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A multiple attribute group decision making method based on generalized interval-valued trapezoidal fuzzy numbers*

by

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Abstract: A ranking approach based on grey correlative coefficient is presented to solve the multiple attribute decision making problems in which the attribute values and the weights take the form of the generalized interval-valued trapezoidal fuzzy number (GIVTFN). Firstly, the concept and the calculation rules of GIVTFN are introduced, the distance of GIVTFN is proposed. Secondly, the method of linguistic terms transformed into GIVTFN and the normalization method of GIVTFN is illustrated, and a grey relational decision making method based on the GIVTFN is presented in detail. The alternatives are ranked based on the grey correlative coefficient. Finally, an illustrative example is given to show the effectiveness of this method and the decision making steps.

Keywords: interval-valued fuzzy number, grey correlative coefficient, multiple attribute group decision making.

1. Introduction

Multiple attribute decision making (MADM) is an important part of modern decision science. It has been extensively applied to various areas such as society, economics, management, military and engineering technology. Examples include investment decision-making, project evaluation, economic evaluation, personnel evaluation etc. Since the objects of decisions are fuzzy and uncertain, and human thinking is ambiguous, the majority of multi-attribute decision-making must account for uncertainty and ambiguity, the respective approach being called fuzzy multiple attribute decision-making (FMADM). Since Bellman and Zadeh (1970) initially proposed the basic model of the fuzzy decision making based on the theory of fuzzy mathematics, many research achievements have been made on FMADM problems based on the various attribute values,

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such as interval numbers, triangular fuzzy numbers, and trapezoidal fuzzy numbers etc. Jahanshahloo et al. (2006), Wang and Elhag (2006), Zhu (2007), Liu and Zeng (2008), Liu (2009b) proposed some extended TOPSIS methods for different types of attribute values, such as, respectively, interval numbers, triangular fuzzy numbers, trapezoidal fuzzy numbers etc. Then, Men and Ji (2008), Wei and Wei (2008) proposed the grey relational analysis methods for different attribute values, such as interval numbers and triangular fuzzy numbers.

The concept of the interval-valued fuzzy set was proposed by Gorzalczany (1987) and Turksen (1996). Wang and Li (1998, 2001) defined the expansion operation of the interval-valued fuzzy numbers, and proposed the concept and properties of similarity coefficient based on the interval-valued fuzzy numbers. Hong and Lee (2002) proposed the distance of the interval-valued fuzzy numbers. Ashtiani et al. (2009) proposed an extended TOPSIS method for group decision making problems based on the interval-valued triangular fuzzy numbers. Wei and Chen (2009) proposed similarity measures between the generalized interval-valued trapezoidal fuzzy numbers (GIVTFN) for risk analysis. Liu (2011) proposed some weighted aggregation operators to solve the multi-attribute group decision-making problems based on GIVTFN.

This paper proposes a decision making method based on the grey correlative coefficient for solving the MADM problems in which the attribute weights and attribute values are given with the form of GIVTFN.

2. The basic concept of the interval-valued trapezoidal fuzzy numbers

2.1. The generalized trapezoidal fuzzy numbers

Definition 1 (Chen, 1985) *Generalized trapezoidal fuzzy number can be defined as a vector $\tilde{A} = (a_1, a_2, a_3, a_4; w_{\tilde{A}})$ (as shown in Fig. 1), with the membership function $a(x) : R \to [0, 1]$ defined as follows:*

$$a(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} \times w_{\tilde{A}}, & x \in (a_1, a_2) \\ w_{\tilde{A}}, & x \in (a_2, a_3) \\ \dfrac{x - a_4}{a_3 - a_4} \times w_{\tilde{A}}, & x \in (a_3, a_4) \\ 0, & x \in (-\infty, a_1) \cup (a_4, \infty) \end{cases} \tag{1}$$

where $a_1 \leq a_2 \leq a_3 \leq a_4$ and $w_{\tilde{A}} \in [0, 1]$.

The elements of a generalized trapezoidal fuzzy number $x \in R$ are real numbers, and its membership function $a(x)$ is a regular and continuous convex function, corresponding to the membership degree in the fuzzy sets. If $-1 \leq a_1 \leq a_2 \leq a_3 \leq a_4 \leq 1$, then $\tilde{A}$ is called normalized trapezoidal fuzzy number.

Especially, if $w_{\tilde{A}} = 1$, then $\tilde{A}$ is a trapezoidal fuzzy number (a_1, a_2, a_3, a_4); if $a_1 < a_2 = a_3 < a_4$, then $\tilde{A}$ is reduced to a triangular fuzzy number. If $a_1 = a_2 = a_3 = a_4$, then $\tilde{A}$ is reduced to a real number.

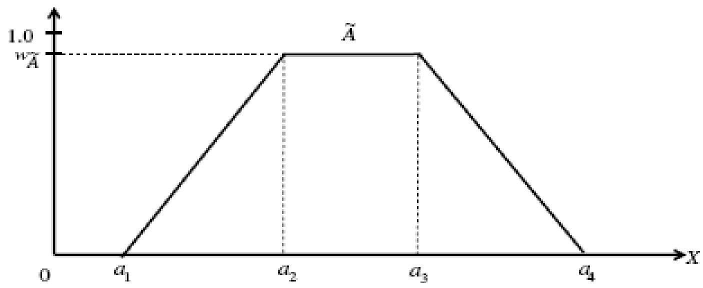

Figure 1. A generalized trapezoidal fuzzy number $\widetilde{A}$.

2.2. The interval-valued trapezoidal fuzzy numbers

(1) *The interval-valued trapezoidal fuzzy numbers* (Wei and Chen, 2009)

Wang and Li (2001) represented the interval-valued trapezoidal fuzzy number $\tilde{\tilde{A}} = \left[\tilde{\tilde{A}}^L, \tilde{\tilde{A}}^U\right] = \left[(a_1^L, a_2^L, a_3^L, a_4^L; w_{\tilde{\tilde{A}}^L}), (a_1^U, a_2^U, a_3^U, a_4^U; w_{\tilde{\tilde{A}}^U})\right]$, as shown in Fig. 2, where $0 \le a_1^L \le a_2^L \le a_3^L \le a_4^L \le 1$, $0 \le a_1^U \le a_2^U \le a_3^U \le a_4^U \le 1$, $0 \le w_{\tilde{\tilde{A}}^L} \le w_{\tilde{\tilde{A}}^U} \le 1$ and $\tilde{\tilde{A}}^L \subset \tilde{\tilde{A}}^U$.

From Fig. 2 we can conclude that interval-valued trapezoidal fuzzy number $\tilde{\tilde{A}}$ consists of the lower values of interval-valued trapezoidal fuzzy number $\tilde{\tilde{A}}^L$ and the upper values of interval-valued trapezoidal fuzzy number$\tilde{\tilde{A}}^U$.

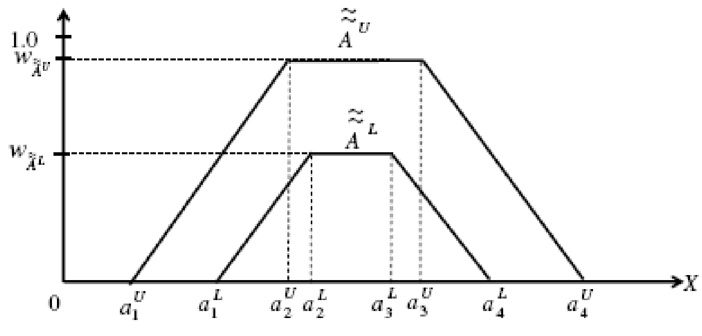

Figure 2. An interval-valued trapezoidal fuzzy number.

(2) *The operational rules of the interval-valued trapezoidal fuzzy numbers* (Wei and Chen, 2009)

Suppose that

$$\tilde{\tilde{A}} = \left[\tilde{\tilde{A}}^L, \tilde{\tilde{A}}^U\right] = \left[(a_1^L, a_2^L, a_3^L, a_4^L; w_{\tilde{\tilde{A}}^L}), (a_1^U, a_2^U, a_3^U, a_4^U; w_{\tilde{\tilde{A}}^U})\right],$$
$$\tilde{\tilde{B}} = \left[\tilde{\tilde{B}}^L, \tilde{\tilde{B}}^U\right] = \left[(b_1^L, b_2^L, b_3^L, b_4^L; w_{\tilde{\tilde{B}}^L}), (b_1^U, b_2^U, b_3^U, b_4^U; w_{\tilde{\tilde{B}}^U})\right]$$

are two interval-valued trapezoidal fuzzy numbers, where $0 \le a_1^L \le a_2^L \le a_3^L \le a_4^L \le 1$, $0 \le a_1^U \le a_2^U \le a_3^U \le a_4^U \le 1$, $0 \le w_{\tilde{\tilde{A}}^L} \le w_{\tilde{\tilde{A}}^U} \le 1$, $\tilde{\tilde{A}}^L \subset \tilde{\tilde{A}}^U$, $0 \le b_1^L \le b_2^L \le b_3^L \le b_4^L \le 1$, $0 \le b_1^U \le b_2^U \le b_3^U \le b_4^U \le 1$, $0 \le w_{\tilde{\tilde{B}}^L} \le w_{\tilde{\tilde{B}}^U} \le 1$, $w_{\tilde{\tilde{B}}^L} \subset w_{\tilde{\tilde{B}}^U}$. The operational rules are defined as follows:

