Control and Cybernetics

vol. **41** (2012) No. 2

Fuzzy global sensitivity analysis of fuzzy net present value*†

by

İrem Uçal Sarı[1] **and Dorota Kuchta**[2]

[1] Istanbul Technical University, Department of Industrial Engineering
Macka, Istanbul, 34367, Turkey
email: ucal@itu.edu.tr

[2] Wroclaw University of Technology, Institute of Organisation and Management
Wybrzeze Wyspanskiego 27, Wroclaw, Poland
email: dorota.kuchta@pwr.wroc.pl

Abstract: The cash flows of an investment project are influenced by several factors. All the factors influencing fuzzy net present value of a project should be taken into account, together with their variability, in order to analyze the whole project, and to make the best decisions during the project's life. In this paper a fuzzy global sensitivity analysis method is proposed for fuzzy net present value to determine the influences of the factors on the worth of an investment project. The results of the proposed method for an application are also interpreted.

Keywords: fuzzy sensitivity analysis, fuzzy net present value

1. Introduction

Investment decisions, which play important role in the survival of a company, are based on various assumptions due to the uncertainty resulting from the lack information on future data. There are different approaches to deal with this uncertainty, such as probabilistic, deterministic and fuzzy approaches. Deterministic approaches consider just one of the estimations of a parameter, being an insufficient information on uncertain data. Stochastic approaches are suitable when previous data are accessible and have a probabilistic well known distribution. When the analysts cannot get previous data, stochastic approaches became useless in defining uncertainty. Fuzzy approaches enable analysts to model linguistic variables and uncertainty.

*Submitted: June 2011; Accepted: March 2012

†This research paper was developed under the financial support of TUBITAK (The Scientific and Technological Research Council of Turkey)

Sensitivity analysis is an important technique for determining the effects of changes of factors on the system. It helps to find out which of the input variables have bigger influence on the output of the model. Especially, in uncertain and risky environments the robustness of the system is determined by sensitivity analysis.

Taylor and West (1992) designed an economic analysis and sensitivity evaluation model to assist investment decisions with multiple parameters. Jovanovic (1999) summarized the techniques used in investment analysis in uncertainty and applied sensitivity analysis on an example of investment decision making procedure. Van Groenendaal and Kleijnen (2002) compared two approaches of sensitivity analysis to determine which factors are important in the investment analysis. Borgonovo and Peccati (2004) determined the differential importance measure and discussed its relation to elasticity and other local sensitivity analysis techniques in the context of discounted cash flow valuation models. Borgonovo and Peccati (2006) used global sensitivity analysis techniques in investment decisions and proposed global importance formulas to determine key factors in net present value and internal rate of return of an investment. Xu et al. (2007) proposed an evaluation methodology based on voltage sensitivity and risk analysis from the perspective of voltage regulation to evaluate reactive power support services. Klingelhöfer (2009) proposed a general approach to valuating investments in end of pipe technologies and showed that tradable permits have several effects on an investment and do not always encourage environmentally beneficial investments by applying sensitivity analysis. Haahtela (2010) used sensitivity analysis on real option valuation to understand the importance of the key parameters. Borgonovo et al. (2010) proposed a methodology based on the differential importance measure to enhance the managerial insights obtained from financial models. Sheen (2005) derived fuzzy net present value and payback year models as decision indexes for cogeneration alternatives decision making by using sensitivity analysis to gain more information on uncertain data.

The purpose of the paper is to develop fuzzy global sensitivity analysis for fuzzy net present value to determine the influences of the factors on fuzzy net present value of an investment project in order to deal with uncertainty. In Section 2, basic knowledge on investment valuation analysis is given. The necessary information on fuzzy numbers and their algebraic operations is given in Section 3. The global sensitivity analysis is determined for crisp case and developed for fuzzy cases in Section 4 and 5, respectively. An application of fuzzy global sensitivity analysis is given in Section 6 and the paper is concluded with the discussions of results.

2. Net present value analysis

Although there are lots of investment analyses methods, net present value (NPV) analysis, which is one of the discounted cash flow analysis methods, is mostly preferred due to its easy applicability and well known procedure. NPV of a project is calculated by summation of present values of all positive and negative cash flows of the project.

Present value of a single future payment ($PV(F)$) after n years from now is calculated as:

$$PV(F) = F\frac{1}{(1+i)^n} \tag{2.1}$$

where F stands for the amount of the payment and i stands for the compound interest rate.

