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Application of Zernike Moments to the problem of General Shape Analysis*†

by

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Abstract: The Zernike Moments (ZM) have been successfully applied to the problem of shape recognition. Their properties allow for solving some fundamental problems in this task. Amongst them the most important one is the invariance to rotation, scaling, translation, and reflectional symmetry. Moreover, the obtained representation can vary according to the level of generalisation of a shape. For this reason in the paper the application of the ZM to the problem of General Shape Analysis (GSA) is proposed and experimentally investigated. The GSA problem is similar to the recognition and retrieval of shapes. However, only the most general classes of shapes (e.g. square, triangle, circle, ellipse) are assumed to perform the role of the basic templates. Moreover, the processed object does not have to belong to any of the template classes, but may be only similar to one of them. This enables us to receive the most general information about a shape, e.g. how square, triangular, round, elliptical, etc. it is. In the paper, in order to evaluate the Zernike Moments applied to the problem of GSA, the performance of this shape descriptor is compared with the results provided by nearly two hundred humans and collected by means of appropriate inquiry forms.

Keywords: Zernike Moments, General Shape Analysis, shape description.

1. Introduction

The application of the Zernike polynomials to digital image analysis has been proposed by M. R. Teague in 1980 (see Teague, 1980). He named the proposed set of complex orthogonal functions the Zernike Moments (abbreviated, ZM).

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The most important properties of the Zernike Moments are the invariance to rotation, scaling, translation, and reflection symmetry. Those advantages resulted in the noticeable popularity of the ZM in digital image processing, analysis, and recognition. They were applied, for example, in the recognition of characters (see Broumandnia and Shanbehzadeh, 2007), sketched symbols (see Hse and Newton, 2004), and aircrafts (see Zhang et al., 2009). Their use in 2D and 3D shape retrieval is also noticeable (e.g. see Yadav et al., 2008). Zhenjiang proposed a novel application in agronomy — the rose variation recognition (see Zhenjiang, 2000). The combination of the ZM with the Sobel algorithm resulted in the creation of a new edge detection method (see Cheng-Song et al., 2005). In Bharathi and Ganesan (2008) the Zernike moment features were applied in distinguishing between normal and abnormal soft liver tissues. The application of the ZM may be also found in the description of moving objects (see Shutler and Nixon, 2006), face recognition (see Haddadnia et al., 2001), palmprint verification (see Pang et al., 2004), image denoising (see Ji et al., 2009), watermarking (see Xin et al., 2004), and many other areas.

In this paper a new application of the discussed algorithm is proposed, described and experimentally investigated. This time, the Zernike Moments are applied to the General Shape Analysis (GSA). This problem is similar to both the recognition and retrieval of shapes. The most important similarity as regards recognition is the use of some pre-determined classes of templates and searching for the most similar one(s) to the shape being processed. The most important difference is that the processed object does not have to belong to any of the template classes. It is only similar to one, or few of them, and it is this similarity that is investigated. On the other hand, from the perspective of shape retrieval the selection of more than one template is applied. Hence, usually three of the most similar templates to the processed shape are selected and presented to the user.

In the General Shape Analysis a small number of template classes (general shapes — circle, triangle, star, etc.) is used. Basing on this assumption one can assign a processed object to one or few general shapes and determine how round, triangular, star-like, etc. it is. This has been useful for example in the identification of type of a stamp (e.g. official, public, institutional) in searching for probable false digital documents stored on a hard drive (see Frejlichowski and Forczmański, 2010), initial, coarse classification in large databases, use of voice commands for shape retrieval in large multimedia databases, etc. (see Frejlichowski, 2010a). The problem is depicted in Fig. 1. An object may be considered similar to the four indicated templates, yet it is definitely not one of them.

