun (2009). Obviously, our results are nontrivial generalizations of recent results reported in Sun (2009) to the case with multiple If-then rules (i.e., $m > 1$ or equivalently; standard T-S fuzzy system).

3. Illustrative example

Example 1 An uncertain T-S fuzzy singularly perturbed system is given by

$$\begin{aligned} x(k+2) &= \begin{bmatrix} -\Delta a \sin(k) & \Delta b \cdot e^{-k} \\ \Delta b \cos(k) & \Delta a \end{bmatrix} x(k+1) + \begin{bmatrix} 1 & 5 \\ 2\Delta c & 2\Delta d \cos(k) \end{bmatrix} z(k) \\ &\quad + \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix} u_1(k), \quad \forall k \ge 0; \\ z(k+2) &= \varepsilon \begin{bmatrix} \Delta c & -\Delta d \\ 0 & \Delta c \cdot e^{-k} \end{bmatrix} x(k+1) + \varepsilon \begin{bmatrix} 0 & \Delta d \sin(k) \\ 1 & 2\Delta d \end{bmatrix} x(k) \\ &\quad + \varepsilon \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} u_2(k), \quad \forall k \ge 0. \end{aligned}$$

where $x(k) = [\ x_1(k) \quad x_2(k)\ ]^T \in \Re^{2\times 1}$, $z(k) = [\ z_1(k) \quad z_2(k)\ ]^T \in \Re^{2\times 1}$, with $-3 \le \Delta a$, $\Delta b \le 0.3$ and $-0.1 \le \Delta c$, $\Delta d \le 0.1$. We use the following fuzzy rules for the fuzzy-model-based control:

If $x{+}2(k)$ is about 0, then $u_1(k) = \begin{bmatrix} 1 & 5 \\ 0 & 0 \end{bmatrix} z(k)$ and $u_2(k) = \begin{bmatrix} 0 & 0 \\ -0.9 & 0 \end{bmatrix} x(k)$.

If $x{+}2(k)$ is about 1, then $u_1(k) = \begin{bmatrix} 0.8 & 5 \\ 0 & 0 \end{bmatrix} z(k)$ and $u_2(k) = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix} x(k)$.

Thus, the T-S fuzzy models can be constructed as follows.

Rule 1:
If $x_2(k)$ is about 0, then

$$x(k+2) = \begin{bmatrix} -\Delta a \sin(k) & \Delta b \cdot e^{-k} \\ \Delta b \cos(k) & \Delta a \end{bmatrix} x(k+1) + \begin{bmatrix} 0 & 0 \\ 2\Delta c & 2\Delta d \cos(k) \end{bmatrix} z(k),$$
$$\forall k \geq 0, \tag{14a}$$
$$z(k+2) = \varepsilon \begin{bmatrix} \Delta c & -\Delta d \\ 0 & \Delta c \cdot e^{-k} \end{bmatrix} x(k+1) + \varepsilon \begin{bmatrix} 0 & \Delta d \sin(k) \\ 0.1 & 2\Delta d \end{bmatrix} x(k),$$
$$\forall k \geq 0. \tag{14b}$$

Rule 2:
If $x_2(k)$ is about 1, then

$$x(k+2) = \begin{bmatrix} -\Delta a \sin(k) & \Delta b \cdot e^{-k} \\ \Delta b \cos(k) & \Delta a \end{bmatrix} x(k+1) + \begin{bmatrix} 0.2 & 0 \\ 2\Delta c & 2\Delta d \cos(k) \end{bmatrix} z(k),$$
$$\forall k \geq 0, \tag{14c}$$
$$z(k+2) = \varepsilon \begin{bmatrix} \Delta c & -\Delta d \\ 0 & \Delta c \cdot e^{-k} \end{bmatrix} x(k+1) + \varepsilon \begin{bmatrix} 0 & \Delta d \sin(k) \\ 0 & 2\Delta d \end{bmatrix} x(k),$$
$$\forall k \geq 0. \tag{14d}$$

Comparing (14) with (1), one has $n = 2$, $m = 2$, $p = q = 2$, and

$$\begin{aligned}
&\|\Delta A_{1,1}\| \leq 0.6, \quad \|\Delta A_{1,2}(k)\| \leq 0.6, \quad \|\Delta B_{0,1}(k)\| \leq 0.2828, \\
&\|\Delta B_{0,2}(k)\| \leq 0.3236, \quad \|\Delta C_{0,1}(k)\| \leq 0.2414, \quad \|\Delta C_{0,2}(k)\| \leq 0.2236, \\
&\|\Delta C_{1,1}(k)\| \leq 0.168, \quad \|\Delta C_{1,2}(k)\| \leq 0.168, \\
&\|\Delta A_{0,1}(k)\| = \|\Delta A_{0,2}(k)\| = \|\Delta B_{1,1}(k)\| = \|\Delta B_{1,2}(k)\| = 0, \\
&\|\Delta D_{0,1}(k)\| = \|\Delta D_{0,2}(k)\| = \|\Delta D_{1,1}(k)\| = \|\Delta D_{1,2}(k)\| = 0.
\end{aligned}$$