Net present value of a cash flow series with the same payments on each time period is calculated (2.2) where A stands for the amount of annual payment, n stands for the time period of the payments and I stands for initial investment of the project:

$$NPV(A,I) = -I + \sum_{j=1}^{n} \frac{(1+i)^j - 1}{i\,(1+i)^j} A. \tag{2.2}$$

The formula for the net present value of a cash flow series ($NPV\,(F_1, \dots F_m)$) with m different payments is given (2.3), where F_i stands for the amount of the payment and n_i stands for the time period of the payment. By formulation of net present value as in (2.3), it is easier to calculate the present value when the discount rates are different for different time periods:

$$NPV\,(I, F_1, \cdots F_m) = \; -I + \left(F_1 \tfrac{1}{(1+i)^{n_1}}\right) + \left(F_2 \tfrac{1}{(1+i)^{n_2}}\right) + \cdots \left(F_m \tfrac{1}{(1+i)^{n_m}}\right). \tag{2.3}$$

In (2.3) the cash flow is taken as a net cash flow in that time period for the calculation, but it contains more than one cash flow occurring during the same time period, such as costs and revenues which could be either negative or positive. Sometimes, especially if decision maker hesitates between accepting or rejecting the project due to its NPV and might change some of the cash flows by changing suppliers, materials or equipments, he/she prefers to know the portion of the cash flows resulting from different factors of project's NPV. For example, if the NPV is small, and cash outflows to one of the suppliers are a big portion of project's NPV, the outflows could be decreased by changing the supplier, or if cash inflows from one of the customers is a small portion of project's NPV, decision maker could try to find another customer which could increase the cash inflows. For the critical situations, factors of the cash flows can be determined as in (2.4), and present values of the factors (PV^k) could be calculated from (2.5)

$$F_1 = F_{11} + F_{12} + F_{13} + \ldots + F_{1k} \tag{2.4}$$

$$PV^k = \sum_{j=1}^{n} \frac{F_{jk}}{(1+i)^{n_j}}. \tag{2.5}$$

As shown in (2.7), NPV of the project is equal to the sum of the NPVs of the factors where PV^I is the present value of the initial investment given by (2.6) (Uçal and Kuchta, 2011):

$$PV^I = PV\,(I_0) + PV\,(I_1) + \cdots + PV(I_z) \tag{2.6}$$

$$NPV = -PV^I + PV^1 + PV^2 + PV^3 + \cdots + PV^k. \tag{2.7}$$

3. Global sensitivity analysis of NPV

Sensitivity analysis can be 'local' or 'global'. Local sensitivity is the sensitivity of model output when only one input factor is changed at a time while all other input factors are at their nominal values. Global sensitivity determines the effect on model output of all the uncertain input factors acting simultaneously over their ranges of uncertainty (Haaker and Verheijen, 2004).

Borgonovo and Peccati (2006) illustrated global sensitivity analysis in the uncertainty management of investment project evaluation. In that paper, non parametric and variance decomposition-based techniques are examined and global importance of a parameter by means of the complete decomposition of the model variance is estimated.

In global sensitivity analysis model of NPV, cash flows are assumed uncorrelated. NPV could be formulated as in (3.1), where F_j, which is the cash flow occurring in year j, is a random variable and i is the discount factor (for details see Borgonovo and Peccati, 2006):

$$Y = NPV\,(I, F_j) = -I + \sum_{j=1}^{n} \frac{F_j}{(1+i)^j}. \tag{3.1}$$

Now, (3.2) results from application of PEAR (Pearson correlation coefficient) to (3.1), where σ_{F_j} denotes standard deviation of F_j and σ_Y denotes standard deviation of NPV:

$$PEAR\,(F_j) = \frac{\sigma_{F_j}}{(1+i)^j\,\sigma_Y}. \tag{3.2}$$

$GI\,(F_j)$, which stands for the global importance of F_j for the NPV of a project, is calculated by (3.3) in the case of (3.1):

$$GI(F_j) \;= PEAR\,(F_j)^2. \tag{3.3}$$

The cash flow which has bigger global importance $GI(F_j)$ has bigger influence on possible changes of the NPV of the project. A change in this cash flow affects NPV more than the same amount of change in the other factors.

4. Fuzzy numbers

Zadeh founded the fuzzy set theory in Zadeh (1965). A fuzzy set is defined as a class of objects with a continuum of grades of membership, characterized by a membership function that assigns to each object a grade of membership ranging between zero and one. A fuzzy set A in U is characterized by a membership function $\mu_A(x)$ which associates with each point in U a real number from $[0,1]$, with the value of $\mu_A(x)$ representing the grade of membership of x in A.