Several shape description algorithms in application to the GSA problem have been investigated and described so far (see for example Frejlichowski 2010a, b). However, the evaluation of particular algorithms is ambiguous, since there is no independent measure for the problem of GSA. In order to avoid this problem an inquiry form was filled in by 187 persons (124 men, 63 women, aged from 9 to 62)

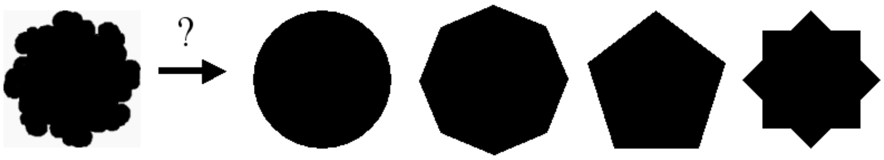

Figure 1. Illustration of the general shape analysis problem — which general shape is the most similar to the processed one?

which enabled to investigate the GSA and was meant to serve as a benchmark. It has also been applied in order to evaluate the results achieved by means of the Zernike Moments.

The remaining part of the paper is organised in the following way. The second section provides a description of the Zernike Moments (basing on Wee and Paramesran, 2007). The third one presents the result of its application to the General Shape Analysis and provides a comparison with human results as well as the results obtained by means of some other methods applied to this problem so far. The last section concludes the paper.

2. The Zernike Moments

For an image intensity function $f(x,y)$ the 2-dimensional complex Zernike Moments of order p (a non-negative integer) and repetition q (an integer value such that $p-|q|$ is even, $|q| \leq p$, and $|\cdot|$ denotes the absolute value) can be defined as:

$$Z_{pq} = \frac{p+1}{\pi} \iint\limits_{x^2+y^2 \leq 1} V^*_{pq}(x,y) f(x,y) \mathrm{d}x \mathrm{d}y, \tag{1}$$

with the orthogonality relation:

$$\iint\limits_{x^2+y^2 \leq 1} V^*_{pq}(x,y) V_{p'q'}(x,y) \mathrm{d}x \mathrm{d}y = \frac{\pi}{p+1} \delta_{pp'} \delta_{qq'}, \tag{2}$$

where p' and q' have the same properties as p and q, respectively.

The function $f(x,y)$ represents the image and * denotes the complex conjugate. The original image stored in a form of a square bitmap, with sides equal to N pixels, should be rescaled in order to obtain the unit square $S = \{(x,y) \in R^2 : -1 \leq x, y \leq 1\}$. Later, from the square S, a round subpart is selected, belonging to the unit disc $D = \{(x,y) \in R^2 : x^2 + y^2 \leq 1\}$, $D \subset S$. In the experiments described is this paper each single shape has to be inscribed in D.

The p-th order Zernike polynomials are defined as:

$$V_{pq}(x,y)=R_{pq}(r)e^{jq\theta}, \tag{3}$$

where:
$r=\sqrt{x^2+y^2}$ — is the distance from the origin of the unit disc to the pixel with coordinates (x,y), $0\leq r\leq 1$,
$\theta=arctan\frac{y}{x}$ — is the angle between the vector from the origin of the unit disc to the pixel (x,y) and the principle X-axis, $x\neq 0, -\pi\leq\theta\leq\pi$,
j — imaginary unit.

The Zernike real-valued radial polynomials can be formulated as:

$$R_{pq}(r)=\sum_{s=0}^{(p-q)/2}(-1)^s\frac{(p-s)!}{s!(\frac{p+|q|}{2}-s)!(\frac{p-|q|}{2}-s)!}r^{p-2s}. \tag{4}$$

For given integers p and q as in formula (1) and for $k=q,\ldots,p, s=\frac{p-k}{2}$, Zernike polynomials can be rewritten as:

$$V_{pq}(x,y)=\sum_{k=q}^{p}B_{pqk}r^k e^{jq\theta}, \tag{5}$$

and the polynomial coefficient B_{pqk} is defined as:

$$B_{pqk}=\frac{(-1)^{\frac{p-k}{2}}(\frac{p+k}{2})!}{(\frac{p-k}{2})!(\frac{k+q}{2})!(\frac{k-q}{2})!}. \tag{6}$$

Now, the ZM can be rewritten in polar form:

$$Z_{pq}=\frac{p+1}{\pi}\int_0^1\int_{-\pi}^{\pi}\sum_{k=q}^{p}B_{pqk}r^k e^{-jq\theta}f(r,\theta)r\mathrm{d}r\mathrm{d}\theta, \tag{7}$$

where: $\mathrm{d}x\mathrm{d}y=r\mathrm{d}r\mathrm{d}\theta$, $-\pi\leq\theta\leq\pi$.