This shows that (A1) is evidently satisfied with

$$\begin{aligned}
&a_{1,1} = 0.6, \quad a_{1,2} = 0.6, \quad b_{0,1} = 0.2828, \\
&b_{0,2} = 0.3236, \quad c_{0,1} = 0.2414, \quad c_{0,2} = 0.2236, \\
&c_{1,1} = 0.168, \quad c_{1,2} = 0.168, \\
&a_{0,1} = a_{0,2} = b_{1,1} = b_{1,2} = d_{0,1} = d_{0,2} = d_{1,1} = d_{1,2} = 0.
\end{aligned}$$

From (2), it follows that

$$p_0 = 0.3236, \quad p_1 = 0.6, \quad q_0 = 0.2414, \quad q_1 = 0.168.$$

Hence, it can be verified that $\sum_{i=0}^{1} p_i = 0.9236 < 1$. By (3), we obtain

$$g(x) = 1 - \sqrt{0.105 + 0.058x^2} - \sqrt{0.36 + 0.028x^2}.$$

The unique positive solution of $g(x) = 0$ is given by $\delta = 0.85$. Consequently, by Theorem 1, we conclude that the uncertain T-S fuzzy singularly perturbed system (14) with $0 < \varepsilon < 0.85$ is globally exponentially stable. The state trajectories of the system (14) are depicted in Figs. 1 and 2.

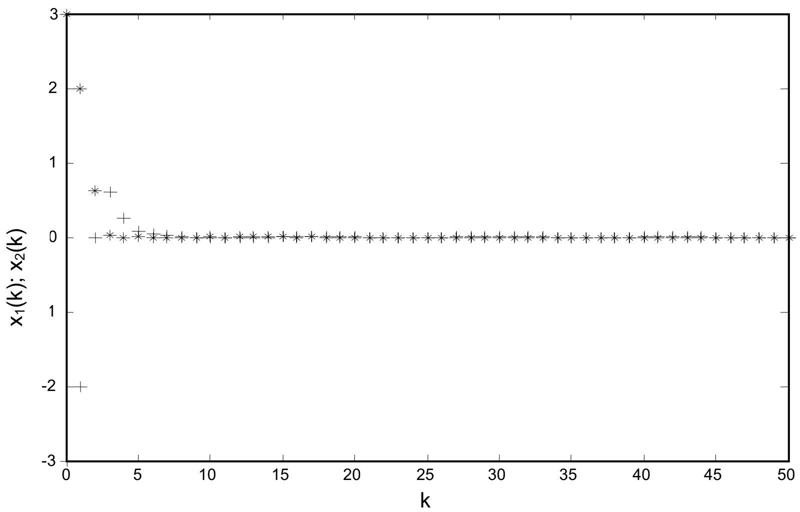

Figure 1. $x\,(k)$ of the system (14)

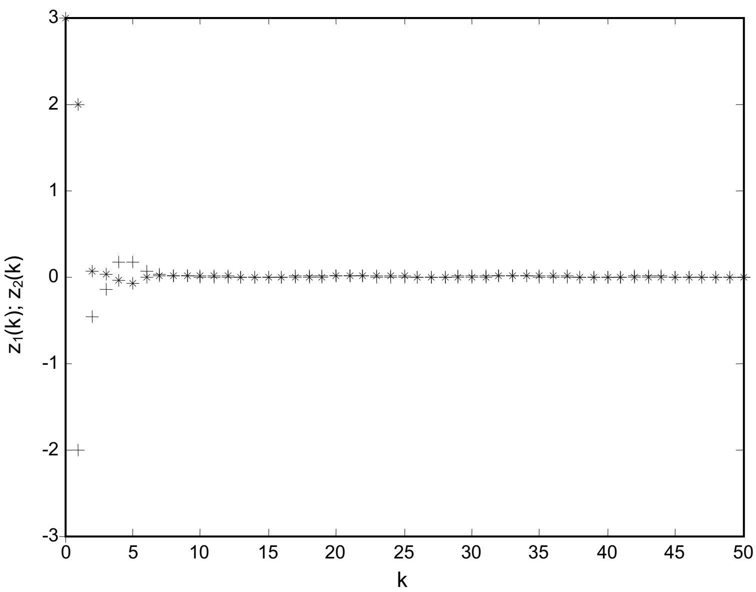

Figure 2. $z\,(k)$ of the system (14)