A formula for a membership function $\mu_{\tilde{A}}(x)$ of a fuzzy number $\tilde{A}$, where a, b and c denote real numbers (Ross, 1995) is:

$$\mu_{\tilde{A}}(x) = \mu_{\tilde{A}}(x; a, b, c) = \begin{cases} \frac{x-a}{b-a}; & a \le x \le b \\ \frac{c-x}{c-b}; & b \le x \le c \\ 0; & x \ge c \text{ or } x \le a \end{cases} . \tag{4.1}$$

Dubois and Prade (1978) proposed the LR representation of fuzzy numbers. A function, usually denoted L or R, is a reference function of fuzzy numbers iff $L(x) = L(-x)$, $L(0) = 1$ and L is nonincreasing on $[0, +\infty)$. A fuzzy number $\tilde{M}$ is said to be an $L - R$ type fuzzy number iff it satisfies (4.2).

$$\mu_{\tilde{M}}(x) = \begin{cases} L((m-x)/\alpha), & \text{for} \quad x \le m,\ \alpha > 0, \\ R((x-m)/\beta), & \text{for} \quad x \ge m,\ \beta > 0. \end{cases} \tag{4.2}$$

In (4.2), L represents left reference and R represents right reference, m is the mean value of $\tilde{M}$, α and β are called left and right spreads, respectively.

A triangular fuzzy number (TFN) is a special type of L − R fuzzy numbers, with linear reference functions on both sides. TFN is one of the most frequently used fuzzy numbers because of its simple membership function. Hanns (2005) defined the membership function for a TFN $\tilde{M} = (m_l, m_m, m_r)$ as:

$$\mu_{\tilde{M}}(x) = \begin{cases} 0 & x \le m_l \\ 1 + \frac{x-m_m}{m_m-m_l} & m_l < x < m_m \\ 1 - \frac{x-m_m}{m_r-m_m} \quad \text{if} & m_m \le x < m_r \\ 0 & x \ge m_r \end{cases} \tag{4.3}$$

4.1. Fuzzy operations

Algebraic operations for *TFN*s $\tilde{M} = (m_l, m_m, m_r)$ and $\tilde{N} = (n_l, n_m, n_r)$ $\tilde{p} = (\bar{x}, \bar{x} - \alpha_l, \bar{x} + \alpha_r) = (a, b, c)$are given by following equations, respectively summation, subtraction, multiplication, division and multiplication by a scalar (Chen et al., 1992):

$$\tilde{M} \oplus \tilde{N} \tilde{=} (m_l + n_l, m_m + n_m, m_r + n_r) \tag{4.4}$$

$$\tilde{M} \ominus \tilde{N} \tilde{=} (m_l - n_r, m_m - n_m, m_r - n_l) \tag{4.5}$$

$$\tilde{M}\otimes\tilde{N}\cong\begin{cases}(m_l n_l, m_m n_m, m_r n_r) & (m_l, m_m, m_r)\geq 0, (n_l, n_m, n_r)\geq 0\\ (m_l n_r, m_m n_m, m_r n_l) \quad \text{if} & (m_l, m_m, m_r)\leq 0, (n_l, n_m, n_r)\geq 0\\ (m_r n_r, m_m n_m, m_l n_l) & (m_l, m_m, m_r)\leq 0, (n_l, n_m, n_r)\leq 0\end{cases} \tag{4.6}$$

$$\tilde{M}\oslash\tilde{N}\cong\begin{cases}\left(\frac{m_l}{n_r}, \frac{m_m}{n_m}, \frac{m_r}{n_l}\right) & (m_l, m_m, m_r)\geq 0, (n_l, n_m, n_r)\geq 0\\ \left(\frac{m_r}{n_r}, \frac{m_m}{n_m}, \frac{m_l}{n_l}\right) \quad \text{if} & (m_l, m_m, m_r)\leq 0, (n_l, n_m, n_r)\geq 0\\ \left(\frac{m_r}{n_l}, \frac{m_m}{n_m}, \frac{m_l}{n_r}\right) & (m_l, m_m, m_r)\leq 0, (n_l, n_m, n_r)\leq 0\end{cases} \tag{4.7}$$

$$\lambda\otimes\tilde{M}\cong\begin{cases}(\lambda m_l, \lambda m_m, \lambda m_r) & \text{if} \quad \lambda\geq 0\\ (\lambda m_r, \lambda m_m, \lambda m_l) & \phantom{\text{if}} \quad \lambda\leq 0\end{cases} \quad \forall\lambda\in R \quad . \tag{4.8}$$