The set of Zernike polynomials $V_{pq}(r,\theta)$ forms a complete orthogonal set on a unit disc $\{(r,\theta):\theta\in[-\pi,\pi]\wedge 0\leq r\,(\theta)\leq 1\}$:

$$\int_0^1\int_{-\pi}^{\pi}V_{pq}(r,\theta)[V_{p'q'}(r,\theta)]^*r\mathrm{d}r\mathrm{d}\theta=\begin{cases}\frac{\pi}{p+1}\delta_{pp'}\delta_{qq'} & p=p',q=q'\\ 0 & \text{otherwise}\end{cases}, \tag{8}$$

and its radial polynomials also satisfy the orthogonality relation as:

$$\int_0^1 R_{pq}(r)[R_{p'q}(r)]^*r\mathrm{d}r=\begin{cases}\frac{1}{2(p+1)}\delta_{pp'} & p=p'\\ 0 & \text{otherwise}\end{cases}, \tag{9}$$

where δ_{ij} is the Kronecker delta.

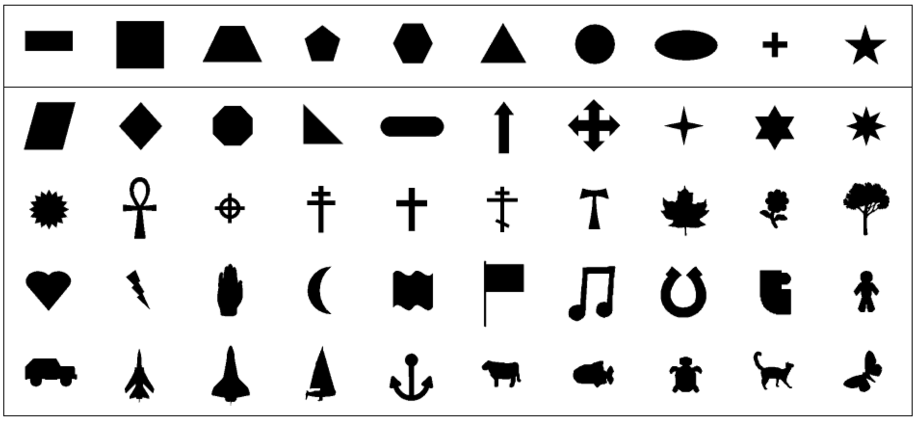

Figure 2. The shapes used in the experiment — 10 templates and 40 test objects

3. Conditions and results of the experiment

The behaviour of the Zernike Moments in application to the General Shape Analysis problem has been investigated by means of the same objects as in Frejlichowski (2010a). The dataset consisted of 10 templates (the general shapes) and 40 tested objects (see Fig. 2). They were stored in the form of binary (black and white) bitmaps, sized 200×200 pixels. For each bitmap containing a shape object the Zernike Moments were calculated, using freeware code by Kuiyu Chang (Chang, 2000). For this purpose, the co-ordinates of particular points belonging to a region were applied. In general, when the moments are applied to greyscale images, the grey-tones for particular pixels are used. However, in the case of binary images, only two values are possible. In the experiments presented in the paper the zero values (white) belong to the background, and ones (black) to the processed shape.

One of the advantages of the Zernike Moments was especially useful in the problem of the General Shape Analysis, namely that the analysed shape descriptor is reversible. It means that the original shape can be obtained from its description. What is more important, the precision of such a reconstruction can be controlled by means of the parameters of the algorithm. This property makes it possible to control the generality of the obtained shape representation, which may be very useful in the described problem. This issue is illustrated in Fig. 3. As may be seen the reconstructed shapes for particular orders of moments differ from one another. The lower the order, the more general the shape reconstruction. In contrast, the higher the order, the more precise the reconstructed shape.