(i) Interval-Valued Fuzzy Numbers Addition$\tilde{\tilde{A}} \oplus \tilde{\tilde{B}}$:

$$\begin{aligned}\tilde{\tilde{A}} \oplus \tilde{\tilde{B}} &= \left[(a_1^L, a_2^L, a_3^L, a_4^L; w_{\tilde{\tilde{A}}^L}), (a_1^U, a_2^U, a_3^U, a_4^U; w_{\tilde{\tilde{A}}^U})\right] \\ &\quad \oplus \left[(b_1^L, b_2^L, b_3^L, b_4^L; w_{\tilde{\tilde{B}}^L}), (b_1^U, b_2^U, b_3^U, b_4^U; w_{\tilde{\tilde{B}}^U})\right] \\ &= \Big[(a_1^L + b_1^L, a_2^L + b_2^L, a_3^L + b_3^L, a_4^L + b_4^L; \min(w_{\tilde{\tilde{A}}^L}, w_{\tilde{\tilde{B}}^L})), \\ &\quad (a_1^U + b_1^U, a_2^U + b_2^U, a_3^U + b_3^U, a_4^U + b_4^U; \min(w_{\tilde{\tilde{A}}^U}, w_{\tilde{\tilde{B}}^U}))\Big]\end{aligned} \tag{2}$$

(ii) Interval-Valued Fuzzy Numbers Subtraction $\tilde{\tilde{A}} - \tilde{\tilde{B}}$

$$\begin{aligned}\tilde{\tilde{A}} - \tilde{\tilde{B}} &= \left[(a_1^L, a_2^L, a_3^L, a_4^L; w_{\tilde{\tilde{A}}^L}), (a_1^U, a_2^U, a_3^U, a_4^U; w_{\tilde{\tilde{A}}^U})\right] \\ &\quad - \left[(b_1^L, b_2^L, b_3^L, b_4^L; w_{\tilde{\tilde{B}}^L}), (b_1^U, b_2^U, b_3^U, b_4^U; w_{\tilde{\tilde{B}}^U})\right] \\ &= \Big[(a_1^L - b_4^L, a_2^L - b_3^L, a_3^L - b_2^L, a_4^L - b_1^L; \min(w_{\tilde{\tilde{A}}^L}, w_{\tilde{\tilde{B}}^L})), \\ &\quad (a_1^U - b_4^U, a_2^U - b_3^U, a_3^U - b_2^U, a_4^U - b_1^U; \min(w_{\tilde{\tilde{A}}^U}, w_{\tilde{\tilde{B}}^U}))\Big]\end{aligned} \tag{3}$$

(iii) Interval-Valued Fuzzy Numbers Multiplication $\tilde{\tilde{A}} \otimes \tilde{\tilde{B}}$

$$\begin{aligned}\tilde{\tilde{A}} \otimes \tilde{\tilde{B}} &= \left[(a_1^L, a_2^L, a_3^L, a_4^L; w_{\tilde{\tilde{A}}^L}), (a_1^U, a_2^U, a_3^U, a_4^U; w_{\tilde{\tilde{A}}^U})\right] \\ &\quad \otimes \left[(b_1^L, b_2^L, b_3^L, b_4^L; w_{\tilde{\tilde{B}}^L}), (b_1^U, b_2^U, b_3^U, b_4^U; w_{\tilde{\tilde{B}}^U})\right] \\ &= \Big[(a_1^L \times b_1^L, a_2^L \times b_2^L, a_3^L \times b_3^L, a_4^L \times b_4^L; \min(w_{\tilde{\tilde{A}}^L}, w_{\tilde{\tilde{B}}^L})), \\ &\quad (a_1^U \times b_1^U, a_2^U \times b_2^U, a_3^U \times b_3^U, a_4^U \times b_4^U; \min(w_{\tilde{\tilde{A}}^U}, w_{\tilde{\tilde{B}}^U}))\Big]\end{aligned} \tag{4}$$

(iv) Generalized Fuzzy Numbers Division $\tilde{\tilde{A}}/\tilde{\tilde{B}}$:

$$\begin{aligned}\tilde{\tilde{A}}/\tilde{\tilde{B}} &= \Big[(a_1^L, a_2^L, a_3^L, a_4^L; w_{\tilde{\tilde{A}}^L}), (a_1^U, a_2^U, a_3^U, a_4^U; w_{\tilde{\tilde{A}}^U})\Big] / \\ &\quad \Big[(b_1^L, b_2^L, b_3^L, b_4^L; w_{\tilde{\tilde{B}}^L}), (b_1^U, b_2^U, b_3^U, b_4^U; w_{\tilde{\tilde{B}}^U})\Big] \\ &= \Big[(a_1^L/b_4^L, a_2^L/b_3^L, a_3^L/b_2^L, a_4^L/b_1^L; \min(w_{\tilde{\tilde{A}}^L}, w_{\tilde{\tilde{B}}^L})), \\ &\quad (a_1^U/b_4^U, a_2^U/b_3^U, a_3^U/b_2^U, a_4^U/b_1^U; \min(w_{\tilde{\tilde{A}}^U}, w_{\tilde{\tilde{B}}^U}))\Big] \qquad (5)\end{aligned}$$

(v) The multiplication for an interval-valued trapezoidal fuzzy number $\tilde{\tilde{A}}$ and a constant λ:

$$\begin{aligned}\lambda\tilde{\tilde{A}} &= \lambda \times \Big[(a_1^L, a_2^L, a_3^L, a_4^L; w_{\tilde{\tilde{A}}^L}), (a_1^U, a_2^U, a_3^U, a_4^U; w_{\tilde{\tilde{A}}^U})\Big] \\ &= \Big[(\lambda a_1^L, \lambda a_2^L, \lambda a_3^L, \lambda a_4^L; w_{\tilde{\tilde{A}}^L}), (\lambda a_1^U, \lambda a_2^U, \lambda a_3^U, \lambda a_4^U; w_{\tilde{\tilde{A}}^U})\Big], \quad \lambda > 0 \qquad (6)\end{aligned}$$

For example, suppose

$$\tilde{\tilde{A}} = [(0.2, 0.3, 0.7, 0.8; 0.8), (0, 0.2, 0.8, 0.9; 1.0)]$$

and

$$\tilde{\tilde{B}} = [(0.2, 0.3, 0.4, 0.5; 0.6), (0.1, 0.2, 0.5, 0.7; 0.8)]$$

are two interval-valued trapezoidal fuzzy numbers, then the Addition, Subtraction, Multiplication and Division of two generalized fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ are performed as follows.

$$\begin{aligned}\tilde{\tilde{A}} \oplus \tilde{\tilde{B}} &= [(0.2, 0.3, 0.7, 0.8; 0.8), (0, 0.2, 0.8, 0.9; 1.0)] \\ &\quad \oplus [(0.2, 0.3, 0.4, 0.5; 0.6), (0.1, 0.2, 0.5, 0.7; 0.8)] \\ &= [(0.4, 0.6, 1.1, 1.3; 0.6), (0.1, 0.4, 1.3, 1.6; 0.8]\end{aligned}$$

$$\begin{aligned}\tilde{\tilde{A}} - \tilde{\tilde{B}} &= [(0.2, 0.3, 0.7, 0.8; 0.8), (0, 0.2, 0.8, 0.9; 1.0)] \\ &\quad - [(0.2, 0.3, 0.4, 0.5; 0.6), (0.1, 0.2, 0.5, 0.7; 0.8)] \\ &= [(-0.3, -0.1, 0.4, 0.6; 0.6), (-0.7, -0.3, 0.6, 0.8; 0.8]\end{aligned}$$

$$\begin{aligned}\tilde{\tilde{A}} \otimes \tilde{\tilde{B}} &= [(0.2, 0.3, 0.7, 0.8; 0.8), (0, 0.2, 0.8, 0.9; 1.0)] \\ &\quad \otimes [(0.2, 0.3, 0.4, 0.5; 0.6), (0.1, 0.2, 0.5, 0.7; 0.8)] \\ &= [(0.04, 0.09, 0.28, 0.4; 0.6), (0, 0.04, 0.4, 0.63; 0.8]\end{aligned}$$

$$\begin{aligned}\tilde{\tilde{A}}/\tilde{\tilde{B}} &= [(0.2, 0.3, 0.7, 0.8; 0.8), (0, 0.2, 0.8, 0.9; 1.0)]/ \\ &\quad [(0.2, 0.3, 0.4, 0.5; 0.6), (0.1, 0.2, 0.5, 0.7; 0.8)] \\ &= [(0.4, 0.74, 2.33, 4; 0.6), (0, 0.4, 4, 9; 0.8]\end{aligned}$$

$$\begin{aligned}0.5 \times \tilde{\tilde{A}} &= 0.5 \times [[(0.2, 0.3, 0.7, 0.8; 0.8), (0, 0.2, 0.8, 0.9; 1.0)]] \\ &= [(0.1, 0.15, 0.35, 0.4; 0.8), (0, 0.1, 0.4, 0.45; 1.0)]\end{aligned}$$

The interval-valued trapezoidal fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$, and the results of their Addition, Subtraction, Multiplication and Division are shown in the following Figs. 3 through 9.

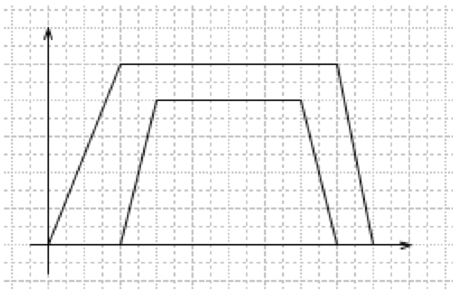

Figure 3. The interval-valued trapezoidal fuzzy number $\tilde{\tilde{A}}$.

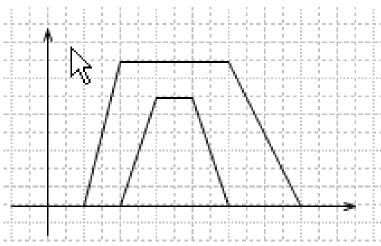

Figure 4. The interval-valued trapezoidal fuzzy number $\tilde{\tilde{B}}$.