A γ-cut of a fuzzy number $\tilde{A}$, one of the most important concepts of fuzzy sets, symbolized as A^γ is determined as a crisp set that contains all the elements of the universal set X whose membership grades in $\tilde{A}$ are greater than or equal to the value of γ (Klir & Yuan, 1995):

$$A^\gamma = \{x \,|\, A(x) \geq\propto\}\,. \tag{4.9}$$

A fuzzy number $\tilde{A}$ can be expressed as a family of A^γs for $\gamma\epsilon\,[0,1]$:

$$\tilde{A} = \{A^\gamma\}_{\gamma\epsilon[0,1]}\,. \tag{4.10}$$

For each $\gamma\epsilon[0,1]$ we can consider $\underline{A}^\gamma = \inf\,\{x\in A^\gamma\}$and $\overline{A}^\alpha = \sup\{x\in A^\gamma\}$. If the fuzzy number $\tilde{A}$ represents a magnitude whose higher values are considered better (e.g. profit), than an optimist would assume, on the γ-level of possibility, that rather the values closer to (and of course not greater than) $\overline{A}^\gamma$ will occur, the pessimist would rather suspect that values closer to (and of course not smaller than) $\underline{A}^\gamma$ will be the actual values of the magnitude represented by $\tilde{A}$. In case the fuzzy number $\tilde{A}$ represents a magnitude that should be as small as possible (e.g. cost), the expectations of the pessimist and optimist would be exactly the other way round (Uçal and Kuchta, 2011).

4.2. Variance of fuzzy numbers

There are several approaches to calculate an equivalent of variance in the random case. In this paper we will use fuzzy numbers and a crisp equivalent of variance for fuzzy numbers.

Carlsson and Fuller (2001) defined variance of a triangular fuzzy number $(\tilde{A} = (A_l A_m A_r)$ as:

$$Var\,(A) = \sigma_A^2 = \frac{1}{2}\int_0^1 \gamma((A_m - A_l)\,\gamma + (A_l - A_r) + (A_r - A_m)\,\gamma)^2$$
$$= \frac{(A_r - A_l)^2}{24} \tag{4.11}$$

where σ_A denotes standard deviation of $\tilde{A}$ and γ denotes the γ- cut level.

4.3. Ranking fuzzy numbers

In the literature, many methods are proposed to compare fuzzy numbers (Chen et al., 1992). Each method is different, because it is based on other features and preferences of the decision maker. In this paper, we need a ranking method to compare fuzzy numbers, as we will compare fuzzy global importance values of the cash flows to decide which cash flow has bigger influence on fuzzy net present value of the project.

We will use one of the simplest ranking methods, proposed by Chiu and Park (1994) to order fuzzy numbers according to their preference values. The preference value of a fuzzy number is given in (4.12), where A_{CP} denotes the preference value of the fuzzy number $\tilde{A} = (A_l A_m A_r)$, A_l is the lowest possible value of $\tilde{A}$, A_m is the most promising value (the mean) of $\tilde{A}$, A_r is the highest possible value of $\tilde{A}$, and w denotes the weight determined by the magnitude of the most promising value. If the magnitude of the most promising value is important, a higher weight, such as $w = 0.3$ is recommended, otherwise smaller weight such as $w = 0.1$ is recommended.

$$A_{CP} = \left(\frac{A_l + A_m + A_r}{3}\right) + wA_m. \tag{4.12}$$

5. Fuzzy net present value analysis

Fuzzy present value of a single future payment $(\tilde{PV}(F))$, occurring at the end of the n^{th} year from now is given in (5.1) where $\tilde{F}$ stands for fuzzy payment and i for the compound interest rate:

$$\widetilde{PV}(\tilde{F}) = \frac{\tilde{F}}{(1+i)^n}. \tag{5.1}$$

Kuchta (2000) defined the general formula for fuzzy net present value as in (5.2), where $\tilde{F}_i$ denotes net cash flows in time period i and $\tilde{\imath}$ denotes the fuzzy interest rate:

$$\widetilde{NPV} = -\tilde{I} + \sum_{i=0}^{n} \frac{\tilde{F}_i}{(1+\tilde{\imath})^i}. \tag{5.2}$$

The formula of fuzzy net present value of a project $(\widetilde{NPV})$, which has m different payments and has an initial investment at the beginning of the project is given by (5.3) if the discount rates are different for different cash flows.