Keeping in mind the above discussion, concerning the usefulness of the Zernike Moments in the GSA problem and the changing appearance of the

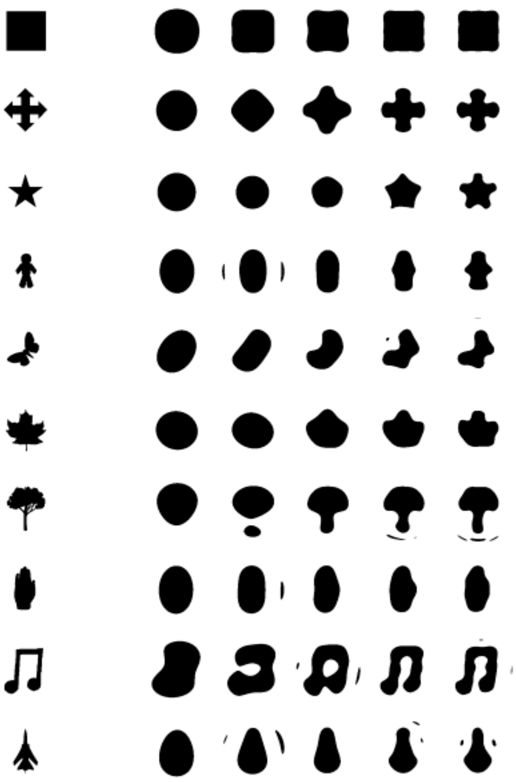

Figure 3. Illustration of precision of the reconstruction for increasing values of the Zernike Moments order — 3rd, 5th, 9th, 12th, 15th. In the left column the original shape is provided

reconstructed shape for varying parameters of this algorithm, the experiments were separately performed for the set of moments with repetition and order varying from 1 to 15. In each case every test object was represented by a particular set of moments, and so were all the templates (general shapes). The description of a test shape was matched by means of the Euclidean distance with all of the descriptions of the templates. The three smallest dissimilarity values indicated the general shapes that were closest to test shape.

As mentioned in the introductory part of the paper, in order to evaluate the investigated shape descriptor, it was compared with the benchmark provided by humans. This benchmark is presented pictorially in Fig. 4. Since the inquiry forms could be analysed in a variety of ways, a certain assumption had to be made. Firstly, a more difficult measure was used, namely that general shape was selected which had been most frequently indicated by humans. In Fig. 4, for each of the forty test shapes the three indicated templates are provided, according

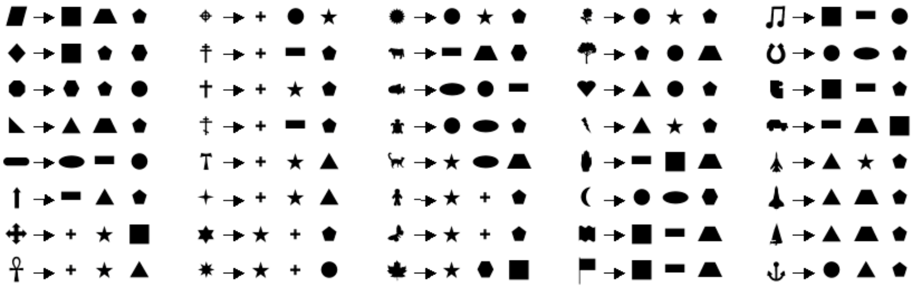

Figure 4. Results of the General Shape Analysis performed by humans

Table 1. The comparison of the GSA performed by humans and the Zernike Moments — the percentage of coincidence between the algorithm and the benchmark human results

Order	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1st indication	32.5	35	37.5	40	**40**	40	40	40	37.5	37.5	37.5	35	32.5	32.5	27.5
2nd indication	17.5	17.5	17.5	17.5	**22.5**	20	22.5	20	22.5	22.5	30	30	30	25	25
3rd indication	20	22.5	25	17.5	**20**	12.5	17.5	17.5	25	15	17.5	10	12.5	7.5	7.5

to the most popular results provided by humans. For example, according to results from the inquiry forms the parallelogram (test object no. 1) is the most similar to the square (1st indication), the regular hexagon (2nd indication) and circle (3rd indication), out of the ten templates.

The results for the three first indications — the correspondence between the Zernike Moments and the increasing order (corresponding to the property of the reconstruction precision) and benchmark human results — are provided in Table 1.