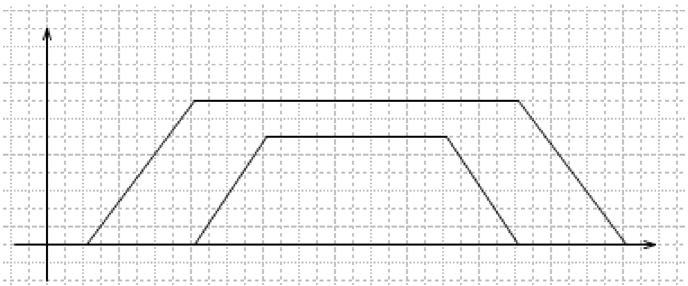

Figure 5. Addition of two interval-valued trapezoidal fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$.

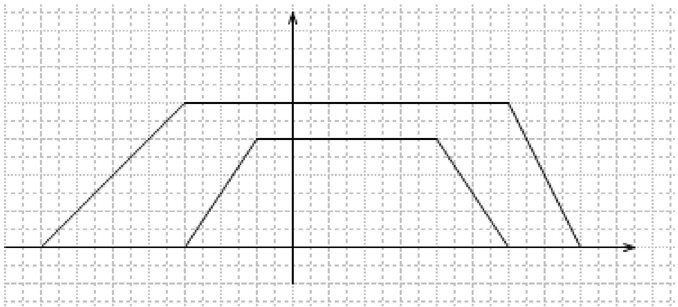

Figure 6. Subtraction of two interval-valued trapezoidal fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$.

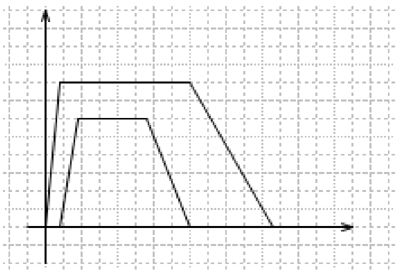

Figure 7. Multiplication of two interval-valued trapezoidal fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$.

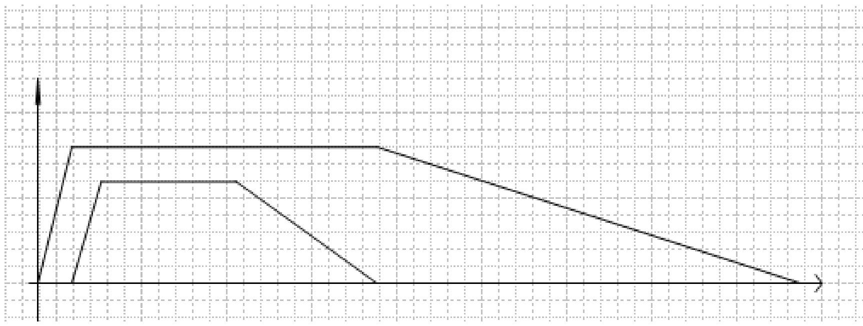

Figure 8. Division of two interval-valued trapezoidal fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ (Note: reduced 4 times in the abscissa).

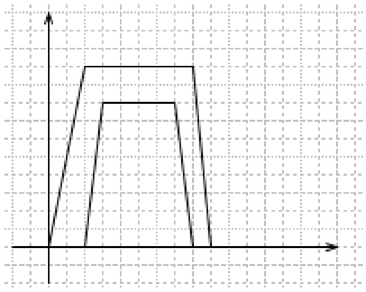

Figure 9. Multiplication of the interval-valued trapezoidal fuzzy number $\tilde{\tilde{A}}$ and constant λ ($\lambda = 0.5$).

2.3. The distance of interval-valued trapezoidal fuzzy numbers

Suppose that

$$\begin{aligned}
\tilde{\tilde{A}} &= \left[\tilde{\tilde{A}}^L, \tilde{\tilde{A}}^U\right] = \left[(a_1^L, a_2^L, a_3^L, a_4^L; w_{\tilde{\tilde{A}}^L}), (a_1^U, a_2^U, a_3^U, a_4^U; w_{\tilde{\tilde{A}}^U})\right],\\
\tilde{\tilde{B}} &= \left[\tilde{\tilde{B}}^L, \tilde{\tilde{B}}^U\right] = \left[(b_1^L, b_2^L, b_3^L, b_4^L; w_{\tilde{\tilde{B}}^L}), (b_1^U, b_2^U, b_3^U, b_4^U; w_{\tilde{\tilde{B}}^U})\right],\\
\tilde{\tilde{C}} &= \left[\tilde{\tilde{C}}^L, \tilde{\tilde{C}}^U\right] = \left[(c_1^L, c_2^L, c_3^L, c_4^L; w_{\tilde{\tilde{C}}^L}), (c_1^U, c_2^U, c_3^U, c_4^U; w_{\tilde{\tilde{C}}^U})\right]
\end{aligned}$$

are three generalized trapezoidal fuzzy numbers, then the distance of two interval-valued trapezoidal fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ is calculated as follows:

$$\begin{aligned}
d(\tilde{\tilde{A}}, \tilde{\tilde{B}}) =& \frac{1}{8} \times \Big(\left|w_{\tilde{\tilde{A}}^L} \times a_1^L - w_{\tilde{\tilde{B}}^L} \times b_1^L\right| + \left|w_{\tilde{\tilde{A}}^L} \times a_2^L - w_{\tilde{\tilde{B}}^L} \times b_2^L\right|\\
&+ \left|w_{\tilde{\tilde{A}}^L} \times a_3^L - w_{\tilde{\tilde{B}}^L} \times b_3^L\right| + \left|w_{\tilde{\tilde{A}}^L} \times a_4^L - w_{\tilde{\tilde{B}}^L} \times b_4^L\right|\\
&+ \left|w_{\tilde{\tilde{A}}^U} \times a_1^U - w_{\tilde{\tilde{B}}^U} \times b_1^U\right| + \left|w_{\tilde{\tilde{A}}^U} \times a_2^U - w_{\tilde{\tilde{B}}^U} \times b_2^U\right|\\
&+ \left|w_{\tilde{\tilde{A}}^U} \times a_3^U - w_{\tilde{\tilde{B}}^U} \times b_3^U\right| + \left|w_{\tilde{\tilde{A}}^U} \times a_4^U - w_{\tilde{\tilde{B}}^U} \times b_4^U\right|\Big) \qquad (7)
\end{aligned}$$

with $d(\tilde{\tilde{A}}, \tilde{\tilde{B}})$ satisfying the following properties:

(i) if $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ are the normalized interval-valued trapezoidal fuzzy numbers, then $0 \le d(\tilde{\tilde{A}}, \tilde{\tilde{B}}) \le 1$;

(ii) $\tilde{\tilde{A}} = \tilde{\tilde{B}} \Leftrightarrow d(\tilde{\tilde{A}}, \tilde{\tilde{B}}) = 0$;

(iii) $d(\tilde{\tilde{A}}, \tilde{\tilde{B}}) = d(\tilde{\tilde{B}}, \tilde{\tilde{A}})$;

(iv) $d(\tilde{\tilde{A}}, \tilde{\tilde{C}}) + d(\tilde{\tilde{C}}, \tilde{\tilde{B}}) \ge d(\tilde{\tilde{A}}, \tilde{\tilde{B}})$.

The properties (i), (ii) and (iii) are obviously satisfied.

For (iv):

$$d(\tilde{\tilde{A}},\tilde{\tilde{C}})+d(\tilde{\tilde{C}},\tilde{\tilde{B}})=\frac{1}{8}\times\Big(\left|w_{\tilde{\tilde{A}}^L}\times a_1^L-w_{\tilde{\tilde{C}}^L}\times c_1^L\right|+\left|w_{\tilde{\tilde{A}}^L}\times a_2^L-w_{\tilde{\tilde{C}}^L}\times c_2^L\right|$$
$$+\left|w_{\tilde{\tilde{A}}^L}\times a_3^L-w_{\tilde{\tilde{C}}^L}\times c_3^L\right|+\left|w_{\tilde{\tilde{A}}^L}\times a_4^L-w_{\tilde{\tilde{C}}^L}\times c_4^L\right|+\left|w_{\tilde{\tilde{A}}^U}\times a_1^U-w_{\tilde{\tilde{C}}^U}\times c_1^U\right|$$
$$+\left|w_{\tilde{\tilde{A}}^U}\times a_2^U-w_{\tilde{\tilde{C}}^U}\times c_2^U\right|+\left|w_{\tilde{\tilde{A}}^U}\times a_3^U-w_{\tilde{\tilde{C}}^U}\times c_3^U\right|+\left|w_{\tilde{\tilde{A}}^U}\times a_4^U-w_{\tilde{\tilde{C}}^U}\times c_4^U\right|\Big)$$
$$+\frac{1}{8}\times\Big(\left|w_{\tilde{\tilde{B}}^L}\times b_1^L-w_{\tilde{\tilde{C}}^L}\times c_1^L\right|+\left|w_{\tilde{\tilde{B}}^L}\times b_2^L-w_{\tilde{\tilde{C}}^L}\times c_2^L\right|$$
$$+\left|w_{\tilde{\tilde{B}}^L}\times b_3^L-w_{\tilde{\tilde{C}}^L}\times c_3^L\right|+\left|w_{\tilde{\tilde{B}}^L}\times b_4^L-w_{\tilde{\tilde{C}}^L}\times c_4^L\right|+\left|w_{\tilde{\tilde{B}}^U}\times b_1^U-w_{\tilde{\tilde{C}}^U}\times c_1^U\right|$$
$$+\left|w_{\tilde{\tilde{B}}^U}\times b_2^U-w_{\tilde{\tilde{C}}^U}\times c_2^U\right|+\left|w_{\tilde{\tilde{B}}^U}\times b_3^U-w_{\tilde{\tilde{C}}^U}\times c_3^U\right|+\left|w_{\tilde{\tilde{B}}^U}\times b_4^U-w_{\tilde{\tilde{C}}^U}\times c_4^U\right|\Big)$$