EXAMPLE 2 Consider a DC servo motor with the transfer function

$$\frac{Y(s)}{U(s)} = \frac{b}{s(s+a)}.$$

We can write, in matrix form,

$$\dot{x} = \begin{bmatrix} 0 & 0 \\ 0 & -a \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u, \quad y = \begin{bmatrix} \frac{b}{a} & \frac{-b}{a} \end{bmatrix} x.$$

Thus, we have found the discrete state model to be

$$\begin{bmatrix} x(k+1) \\ z(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & e^{aT} \end{bmatrix} \begin{bmatrix} x(k) \\ z(k) \end{bmatrix} + \begin{bmatrix} T \\ a(1-e^{-aT})^{-1} \end{bmatrix} u(k),$$

$$y(k) = \begin{bmatrix} \frac{b}{a} & \frac{-b}{a} \end{bmatrix} x(k),$$

where T is the sample period. For $a = 20$, $T = 0.1$, we use the following fuzzy rules for fuzzy-model-based control:

If $z(k)$ is about 0, then $u(k) = \begin{bmatrix} -10 & 0 \end{bmatrix} \begin{bmatrix} x(k) \\ z(k) \end{bmatrix}$;

If $z(k)$ is about 1, then $u(k) = \begin{bmatrix} -10 & 2 \end{bmatrix} \begin{bmatrix} x(k) \\ z(k) \end{bmatrix}$.

Thus, the T-S fuzzy models can be constructed as follows.

Rule 1:

If $z(k)$ is about zero, then

$$\begin{bmatrix} x(k+1) \\ z(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -0.5\varepsilon & 0.156\varepsilon \end{bmatrix} \begin{bmatrix} x(k) \\ z(k) \end{bmatrix}. \tag{15a}$$

Rule 2:

If $z(k)$ is not about zero, then

$$\begin{bmatrix} x(k+1) \\ z(k+1) \end{bmatrix} = \begin{bmatrix} 0 & -0.2 \\ -0.5\varepsilon & 0.056\varepsilon \end{bmatrix} \begin{bmatrix} x(k) \\ z(k) \end{bmatrix}, \tag{15b}$$

with $\varepsilon := 1 - e^{-aT} = 0.865$. By comparing (15) and (1), one has $n = 1$, $m = 1$, $p = q = 1$, and (A1) is evidently satisfied with

$$a_{0,1} = 0, \quad b_{0,1} = 0, \quad c_{0,1} = 0.5, \quad d_{0,1} = 0.156,$$
$$a_{0,2} = 0, \quad b_{0,2} = 0.2, \quad c_{0,2} = 0.5, \quad d_{0,2} = 0.056.$$

From (2), it results that

$$p_0 = 0.2, \quad q_0 = 0.656.$$

Thus, it can be verified that $\sum_{i=0}^{0} p_i = 0.2 < 1$. By (3), we obtain

$$g(x) = 1 - \sqrt{0.04 + 0.43x^2}.$$

The unique positive solution of $g(x) = 0$ is given by $\delta = 1.494$. Consequently, by Theorem 1, we conclude that the controlled T-S fuzzy singularly perturbed system (15) is globally exponentially stable. The state trajectories of the system (15) are depicted in Fig. 3. Simulation results reveal the effectiveness and accuracy of the main result.

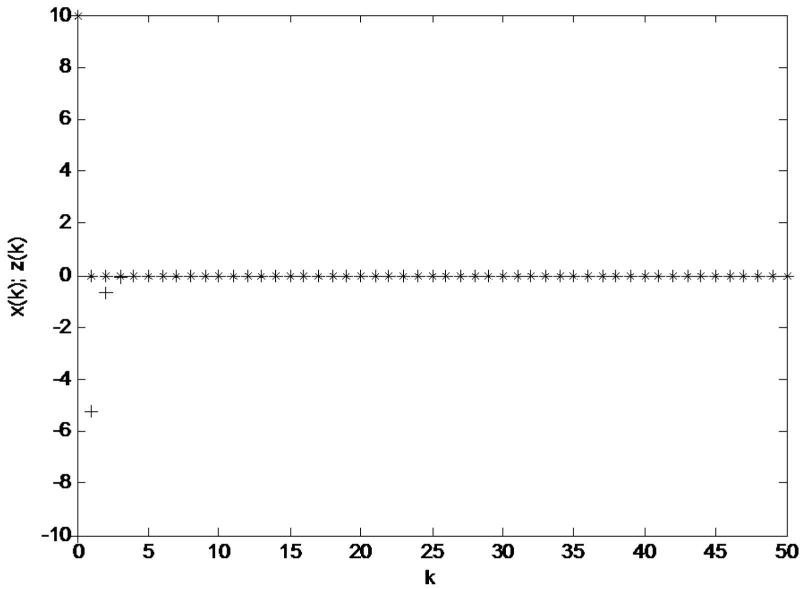

Figure 3. $x(k)$ and $z(k)$ of the system (15)