The analysis of Table 1 provides us with several important conclusions. First of all, the lowest orders (from 1 to 3) gave the worst results. This may be explained by the fact that the level of the obtained generalisation was too high. This phenomenon is also visible in Fig. 3, for the reconstructed shapes for low orders of the Zernike Moments. The results for orders from the 4th to 8th are much better, with the 5th order prevailing over the rest. Above the 8th order the results are getting worse. This may be easily explained by the increasing precision of the obtained representation, which is equivalent to its decreasing generality. This property may be also observed in Fig. 3. Therefore, the Zernike Moments of the 5th order were selected for the representation of shapes for the GSA problem (see Fig. 5 for pictorial representation of the results, provided in the same way as the benchmark human results, presented in Fig. 4). In this case, the degree of similarity between the Zernike Moments and the results provided by humans was as follows. The first indication was the same in the

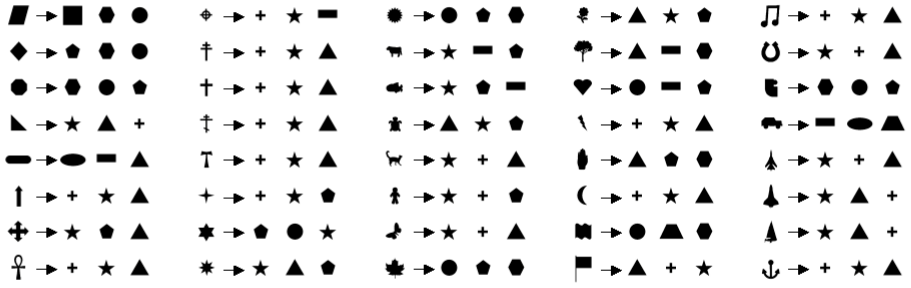

Figure 5. Results of the experiment on General Shape Analysis using the 5th order Zernike Moments

Table 2. The comparison of the GSA performed by humans and various shape descriptors — the percentage of coincidence between the algorithm and benchmark human results

Shape descriptor	1st indication	2nd indication	3rd indication
1. Zernike Moments	40%	22.5%	20%
2. Fourier Descriptors	35%	22.5%	17.5%
3. Point Distance Histogram	25%	15%	27.5%
4. Roundness	25%	12.5%	17.5%
5. Moment Invariants	20%	17.5%	5%
6. UNL-Fourier	17.5%	10%	12.5%
7. UNL	15%	5%	12.5%
8. Contour Sequence Moments	12.5%	12.5%	10%
9. X/Y Feret	10%	7.5%	2.5%

case of 40 % of the test objects. The second one was identical for 22.5 %, and the third — for 20 % of the test objects. Such a result can be considered very good, since the best method investigated so far (see Frejlichowski, 2010a) — Fourier Descriptors — is less effective (35 %, 22.5 %, 17.5 %, respectively). In order to show the superiority of the ZM amongst other shape descriptors tested in the problem of General Shape Analysis, in Table 2 their results are compared with the achievements of eight other popular algorithms, examined in Frejlichowski (2010a, b) — Fourier Descriptors (FD), Point Distance Histogram (PDH), Roundness, geometrical Moment Invariants (MI), UNL-Fourier (UNL-F), UNL, Contour Sequence Moments (CSM), and X/Y Feret.

The conclusions presented above were based on the comparison of the results of the Zernike Moments with the ones provided by humans. For this purpose a challenging assumption was applied, namely the percentage of proper indications, separately for each of the three templates which have been selected first. However, the analysis of the inquiry forms led to the conclusion that in many cases the indications made by humans vary. In some cases the sequence is different, yet the selected set of templates remains the same. Therefore, a second experiment was performed. This time, the sequence was not taken into account, only the presence of the particular indication obtained by means of a shape descriptor in the whole benchmark set consisting of three elements.

Similarly to the previously described experiment, firstly the results obtained by means of the Zernike Moments are provided (see Table 3). Again, the results for moments from 1st to 15th order are presented. For each indication the percentage of the correspondence to the benchmark human results was calculated basing on the assumption that the single artificial result was proper if it was the same as any of the three indications made by humans. As a result, the sequence of the indications was no longer important. Only the appearance of the considered resultant shape, indicated by the artificial algorithm, in the three element benchmark set, was taken into account.