$$=\frac{1}{8}\times\Big(\left|w_{\tilde{\tilde{A}}^L}\times a_1^L-w_{\tilde{\tilde{C}}^L}\times c_1^L\right|+\left|w_{\tilde{\tilde{B}}^L}\times b_1^L-w_{\tilde{\tilde{C}}^L}\times c_1^L\right|+\left|w_{\tilde{\tilde{A}}^L}\times a_2^L-w_{\tilde{\tilde{C}}^L}\times c_2^L\right|$$
$$+\left|w_{\tilde{\tilde{B}}^L}\times b_2^L-w_{\tilde{\tilde{C}}^L}\times c_2^L\right|+\left|w_{\tilde{\tilde{A}}^L}\times a_3^L-w_{\tilde{\tilde{C}}^L}\times c_3^L\right|+\left|w_{\tilde{\tilde{B}}^L}\times b_3^L-w_{\tilde{\tilde{C}}^L}\times c_3^L\right|$$
$$+\left|w_{\tilde{\tilde{A}}^L}\times a_4^L-w_{\tilde{\tilde{C}}^L}\times c_4^L\right|+\left|w_{\tilde{\tilde{B}}^L}\times b_4^L-w_{\tilde{\tilde{C}}^L}\times c_4^L\right|+\left|w_{\tilde{\tilde{A}}^U}\times a_1^U-w_{\tilde{\tilde{C}}^U}\times c_1^U\right|$$
$$+\left|w_{\tilde{\tilde{B}}^U}\times b_1^U-w_{\tilde{\tilde{C}}^U}\times c_1^U\right|+\left|w_{\tilde{\tilde{A}}^U}\times a_2^U-w_{\tilde{\tilde{C}}^U}\times c_2^U\right|+\left|w_{\tilde{\tilde{B}}^U}\times b_2^U-w_{\tilde{\tilde{C}}^U}\times c_2^U\right|$$
$$+\left|w_{\tilde{\tilde{A}}^U}\times a_3^U-w_{\tilde{\tilde{C}}^U}\times c_3^U\right|+\left|w_{\tilde{\tilde{B}}^U}\times b_3^U-w_{\tilde{\tilde{C}}^U}\times c_3^U\right|+\left|w_{\tilde{\tilde{A}}^U}\times a_4^U-w_{\tilde{\tilde{C}}^U}\times c_4^U\right|$$
$$+\left|w_{\tilde{\tilde{B}}^U}\times b_4^U-w_{\tilde{\tilde{C}}^U}\times c_4^U\right|\Big)$$

$$\geq\frac{1}{8}\times\Big(\left|w_{\tilde{\tilde{A}}^L}\times a_1^L-c_1^L\right|+\left|w_{\tilde{\tilde{B}}^L}\times b_1^L-c_1^L\right|+\left|w_{\tilde{\tilde{A}}^L}\times a_2^L-c_2^L\right|+\left|w_{\tilde{\tilde{B}}^L}\times b_2^L-c_2^L\right|$$
$$+\left|w_{\tilde{\tilde{A}}^L}\times a_3^L-c_3^L\right|+\left|w_{\tilde{\tilde{B}}^L}\times b_3^L-c_3^L\right|+\left|w_{\tilde{\tilde{A}}^L}\times a_4^L-c_4^L\right|+\left|w_{\tilde{\tilde{B}}^L}\times b_4^L-c_4^L\right|$$
$$+\left|w_{\tilde{\tilde{A}}^U}\times a_1^U-c_1^U\right|+\left|w_{\tilde{\tilde{B}}^U}\times b_1^U-c_1^U\right|+\left|w_{\tilde{\tilde{A}}^U}\times a_2^U-c_2^U\right|+\left|w_{\tilde{\tilde{B}}^U}\times b_2^U-c_2^U\right|$$
$$+\left|w_{\tilde{\tilde{A}}^U}\times a_3^U-c_3^U\right|+\left|w_{\tilde{\tilde{B}}^U}\times b_3^U-c_3^U\right|+\left|w_{\tilde{\tilde{A}}^U}\times a_4^U-c_4^U\right|+\left|w_{\tilde{\tilde{B}}^U}\times b_4^U-c_4^U\right|\Big)$$

$$\geq\frac{1}{8}\times\Big(\left|\left(w_{\tilde{\tilde{A}}^L}\times a_1^L-c_1^L\right)-\left(w_{\tilde{\tilde{B}}^L}\times b_1^L-c_1^L\right)\right|+\left|\left(w_{\tilde{\tilde{A}}^L}\times a_2^L-c_2^L\right)-\left(w_{\tilde{\tilde{B}}^L}\times b_2^L-c_2^L\right)\right|$$
$$+\left|\left(w_{\tilde{\tilde{A}}^L}\times a_3^L-c_3^L\right)-\left(w_{\tilde{\tilde{B}}^L}\times b_3^L-c_3^L\right)\right|+\left|\left(w_{\tilde{\tilde{A}}^L}\times a_4^L-c_4^L\right)-\left(w_{\tilde{\tilde{B}}^L}\times b_4^L-c_4^L\right)\right|$$
$$+\left|\left(w_{\tilde{\tilde{A}}^U}\times a_1^U-c_1^U\right)-\left(w_{\tilde{\tilde{B}}^U}\times b_1^U-c_1^U\right)\right|+\left|\left(w_{\tilde{\tilde{A}}^U}\times a_2^U-c_2^U\right)-\left(w_{\tilde{\tilde{B}}^U}\times b_2^U-c_2^U\right)\right|$$
$$+\left|\left(w_{\tilde{\tilde{A}}^U}\times a_3^U-c_3^U\right)-\left(w_{\tilde{\tilde{B}}^U}\times b_3^U-c_3^U\right)\right|+\left|\left(w_{\tilde{\tilde{A}}^U}\times a_4^U-c_4^U\right)-\left(w_{\tilde{\tilde{B}}^U}\times b_4^U-c_4^U\right)\right|\Big)$$

$$\because\frac{1}{8}\times\Big(\left|\left(w_{\tilde{\tilde{A}}^L}\times a_1^L-c_1^L\right)-\left(w_{\tilde{\tilde{B}}^L}\times b_1^L-c_1^L\right)\right|+\left|\left(w_{\tilde{\tilde{A}}^L}\times a_2^L-c_2^L\right)-\left(w_{\tilde{\tilde{B}}^L}\times b_2^L-c_2^L\right)\right|$$
$$+\left|\left(w_{\tilde{\tilde{A}}^L}\times a_3^L-c_3^L\right)-\left(w_{\tilde{\tilde{B}}^L}\times b_3^L-c_3^L\right)\right|+\left|\left(w_{\tilde{\tilde{A}}^L}\times a_4^L-c_4^L\right)-\left(w_{\tilde{\tilde{B}}^L}\times b_4^L-c_4^L\right)\right|$$
$$+\left|\left(w_{\tilde{\tilde{A}}^U}\times a_1^U-c_1^U\right)-\left(w_{\tilde{\tilde{B}}^U}\times b_1^U-c_1^U\right)\right|+\left|\left(w_{\tilde{\tilde{A}}^U}\times a_2^U-c_2^U\right)-\left(w_{\tilde{\tilde{B}}^U}\times b_2^U-c_2^U\right)\right|$$
$$+\left|\left(w_{\tilde{\tilde{A}}^U}\times a_3^U-c_3^U\right)-\left(w_{\tilde{\tilde{B}}^U}\times b_3^U-c_3^U\right)\right|+\left|\left(w_{\tilde{\tilde{A}}^U}\times a_4^U-c_4^U\right)-\left(w_{\tilde{\tilde{B}}^U}\times b_4^U-c_4^U\right)\right|\Big)$$

$$=d(\tilde{\tilde{A}},\tilde{\tilde{B}})$$

$$\therefore d(\tilde{\tilde{A}},\tilde{\tilde{C}})+d(\tilde{\tilde{C}},\tilde{\tilde{B}})\geq d(\tilde{\tilde{A}},\tilde{\tilde{B}}).$$

2.4. Using interval-valued trapezoidal fuzzy numbers to represent linguistic terms

In the real decision making, it is difficult to get the generalized interval-valued trapezoidal fuzzy numbers for the attribute values and weights directly by the decision makers, however, linguistic variables can easily be used to express fuzzy information. In the traditional multi-attribute decision making, linguistic variables are usually converted into the interval numbers, triangular fuzzy numbers, and trapezoidal fuzzy numbers (Liu, 2009a, Liu and Zhang, 2010). However, the interval-valued trapezoidal fuzzy numbers can more precisely express fuzzy information than the interval numbers, triangular fuzzy numbers, and trapezoidal fuzzy numbers. So, it becomes necessary to convert the linguistic variables into the interval-valued trapezoidal fuzzy numbers. Wei and Chen (2009) utilized the interval-valued trapezoidal fuzzy numbers to represent the nine-member linguistic terms (see Table 1 and Fig. 10).