4. Conclusions

In this paper, the global exponential stability of a class of uncertain T-S fuzzy singularly perturbed systems has been investigated. Based on time-domain approach with difference inequality technique, a simple criterion has been derived to guarantee the global exponential stability of such systems. The upper bound of the singular perturbation parameter has also been provided by estimating the unique positive zero of the specific function. Finally, two numerical examples have been given to demonstrate the feasibility and effectiveness of the obtained result.

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References

Alwan, M.S. and Liu, X. (2009) Stability of singularly perturbed switched systems with time delay and impulsive effects. *Nonlinear Analysis: Theory, Methods & Applications*, **71**, 4297-4308.

Chen, C.W. (2009) The stability of an oceanic structure with T-S fuzzy models. *Mathematics and Computers in Simulation*, **80**, 402-426.

Chen, C.F., Pan, S.T. and Hsieh, J.G. (2002a) Stability of nominally stable uncertain singularly perturbed systems with multiple time delays. *Control and Cybernetics*, **31**, 5-16.

Chen, C.F., Pan, S.T. and Hsieh, J.G. (2002b) Stability analysis for a class of uncertain discrete singularly perturbed systems with multiple time delays. *ASME Journal of Dynamic Systems, Measurement, and Control*, **124**, 467-472.

Chung, H.Y. and Wu, S.M. (2009) Hybrid approaches for regional Takagi–Sugeno static output feedback fuzzy controller design. *Expert Systems with Applications*, **36**, 1720-1730.

Farzaneh, Y. and Tootoonchi, A.A. (2009) A novel data reduction method for Takagi–Sugeno fuzzy system design based on statistical design of experiment. *Applied Soft Computing*, **9**, 1367-1376.

Lagrat, I., Ouakka, H. and Boumhidi, I. (2008) Fuzzy adaptive control of a class of MISO nonlinear systems. *Control and Cybernetics*, **37**, 177-190.

Ledermann, W. and Vajda, S. (1980) *Handbook of Applicable Mathematics*. John Wiley & Sons, New York.

Lien, C.H. (2006) Further results on delay-dependent robust stability of uncertain fuzzy systems with time-varying delay. *Chaos, Solitons & Fractals*, **28**, 422-427.

Meng, B. and Jing, Y.W. (2009) Robust semiglobally practical stabilization for nonlinear singularly perturbed systems. *Nonlinear Analysis: Theory, Methods & Applications*, **70**, 2691-2699.

Prljaca, N. and Gajic Z. (2008) A method for optimal control and filtering of multitime-scale linear singularly-perturbed stochastic systems. *Automatica*, **44**, 2149-2156.

Rhee, B.J. and Won, S. (2006) A new fuzzy Lyapunov function approach for a Takagi-Sugeno fuzzy control system design. *Fuzzy Sets and Systems*, **157**, 1211-1228.

Sun, Y.J., Cheng, J.S., Hsieh, J.G. and Chen, C.C. (1996) Feedback control of a class of nonlinear singularly perturbed systems with time delay. *International Journal of System Science*, **27**, 589-596.

SUN, Y.J. (2009) On the C(R) stability of uncertain singularly perturbed systems. *Chaos, Solitons & Fractals*, **39**, 182-187.

TAKAGI, T. and SUGENO, M. (1985) Fuzzy identification of systems and its applications to modeling and control. *IEEE Transactions on Systems, Man, and Cybernetics*, **15**, 116-132.

TANAKA, K. and SUGENO, M. (1992) Stability analysis and design of fuzzy control systems. *Fuzzy Sets and Systems*, **45**, 135-156.

XU, S. and FENG, G. (2009) New results on H^∞ control of discrete singularly perturbed systems. *Automatica*, **45**, 2339-2343.

YU, F.M. (2009) The compact fuzzy filter design via Takagi-Sugeno fuzzy models. *Expert Systems with Applications*, **36**, 4412-4416.

ZHANG, B., LAM, J., XU, S. and SHU, Z. (2009) Robust stabilization of uncertain T-S fuzzy time-delay systems with exponential estimates. *Fuzzy Sets and Systems*, **160**, 1720-1737.