Table 3. The comparison of the GSA performed by humans and the Zernike Moments — the percentage of coincidence between the artificial algorithm and benchmark human results. Here, the particular indication obtained by means of the ZM is considered proper if it is the same as any of the three benchmark human indications for the considered test shape

Order	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1st indication	55	60	60	55	57.5	60	55	60	60	62.5	55	52.5	50	50	45
2nd indication	45	42.5	40	40	42.5	35	42.5	45	45	47.5	57.5	50	47.5	42.5	45
3rd indication	40	37.5	35	42.5	42.5	42.5	42.5	42.5	45	32.5	37.5	27.5	35	27.5	30

The analysis of Table 3 provides us with the main conclusion that the results for lower orders are more similar as compared to the previous experiment. For example, for the first indication the percentage of the correspondence with the benchmark varies from 55 to 60. The results are getting worse starting from the 12th order. In general, it is more difficult to select the best order of the ZM. If we assume that the 1st indication is the most important one, the 10th order (62.5 %) may be considered as the best one.

The results obtained by means of the ZM were again compared with several other shape descriptors investigated in the General Shape Analysis so far (see Frejlichowski (2010a, b). This time, the 10th order was selected for the representation of a shape, basing on the Zernike Moments. The comparison is provided in Table 4. Again, the Zernike Moments outperformed the other algorithms for shape representation applied to the problem of General Shape Analysis.

Table 4. The comparison of the GSA performed by humans and shape descriptors — the percentage of coincidence between artificial algorithms and benchmark human results. Here, the particular obtained indication is considered proper, if it is the same as any of the three benchmark human indications for the considered test shape

Shape descriptor	1st indication	2nd indication	3rd indication
1. Zernike Moments	62.5%	47.5%	32.5%
2. Fourier Descriptors	60%	47.5%	25%
3. X/Y Feret	57.5%	22.5%	32.5%
4. UNL-Fourier	50%	37.5%	40%
5. Contour Sequence Moments	50%	27.5%	37.5%
6. Roundness	45%	30%	40%
7. Point Distance Histogram	42.5%	30%	42.5%
8. Moment Invariants	37.5%	37.5%	27.5%
9. UNL	35%	22.5%	32.5%

4. Conclusions and future work

In the paper the results of the experiment concerning the General Shape Analysis performed by means of the Zernike Moments have been provided. In this problem the most general information about a shape is considered, e.g. how round, elliptical, triangular, square, etc. it is. This can be useful in applications, in which the goal is not to exactly recognise the object, but to obtain general information about it. One example is the analysis of stamp shapes when searching for probable false documents stored on a hard drive in a digital form (see Frejlichowski and Forczmański, 2010). This results from the fact that in Poland the particular types of stamps have an expected shape, e.g. the official ones are round, while the medical ones are rectangular.

The application of the Zernike Moments to the described problem is not accidental. This shape descriptor is successfully applied in a variety of problems in digital image processing and recognition. Apart from its invariance to the most typical shape deformations (e.g. rotation, translation, scaling) the algorithm allows for a more general description of an object. This property proved to be very useful in the novel problem, described in the paper. The results achieved by means of the Zernike Moments were better than for other algorithms which have been investigated so far. The evaluation of their behaviour was based on the comparison with the results provided by nearly two hundred persons, who filled out an inquiry form, similar to the test used in the experiments employing artificial methods.

Future plans include two main aspects. First, other shape descriptors will obviously be studied. The results achieved by means of the ZM prove the best so far, but they are not ideal. Second, the results provided by humans are ambiguous. That is why in future other ways of comparing the artificial results with benchmark will be established. They have to take into account the variety of results provided by particular individuals. The average result for questioned people is sufficient at the beginning. However, future work will also be aimed at the individualisation of results and their connection with particular approaches. It is possible that a relationship between particular shape descriptors and groups of people (basing on age, gender, etc.) may be found. On the other hand, the weights for selected general shapes on first, second and third places can be aggregated, and the total joint measure of popularity for particular test object can be used, which may be very useful for ambiguous results provided by humans.

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