Table 1. A nine-member linguistic term set

linguistic terms (the attribute values)	linguistic terms (weights)	generalized interval-valued trapezoidal fuzzy numbers
absolutely-poor (AP)	absolutely-low(AL)	[(0.00,0.00,0.00,0.00;0.8), (0.00,0.00,0.00,0.00;1.0)]
very-poor (VP)	very-low (VL)	[(0.00,0.00,0.02,0.07;0.8), (0.00,0.00,0.02,0.07;1.0)]
poor (P)	low (L)	[(0.04,0.10,0.18,0.23;0.8), (0.04,0.10,0.18,0.23;1.0)]
medium-poor (MP)	medium-low(ML)	[(0.17,0.22,0.36,0.42;0.8), (0.17,0.22,0.36,0.42;1.0)]
medium (F)	medium (M)	[(0.32,0.41,0.58,0.65;0.8), (0.32,0.41,0.58,0.65;1.0)]
newline medium-good (MG)	medium-high(MH)	[(0.58,0.63,0.80,0.86;0.8), (0.58,0.63,0.80,0.86;1.0)]
good (G)	high(H)	[(0.72,0.78,0.92,0.97;0.8), (0.72,0.78,0.92,0.97;1.0)]
very-good (VG)	very-high (VH)	[(0.93,0.98,1.00,1.00;0.8), (0.93,0.98,1.00,1.00;1.0)]
absolutely-good (AG)	absolutely-high (AH)	[(1.00,1.00,1.00,1.00;0.8), (1.00,1.00,1.00,1.00;1.0)]

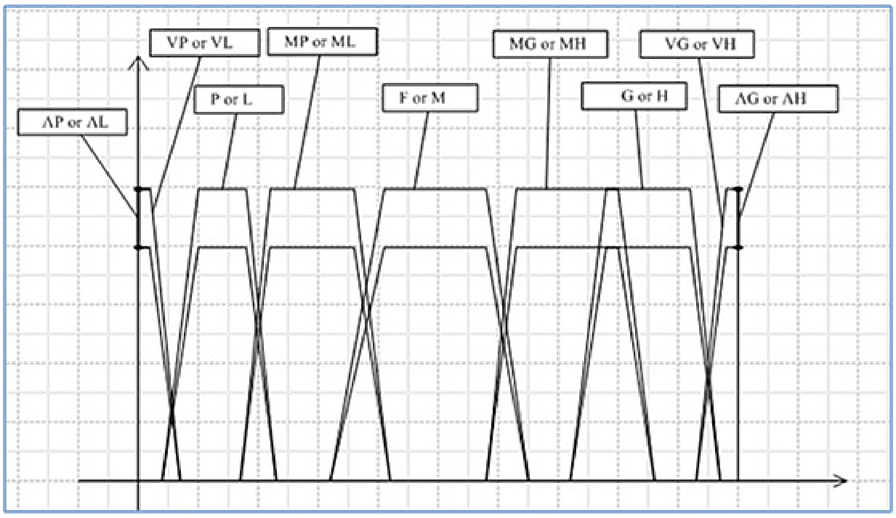

Figure 10. Interval-valued trapezoidal fuzzy numbers of the nine-member linguistic term set.

3. Group decision making method

3.1. Description the decision making problems

Let $E = \{e_1, e_2, \cdots, e_q\}$ be the set of decision makers in the group decision making, $A = \{A_1, A_2, \cdots, A_m\}$ be the set of alternatives, and $C = \{C_1, C_2, \cdots, C_n\}$ be the set of attributes. Suppose that

$$\tilde{\tilde{a}}_{ijk} = \left[(a^L_{ijk1}, a^L_{ijk2}, a^L_{ijk3}, a^L_{ijk4}; w^L_{ijk}), (a^U_{ijk1}, a^U_{ijk2}, a^U_{ijk3}, a^U_{ijk4}; w^U_{ijk})\right]$$

is an attribute value given by the decision maker e_k, where $\tilde{\tilde{a}}_{ijk}$ is an interval-valued trapezoidal fuzzy number for the alternative A_i with respect to the attribute C_j, $\tilde{\tilde{\omega}}_{kj} = \left[(\omega^L_{kj1}, \omega^L_{kj2}, \omega^L_{kj3}, \omega^L_{kj4}; \eta^L_{kj}), (\omega^U_{kj1}, \omega^U_{kj2}, \omega^U_{kj3}, \omega^U_{kj4}; \eta^U_{kj})\right]$ is an attribute weight given by the decision maker e_k, where $\tilde{\tilde{\omega}}_{kj}$ is also an interval-valued trapezoidal fuzzy number. Let $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_q)$ be the vector of decision makers, where λ_k is a real number, and $\sum_{k=1}^{q} \lambda_k = 1$. Then we use the attribute weights, the decision makers' weights, and the attribute values to rank the alternatives.

3.2. Normalizing the decision-making information

In order to eliminate the impact of different physical dimensions on the decision-making result, we need to normalize the decision-making information. Consider that there are generally benefit attributes (I_1) and cost attributes (I_2). The normalizing methods used are as follows:

(1) For benefit attributes

$$\begin{aligned}\tilde{\tilde{x}}_{ijk} &= \left[(x^L_{ijk1}, x^L_{ijk2}, x^L_{ijk3}, x^L_{ijk4}; w^L_{ijk}), (x^U_{ijk1}, x^U_{ijk2}, x^U_{ijk3}, x^U_{ijk4}; w^U_{ijk})\right] \\ &= \left[\left((\frac{a^L_{ijk1}}{m_{jk}}, \frac{a^L_{ijk2}}{m_{jk}}, \frac{a^L_{ijk3}}{m_{jk}}, \frac{a^L_{ijk4}}{m_{jk}}; w^L_{ijk}), (\frac{a^U_{ijk1}}{m_{jk}}, \frac{a^U_{ijk2}}{m_{jk}}, \frac{a^U_{ijk3}}{m_{jk}}, \frac{a^U_{ijk4}}{m_{jk}}; w^U_{ijk}) \right)\right] \quad (8)\end{aligned}$$

where $m_{jk} = \underset{i}{max}(a^U_{ijk4})$.

(2) For cost attributes

$$\begin{aligned}\tilde{\tilde{x}}_{ijk} &= \left[(x^L_{ijk1}, x^L_{ijk2}, x^L_{ijk3}, x^L_{ijk4}; w^L_{ijk}), (x^U_{ijk1}, x^U_{ijk2}, x^U_{ijk3}, x^U_{ijk4}; w^U_{ijk})\right] \\ &= \left[\left((\frac{n_{jk}}{a^L_{ijk1}}, \frac{n_{jk}}{a^L_{ijk2}}, \frac{n_{jk}}{a^L_{ijk3}}, \frac{n_{jk}}{a^L_{ijk4}}; w^L_{ijk}), (\frac{n_{jk}}{a^U_{ijk1}}, \frac{n_{jk}}{a^U_{ijk2}}, \frac{n_{jk}}{a^U_{ijk3}}, \frac{n_{jk}}{a^U_{ijk4}}; w^U_{ijk}) \right)\right] \quad (9)\end{aligned}$$

where $n_{jk} = \min_i(a^L_{ijk1})$.

3.3. Combining the evaluation information of each decision maker

According to the different project attribute values and weights, which were given by different experts for particular attributes, we can the aggregate them to obtain the collective attribute values and weights.

The combining steps are shown as follows:

$$\begin{aligned}&\tilde{\tilde{x}}_{ij} = \left[(x^L_{ij1}, x^L_{ij2}, x^L_{ij3}, x^L_{ij4}; w^L_{ij}), (x^U_{ij1}, x^U_{ij2}, x^U_{ij3}, x^U_{ij4}; w^U_{ij})\right] \\ &= \sum_{k=1}^{q} \left(\lambda_k \tilde{\tilde{x}}_{ijk}\right) \\ &= \sum_{k=1}^{q} \left\{\lambda_k \times \left[(x^L_{ijk1}, x^L_{ijk2}, x^L_{ijk3}, x^L_{ijk4}; w^L_{ijk}), (x^U_{ijk1}, x^U_{ijk2}, x^U_{ijk3}, x^U_{ijk4}; w^U_{ijk})\right]\right\} \\ &= \Big[\Big(\sum_{k=1}^{q}\left(\lambda_k x^L_{ijk1}\right), \sum_{k=1}^{q}\left(\lambda_k x^L_{ijk2}\right), \sum_{k=1}^{q}\left(\lambda_k x^L_{ijk3}\right), \sum_{k=1}^{q}\left(\lambda_k x^L_{ijk4}\right); \min_k(w^L_{ijk})\Big), \\ &\quad \Big(\sum_{k=1}^{q}\left(\lambda_k x^U_{ijk1}\right), \sum_{k=1}^{q}\left(\lambda_k x^U_{ijk2}\right), \sum_{k=1}^{q}\left(\lambda_k x^U_{ijk3}\right), \sum_{k=1}^{q}\left(\lambda_k x^U_{ijk4}\right); \min_k(w^U_{ijk})\Big)\Big] \quad (10)\end{aligned}$$

$$\begin{aligned}&\tilde{\tilde{\omega}}_j = \left[(\omega^L_{j1}, \omega^L_{j2}, \omega^L_{j3}, \omega^L_{j4}; \eta^L_j), (\omega^U_{j1}, \omega^U_{j2}, \omega^U_{j3}, \omega^U_{j4}; \eta^U_j)\right] \\ &= \sum_{k=1}^{q} \left(\lambda_k \times \tilde{\tilde{\omega}}_{kj}\right)\end{aligned}$$

$$= \sum_{k=1}^{q} \left(\lambda_k \times \left[(\omega_{kj1}^L, \omega_{kj2}^L, \omega_{kj3}^L, \omega_{kj4}^L; \eta_{kj}^L), (\omega_{kj1}^U, \omega_{kj2}^U, \omega_{kj3}^U, \omega_{kj4}^U; \eta_{kj}^U)\right]\right)$$

$$= \Big[\Big(\sum_{k=1}^{q}\left(\lambda_k\omega_{kj1}^L\right), \sum_{k=1}^{q}\left(\lambda_k\omega_{kj2}^L\right), \sum_{k=1}^{q}\left(\lambda_k\omega_{kj3}^L\right), \sum_{k=1}^{q}\left(\lambda_k\omega_{kj4}^L\right); \min_k(\eta_{kj}^L)\Big),$$
$$\Big(\sum_{k=1}^{q}\left(\lambda_k\omega_{kj1}^U\right), \sum_{k=1}^{q}\left(\lambda_k\omega_{kj2}^U\right), \sum_{k=1}^{q}\left(\lambda_k\omega_{kj3}^U\right), \sum_{k=1}^{q}\left(\lambda_k\omega_{kj4}^U\right); \min_k(\eta_{kj}^U)\Big)\Big] \tag{11}$$

3.4. Constructing the weighted matrix

Let $\tilde{\tilde{V}} = \left[\tilde{\tilde{v}}_{ij}\right]_{m\times n}$ be the weighted matrix, then:

$$\begin{aligned}\tilde{\tilde{v}}_{ij} &= \left[(v_{ij1}^L, v_{ij2}^L, v_{ij3}^L, v_{ij4}^L; \varpi_{ij}^L), (v_{ij1}^U, v_{ij2}^U, v_{ij3}^U, v_{ij4}^U; \varpi_{ij}^U)\right] = \tilde{\tilde{x}}_{ij} \otimes \tilde{\tilde{\omega}}_j \\ &= \left[(x_{ij1}^L\omega_{j1}^L, x_{ij2}^L\omega_{j2}^L, x_{ij3}^L\omega_{j3}^L, x_{ij4}^L\omega_{j4}^L; \min(w_{ij}^L, \eta_j^L)),\right. \\ &\quad \left.(x_{ij1}^U\omega_{j1}^U, x_{ij2}^U\omega_{j2}^U, x_{ij3}^U\omega_{j3}^U, x_{ij4}^U\omega_{j4}^U; \min(w_{ij}^U, \eta_j^U))\right]\end{aligned} \tag{12}$$

3.5. Ranking the alternatives based on the grey relational theory

(1) *Determining the positive ideal solution and the negative ideal solution of the evaluation objects*

Suppose that the positive ideal solution and the negative ideal solution are $\tilde{\tilde{V}}^+ = \left[\tilde{\tilde{v}}_j^+\right]_{1\times n}, \tilde{\tilde{V}}^- = \left[\tilde{\tilde{v}}_j^-\right]_{1\times n}$, respectively, then:

$$\begin{aligned}\tilde{\tilde{v}}_j^+ &= \left[(v_{j1}^{L+}, v_{j2}^{L+}, v_{j3}^{L+}, v_{j4}^{L+}; \varpi_j^{L+}), (v_{j1}^{U+}, v_{j2}^{U+}, v_{j3}^{U+}, v_{j4}^{U+}; \varpi_j^{U+})\right] \\ &= \Big[(\max_i(v_{ij1}^L), \max_i(v_{ij2}^L), \max_i(v_{ij3}^L), \max_i(v_{ij4}^L); \max_i(\varpi_{ij}^L)), \\ &\quad (\max_i(v_{ij1}^U), \max_i(v_{ij2}^U), \max_i(v_{ij3}^U), \max_i(v_{ij4}^U); \max_i(\varpi_{ij}^U))\Big]\end{aligned} \tag{13}$$

$$\begin{aligned}\tilde{\tilde{v}}_j^- &= \left[(v_{j1}^{L-}, v_{j2}^{L-}, v_{j3}^{L-}, v_{j4}^{L-}; \varpi_j^{L-}), (v_{j1}^{U-}, v_{j2}^{U-}, v_{j3}^{U-}, v_{j4}^{U-}; \varpi_j^{U-})\right] \\ &= \Big[(\min_i(v_{ij1}^L), \min_i(v_{ij2}^L), \min_i(v_{ij3}^L), \min_i(v_{ij4}^L); \min_i(\varpi_{ij}^L)), \\ &\quad (\min_i(v_{ij1}^U), \min_i(v_{ij2}^U), \min_i(v_{ij3}^U), \min_i(v_{ij4}^U); \min_i(\varpi_{ij}^U))\Big]\end{aligned} \tag{14}$$

(2) *Calculating the grey correlative coefficient of the i-th alternative and the positive ideal solution with respect to the j-th attribute (Liu et al., 1999)*

The grey correlative coefficient of the i-th alternative and the positive ideal solution with respect to the j-th attribute is

$$r_{ij}^+ = \frac{m + \xi M}{\Delta_{ij}^+ + \xi M}, \quad \xi \in (0, 1) \tag{15}$$

where $\Delta_{ij}^{+} = d(\tilde{\tilde{v}}_j^{+}, \tilde{\tilde{v}}_{ij})$, $m = \underbrace{\min}_{i}\underbrace{\min}_{j}\Delta_{ij}^{+}$,$M = \underbrace{\max}_{i}\underbrace{\max}_{j}\Delta_{ij}^{+}$. ξ is a resolution coefficient, generally, $\xi = 0.5$; then, the grey correlative coefficient matrix of each alternative and the positive ideal solutions is

$$R^{+} = \begin{bmatrix} r_{11}^{+} & r_{12}^{+} & \cdots & r_{1n}^{+} \\ r_{21}^{+} & r_{22}^{+} & \cdots & r_{2n}^{+} \\ \vdots & \vdots & \vdots & \vdots \\ r_{m1}^{+} & r_{m2}^{+} & \cdots & r_{mn}^{+} \end{bmatrix}.$$

The grey correlative coefficient of the i-th alternative and the positive ideal solutions is:

$$R_i^{+} = \frac{1}{n}\sum_{j=1}^{n} r_{ij}^{+}, (i = 1, 2, \cdots, m) \tag{16}$$

(3) *Calculating the grey correlative coefficient of the i-th alternative and the negative ideal solution with respect to the j-th attribute*

The grey correlative coefficient of the i-th alternative and the negative ideal solution with respect to the j-th attribute is

$$r_{ij}^{-} = \frac{m + \xi M}{\Delta_{ij}^{-} + \xi M}, \quad \xi \in (0, 1) \tag{17}$$

where $\Delta_{ij}^{-} = d(\tilde{\tilde{v}}_j^{-}, \tilde{\tilde{v}}_{ij})$, $m = \underbrace{\min}_{i}\underbrace{\min}_{j}\Delta_{ij}^{-}$,$M = \underbrace{\max}_{i}\underbrace{\max}_{j}\Delta_{ij}^{-}$,$\xi$ is a resolution coefficient, generally, $\xi = 0.5$; then, the grey correlative coefficient matrix of each alternative and the negative ideal solutions is:

$$R^{-} = \begin{bmatrix} r_{11}^{-} & r_{12}^{-} & \cdots & r_{1n}^{-} \\ r_{21}^{-} & r_{22}^{-} & \cdots & r_{2n}^{-} \\ \vdots & \vdots & \vdots & \vdots \\ r_{m1}^{-} & r_{m2}^{-} & \cdots & r_{mn}^{-} \end{bmatrix}.$$

The grey correlative coefficient of the i-th alternative and the negative ideal solutions is

$$R_i^{-} = \frac{1}{n}\sum_{j=1}^{n} r_{ij}^{-}, (i = 1, 2, \cdots, m) \tag{18}$$

(4) *Calculating the grey correlative similarity coefficient of each alternative*

$$C_i = \frac{R_i^{+}}{R_i^{+} + R_i^{-}}, (i = 1, 2, \cdots, m). \tag{19}$$

It can be seen from the grey relational theory (Liu et al., 1999) that any two sequences in a system may not be unrelated strictly. Namely, $0 < R_i^+ \leq 1$, $0 < R_i^- \leq 1$. So, the grey correlative similarity coefficient C_i satisfies the property: $0 < C_i < 1$.

(5) *Ranking the alternatives*

Based on the grey correlative similarity coefficient, we rank the alternatives. The bigger the grey correlative similarity coefficient is, the higher is the rank of the alternative, or vice versa.

4. An illustrative example

Suppose that a Telecommunication Company intends to choose a manager for R&D department among four candidates named A1, A2, A3 and A4. The decision making committee assesses the four concerned persons based on five attributes: (1) proficiency in identifying research areas (C1), (2) proficiency in administration (C2), (3) personality (C3), (4) past experience (C4), and (5) self-confidence (C5). There are three committee members, labeled DM1, DM2, DM3, respectively. Each of these decision makers has presented his/her assessment, based on linguistic terms, for the importance of each attribute and the evaluation information on the four candidates is depicted in Tables 2, 3, 4 and 5 (Ashtiani et al., 2009).

Table 2. The attribute weights given by the three DMs

	c_1	c_2	c_3	c_4	c_5
DM1	VH	H	H	VH	M
DM2	VH	H	MH	H	MH
DM3	VH	MH	MH	VH	M

Table 3. The evaluation information on four candidates given by DM1

	c_1	c_2	c_3	c_4	c_5
a_1	VG	VG	VG	VG	VG
a_2	G	VG	VG	VG	MG
a_3	VG	MG	G	G	G
a_4	G	F	F	G	MG

Table 4. The evaluation information on four candidates given by DM2

	c_1	c_2	c_3	c_4	c_5
a_1	G	MG	G	G	VG
a_2	G	VG	VG	VG	MG
a_3	G	G	MG	VG	G
a_4	VG	F	MG	F	G

Table 5. The evaluation information on four candidates given by DM3

	c_1	c_2	c_3	c_4	c_5
a_1	MG	F	G	VG	VG
a_2	MG	MG	G	MG	G
a_3	VG	VG	VG	VG	MG
a_4	MG	VG	MG	VG	F

Decision steps taken are as follows:

(1) Converting the linguistic terms into interval-valued trapezoidal fuzzy numbers, to get:

$$
\left[\tilde{\tilde{\omega}}_{kj}\right]_{3\times 5} = \left[\begin{array}{l}
[(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00; 1.0)], \\
[(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00; 1.0)], \\
[(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00; 1.0)], \\
[(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97; 1.0)], \\
[(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97; 1.0)], \\
[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)], \\
[(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97; 1.0)], \\
[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)], \\
[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)], \\
[(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00; 1.0)], \\
[(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97; 1.0)], \\
[(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00; 1.0)], \\
[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)] \\
[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)] \\
[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]
\end{array}\right]
$$

$$
\left[\tilde{\tilde{a}}_{ij1}\right]_{4\times 5} = \left[\begin{array}{l}
[(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00; 1.00)], \\
[(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97; 1.00)], \\
[(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00; 1.00)], \\
[(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97; 1.00)], \\
[(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00; 1.00)], \\
[(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00; 1.00)], \\
[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.00)], \\
[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.00)], \\
[(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00; 1.00)], \\
[(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00; 1.00)], \\
[(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97; 1.00)], \\
[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.00)], \\
[(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00; 1.00)], \\
[(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00; 1.00)], \\
[(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97; 1.00)], \\
[(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97; 1.00)], \\
[(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00, 1.0)] \\
[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86, 1.0)] \\
[(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97, 1.0)] \\
[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86, 1.0)]
\end{array}\right]
$$

$$\left[\tilde{\tilde{a}}_{ij2}\right]_{4\times 5}= \begin{bmatrix} [(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97; 1.00)], \\ [(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97; 1.00)], \\ [(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97; 1.00)], \\ [(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00; 1.00)], \\ [(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.00)], \\ [(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00; 1.00)], \\ [(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97; 1.00)], \\ [(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.00)], \\ [(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97; 1.00)], \\ [(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00; 1.00)], \\ [(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.00)], \\ [(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.00)], \\ [(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97; 1.00)], \\ [(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00; 1.00)], \\ [(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00; 1.00)], \\ [(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.00)], \\ [(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00, 1.0)] \\ [(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86, 1.0)] \\ [(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97, 1.0)] \\ [(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97, 1.0)] \end{bmatrix}$$

$$\left[\tilde{\tilde{a}}_{ij3}\right]_{4\times 5}= \begin{bmatrix} [(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.00)], \\ [(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.00)], \\ [(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00; 1.00)], \\ [(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.00)], \\ [(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.00)], \\ [(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.00)], \\ [(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00; 1.00)], \\ [(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00; 1.00)], \\ [(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97; 1.00)], \\ [(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97; 1.00)], \\ [(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00; 1.00)], \\ [(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.00)], \\ [(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00; 1.00)], \\ [(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.00)], \\ [(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00; 1.00)], \\ [(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00; 1.00)], \\ [(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00, 1.0)] \\ [(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97, 1.0)] \\ [(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86, 1.0)] \\ [(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65, 1.0)] \end{bmatrix}$$

(2) Combining the individual preferences in order to obtain a collective preference value for each alternative:

$$[\tilde{\tilde{x}}_{ij}]_{4\times 5} = \begin{bmatrix} [(0.743, 0.797, 0.907, 0.943; 0.800), (0.743, 0.797, 0.907, 0.943; 1.000)], \\ [(0.673, 0.730, 0.880, 0.933; 0.800), (0.673, 0.730, 0.880, 0.933; 1.000)], \\ [(0.860, 0.913, 0.973, 0.990; 0.800), (0.860, 0.913, 0.973, 0.990; 1.000)], \\ [(0.743, 0.797, 0.907, 0.943; 0.800), (0.743, 0.797, 0.907, 0.943; 1.000)], \\ [(0.610, 0.673, 0.793, 0.837; 0.800), (0.610, 0.673, 0.793, 0.837; 1.000)], \\ [(0.813, 0.863, 0.933, 0.953; 0.800), (0.813, 0.863, 0.933, 0.953; 1.000)], \\ [(0.743, 0.797, 0.907, 0.943; 0.800), (0.743, 0.797, 0.907, 0.943; 1.000)], \\ [(0.523, 0.600, 0.720, 0.767; 0.800), (0.523, 0.600, 0.720, 0.767; 1.000)], \\ [(0.790, 0.847, 0.947, 0.980; 0.800), (0.790, 0.847, 0.947, 0.980; 1.000)], \\ [(0.860, 0.913, 0.973, 0.990; 0.800), (0.860, 0.913, 0.973, 0.990; 1.000)], \\ [(0.743, 0.797, 0.907, 0.943; 0.800), (0.743, 0.797, 0.907, 0.943; 1.000)], \\ [(0.493, 0.557, 0.727, 0.790; 0.800), (0.493, 0.557, 0.727, 0.790; 1.000)], \\ [(0.860, 0.913, 0.973, 0.990; 0.800), (0.860, 0.913, 0.973, 0.990; 1.000)], \\ [(0.813, 0.863, 0.933, 0.953; 0.800), (0.813, 0.863, 0.933, 0.953; 1.000)], \\ [(0.860, 0.913, 0.973, 0.990; 0.800), (0.860, 0.913, 0.973, 0.990; 1.000)], \\ [(0.657, 0.723, 0.833, 0.873; 0.800), (0.657, 0.723, 0.833, 0.873; 1.000)], \\ [(0.930, 0.980, 1.000, 1.000; 0.800), (0.930, 0.980, 1.000, 1.000; 1.000)] \\ [(0.627, 0.680, 0.840, 0.897; 0.800), (0.627, 0.680, 0.840, 0.897; 1.000)] \\ [(0.673, 0.730, 0.880, 0.933; 0.800), (0.673, 0.730, 0.880, 0.933; 1.000)] \\ [(0.540, 0.607, 0.767, 0.827; 0.800), (0.540, 0.607, 0.767, 0.827; 1.000)] \end{bmatrix}$$

$$\begin{aligned}[\tilde{\tilde{\omega}}_j]_5 = [&[(0.930, 0.980, 1.000, 1.000; 0.800), (0.930, 0.980, 1.000, 1.000; 1.000)], \\ &[(0.627, 0.680, 0.840, 0.897; 0.800), (0.627, 0.680, 0.840, 0.897; 1.000)], \\ &[(0.627, 0.680, 0.840, 0.897; 0.800), (0.627, 0.680, 0.840, 0.897; 1.000)], \\ &[(0.930, 0.980, 1.000, 1.000; 0.800), (0.930, 0.980, 1.000, 1.000; 1.000)], \\ &[(0.320, 0.410, 0.580, 0.650; 0.800), (0.320, 0.410, 0.580, 0.650; 1.000)]]\end{aligned}$$

(3) Calculating the weighted decision making matrix:

$$[\tilde{\tilde{v}}_{ij}]_{4\times 5} = \begin{bmatrix} [(0.691, 0.781, 0.907, 0.943; 0.800), (0.691, 0.781, 0.907, 0.943; 1.000)], \\ [(0.626, 0.715, 0.880, 0.933; 0.800), (0.626, 0.715, 0.880, 0.933; 1.000)], \\ [(0.800, 0.895, 0.973, 0.990; 0.800), (0.800, 0.895, 0.973, 0.990; 1.000)], \\ [(0.691, 0.781, 0.907, 0.943; 0.800), (0.691, 0.781, 0.907, 0.943; 1.000)], \\ [(0.382, 0.458, 0.666, 0.750; 0.800), (0.382, 0.458, 0.666, 0.750; 1.000)], \\ [(0.510, 0.587, 0.784, 0.855; 0.800), (0.510, 0.587, 0.784, 0.855; 1.000)], \\ [(0.466, 0.542, 0.762, 0.846; 0.800), (0.466, 0.542, 0.762, 0.846; 1.000)], \\ [(0.328, 0.408, 0.605, 0.687; 0.800), (0.328, 0.408, 0.605, 0.687; 1.000)], \\ [(0.495, 0.576, 0.795, 0.879; 0.800), (0.495, 0.576, 0.795, 0.879; 1.000)], \\ [(0.539, 0.621, 0.818, 0.888; 0.800), (0.539, 0.621, 0.818, 0.888; 1.000)], \\ [(0.466, 0.542, 0.762, 0.846; 0.800), (0.466, 0.542, 0.762, 0.846; 1.000)], \\ [(0.309, 0.379, 0.610, 0.708; 0.800), (0.309, 0.379, 0.610, 0.708; 1.000)], \\ [(0.800, 0.895, 0.973, 0.990; 0.800), (0.800, 0.895, 0.973, 0.990; 1.000)], \\ [(0.756, 0.846, 0.933, 0.953; 0.800), (0.756, 0.846, 0.933, 0.953; 1.000)], \\ [(0.800, 0.895, 0.973, 0.990; 0.800), (0.800, 0.895, 0.973, 0.990; 1.000)], \\ [(0.611, 0.709, 0.833, 0.873; 0.800), (0.611, 0.709, 0.833, 0.873; 1.000)], \end{bmatrix}$$

$$\begin{bmatrix} [(0.298, 0.402, 0.580, 0.650; 0.800), (0.298, 0.402, 0.580, 0.650; 1.000)] \\ [(0.201, 0.279, 0.487, 0.583; 0.800), (0.201, 0.279, 0.487, 0.583; 1.000)] \\ [(0.215, 0.299, 0.510, 0.607; 0.800), (0.215, 0.299, 0.510, 0.607; 1.000)] \\ [(0.173, 0.249, 0.445, 0.537; 0.800), (0.173, 0.249, 0.445, 0.537; 1.000)] \end{bmatrix}$$

(4) Determining the positive ideal solution and the negative ideal solution:

$$\begin{aligned} \tilde{\tilde{V}}^{+} = [&[(0.800, 0.895, 0.973, 0.990; 0.800), (0.800, 0.895, 0.973, 0.990; 1.000)], \\ &[(0.510, 0.587, 0.784, 0.855; 0.800), (0.510, 0.587, 0.784, 0.855; 1.000)], \\ &[(0.539, 0.621, 0.818, 0.888; 0.800), (0.539, 0.621, 0.818, 0.888; 1.000)], \\ &[(0.800, 0.895, 0.973, 0.990; 0.800), (0.800, 0.895, 0.973, 0.990; 1.000)], \\ &[(0.298, 0.402, 0.580, 0.650; 0.800), (0.298, 0.402, 0.580, 0.650; 1.000)]] \end{aligned}$$

$$\begin{aligned} \tilde{\tilde{V}}^{-} = [&[(0.626, 0.715, 0.880, 0.933; 0.800), (0.626, 0.715, 0.880, 0.933; 1.000)], \\ &[(0.328, 0.408, 0.605, 0.687; 0.800), (0.328, 0.408, 0.605, 0.687; 1.000)], \\ &[(0.309, 0.379, 0.610, 0.708; 0.800), (0.309, 0.379, 0.610, 0.708; 1.000)], \\ &[(0.611, 0.709, 0.833, 0.873; 0.800), (0.611, 0.709, 0.833, 0.873; 1.000)], \\ &[(0.173, 0.249, 0.445, 0.537; 0.800), (0.173, 0.249, 0.445, 0.537; 1.000)]] \end{aligned}$$

(5) Calculate the grey correlative coefficient matrix:

$$R^{+} = \begin{bmatrix} 0.5609 & 0.4728 & 0.7808 & 1.0000 & 1.0000 \\ 0.4604 & 1.0000 & 1.0000 & 0.7175 & 0.5305 \\ 1.0000 & 0.7808 & 0.6318 & 1.0000 & 0.5907 \\ 0.5609 & 0.3777 & 0.3333 & 0.4046 & 0.4495 \end{bmatrix}$$

$$R^{-} = \begin{bmatrix} 0.7199 & 0.6527 & 0.3678 & 0.4046 & 0.4495 \\ 1.0000 & 0.3777 & 0.3333 & 0.4812 & 0.7465 \\ 0.4604 & 0.4226 & 0.4137 & 0.4046 & 0.6529 \\ 0.7199 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \end{bmatrix}$$

(6) Calculating the grey correlative similarity coefficient of each alternative:

$$C = (0.5952\ 0.5579\ 0.6297\ 0.3106)$$

(7) Ranking the alternatives:

Based on the grey correlative similarity coefficient, we can now rank the alternatives: $a_3 \succ a_1 \succ a_2 \succ a_4$.

(8) Analysis:

In this example, our approach produces the same ranking as shown in the literature (Ashtiani et al., 2009). In addition, we use the method proposed by Liu (2011) to solve the example, and it yields the same ranking result, showing that the approach presented in this paper is effective. But the approach, suggested in this paper is more general in terms of dealing with more complex problems of fuzzy multiple attribute decision making.

5. Discussion

This paper proposes a decision making method for fuzzy multiple attribute decision making problems, in which the attribute values and the weights take the form of the generalized interval-valued trapezoidal fuzzy numbers (GIVTFN). It supposes that attribute values are independent and have additive characteristic, and decision makers are independent of each other. However, in real life problems, it is difficult to meet these conditions. For example, fuel consumption has greater importance at lower car prices (cheap cars are purchased mainly by poor people) and small or even null importance for high car prices (expensive cars are purchased by rich people). Therefore, separate evaluation of attribute importance is a significant simplification, causing errors in decision making.

In addition, multiple attribute decision-making method proposed in this paper is based on the rational choice model. However, in the actual decision-making process, people often are not fully rational decision-makers. Simon (1971) proposed the "bounded rationality" principle, meaning that people's decision-making has only a limited rationality; Kahneman and Tversky (1979) collected many studies of the individual behavior based on Simon's "bounded rationality", using surveys and tests, and they found that people's judgments and decisions in the actual behavior under uncertainty depart form the predictions of the expected utility theory, so they proposed the so-called Prospect Theory in 1979. Let us note that the foundations for substantive propositions behind the prospect theory are empirical and experimental in nature.

Obviously, the decision-making based on prospect theory is more in line with the actual decision-making behavior, and it is an important research topic to devise the ways of using the prospect theory in multiple attribute decision making, and it is also the direction of future research.

6. Conclusion

Fuzzy multiple attribute decision making (FMADM) is widely used in various areas. Against this context - the interval-valued trapezoidal fuzzy numbers can precisely express the attribute values and weights in FMADM. This paper proposes a decision making method based on the grey correlative coefficient for solving the MADM problems, in which the attribute weights and values are given in the form of GIVTFN, and it also proposes the respective decision making steps, forming a coherent procedure. This method is simple and easy to understand. It enriches and broadens the theory and methodology of FMADM, and proposes a new idea for solving the FMADM problems.

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References

Ashtiani, B., Haghighirad, F., Makui, A. et al. (2009) Extension of fuzzy TOPSIS method based on interval-valued fuzzy sets. *Applied Soft Computing* **9** (2),457–461.

Bellman, R.E. and Zadeh, L.A. (1970) Decision-making in a fuzzy environment. *Management Science* **17** (4), 141–164.

Chen, S.H. (1985) Ranking fuzzy numbers with maximizing set and minimizing set. *Fuzzy Sets and Systems* **17** (2), 113–129.

Chen, S.M. and Chen, J.H. (2009) Fuzzy risk analysis based on ranking generalized fuzzy numbers with different heights and different spreads. *Expert Systems with Applications* **36** (3), 6833–6842.

Chen, S.J. and Chen, S.M. (2003) A new method for handing multicriteria fuzzy decision-making problems using FN-IOWA operators. *Cybernetics and Systems* **34** (2), 109–137.

Gorzalczany, M.B. (1987) A method of inference in approximate reasoning based on interval-valued fuzzy sets. *Fuzzy Sets and Systems* **21** (1), 1–17.

Hong, D.H. and Lee, S. (2002) Some algebraic properties and a distance measure for interval-valued fuzzy numbers. *Information Sciences* **148** (1), 1–10.

Jahanshahloo, G.R., Hosseinzadeh, L.F. and Izadikhah, M. (2006) An algorithmic method to extend TOPSIS for decision-making problems with interval data. *Applied Mathematics and Computation* **175** (2), 1375–1384.

Kahneman, D. and Tversky, A. (1979) Prospect theory: an analysis of decision under risk. *Econometrica* **47** (2), 263–292.

Liu, P.D. (2009a) A novel method for hybrid multiple attribute decision making. *Knowledge-Based Systems* **22** (5), 388–391.

Liu, P.D. (2009b) Multi-Attribute Decision-Making Method Research Based on Interval Vague Set and TOPSIS Method. *Technological and Economic Development of Economy* **15** (3), 453–463.

Liu, P.D. (2011) A Weighted Aggregation Operators Multi-attribute Group Decision-making Method based on Interval-valued Trapezoidal Fuzzy Numbers. *Expert Systems with Applications* **38** (1), 1053-1060.

Liu, P.D. and Zhang, X. (2010) The Study on Multi-Attribute Decision-Making with Risk Based on Linguistic Variable. *International Journal of Computational Intelligence Systems* **3** (5), 601–609.

Liu, W.J. and Zeng, L. (2008) A new TOPSIS method for fuzzy multiple attribute group decision making problem. *Journal of Guilin University of Electronic Technology* **28** (1), 59–62.

Liu, S.F., Guo, T.B. and Dang, Y.G. (1999) *Grey System Theory and Application.* Science Press, Beijing.

Men, F. and Ji, S.Q. (2008) An improved FMEA based on Fuzzy Set theory and Grey Relational Theory. Industrial Engineering and Management, (2), 55–59.

Simon, H.A. (1971) *Administrative Behavior - A Study of Decision Making Processes in Administrative Organization.* Macmillan, New York.

Turksen, I.B. (1996) Interval-valued strict preference with Zadeh triples. *Fuzzy Sets and Systems* **78** (2), 183–195.

Wang, G. and Li, X. (1998) The applications of interval-valued fuzzy numbers and interval-distribution numbers. *Fuzzy Sets and Systems* **98** (3), 331–335.

Wang, G. and Li, X. (2001) Correlation and information energy of interval-valued fuzzy numbers. *Fuzzy Sets and Systems* **103** (1), 169–175.

Wang, Y.M. and Elhag Taha, M.S. (2006) Fuzzy TOPSIS method based on alpha level sets with an application to bridge risk assessment. *Expert Systems with Applications* **31** (2), 309–319.

Wei, G.W. and Wei, Y. (2008) Model of Grey Relational Analysis for Interval Multiple Attribute Decision Making with Preference Information on Alternatives. *Chinese Journal of Management Science* **16** (1), 158–162.

Wei, S.H. and Chen, S.M. (2009) Fuzzy risk analysis based on interval-valued fuzzy numbers. *Expert Systems with Applications* **36** (2), 2285–2299.

Zhu, H.P., Zhang, G.J. and Shao, X.Y. (2007) Study on the Application of Fuzzy TOPSIS to Multiple Criteria Group Decision Making Problem. *Industrial Engineering and Management*, (1) 99–102.