$$\widetilde{NPV} = -\tilde{I} + \left(\tilde{F}_1 \frac{1}{(1+i)^{n_1}}\right) + \left(\tilde{F}_2 \frac{1}{(1+i)^{n_2}}\right) + \dots \left(\tilde{F}_m \frac{1}{(1+i)^{n_m}}\right). \tag{5.3}$$

The $\widetilde{NPV}$ of a project which has m different payments and has an initial investment distributed over z years is given by:

$$\widetilde{NPV} = -\tilde{I}_0 - \left(\tilde{I}_1 \frac{1}{(1+i)^{n_1}}\right) - \left(\tilde{I}_2 \frac{1}{(1+i)^{n_2}}\right) - \cdots - \left(\tilde{I}_z \frac{1}{(1+i)^{n_z}}\right)$$
$$+ \left(\tilde{F}_1 \frac{1}{(1+i)^{n_1}}\right) + \left(\tilde{F}_2 \frac{1}{(1+i)^{n_2}}\right) + \cdots + \left(\tilde{F}_m \frac{1}{(1+i)^{n_m}}\right). \tag{5.4}$$

Now, (5.5) gives the fuzzy cash flow defined by its components due to factors. To sort the factors of cash flows, the fuzzy net present value of a factor is given in (5.6).

$$\tilde{F}_1 = \tilde{F}_{11} + \tilde{F}_{12} + \tilde{F}_{13} + \cdots + \tilde{F}_{1k} \tag{5.5}$$

$$\widetilde{PV}^k = \sum_{j=1}^{n} \frac{\tilde{F}_{jk}}{(1+i)^j}. \tag{5.6}$$

As shown in (5.8), $\widetilde{NPV}$ of a project is equal to the sum of $\widetilde{PV}$s of factors, because of the linearity of $\widetilde{NPV}$ and the definition of the addition of fuzzy numbers, where $\widetilde{PV}^I$ denotes the present value of initial investment given in (5.7) (Uçal Sarı and Kuchta, 2011):

$$\widetilde{PV}^I = \widetilde{PV}(I_0) + \widetilde{PV}(I_1) + \cdots + \widetilde{PV}(I_z) \tag{5.7}$$

$$\widetilde{NPV} = -\widetilde{PV}^I + \widetilde{PV}^1 + \widetilde{PV}^2 + \widetilde{PV}^3 + \cdots + \widetilde{PV}^k. \tag{5.8}$$

6. Fuzzy global sensitivity analysis for fuzzy net present value

$\widetilde{NPV}$ can be formulated as in (6.1) where $\tilde{F}_j$ is the fuzzy cash flows of the year j and $\tilde{i}$ is the fuzzy interest rate, and the Pearson correlation coefficient of $\tilde{F}_j$ $\left(PEAR\left(\tilde{F}_j\right)\right)$on NPV is formulated as in (6.2) It is assumed that individual cash flows $(\tilde{F}_j)$ do not depend on other cash flows.

$$\tilde{Y} = f\left(\tilde{F}_j\right) = \sum_{j=1}^{n} \frac{\tilde{F}_j}{(1+\tilde{\imath})^j} \tag{6.1}$$

$$\widetilde{PEAR}\left(\tilde{F}_j\right) = \frac{\sigma_{F_j}}{(1+\tilde{\imath})^j \, \sigma_{NPV}}. \tag{6.2}$$

The upper, most promising and lower values of the TFN $\widetilde{PEAR}\left(\tilde{F}_j\right)$, respectively, are:

$$PEAR\left(\tilde{F}_j\right)_r = \frac{\sigma_{F_j}}{(1+i_l)^j \, \sigma_Y} \tag{6.3}$$

$$PEAR\left(\tilde{F}_j\right)_m = \frac{\sigma_{F_j}}{(1+i_m)^j \sigma_y} \tag{6.4}$$

$$PEAR\left(\tilde{F}_j\right)_l = \frac{\sigma_{F_j}}{(1+i_r)^j \, \sigma_Y}. \tag{6.5}$$

Global importance of fuzzy cash flows $\tilde{F}_j$ on NPV of the investment is formulated as:

$$GI(\tilde{F}_j) = PEAR\left(\tilde{F}_j\right)^2 = \frac{\sigma_{F_j}^2}{(1+i_l)^{2j} \, \sigma_Y^2} = \frac{(F_{j_r} - F_{j_l})^2}{(NPV_r - NPV_l)^2 \, (1+i_l)^{2j}}. \tag{6.6}$$

The lower, most promising and upper values of global importance of a fuzzy cash flow from year j on $\widetilde{NPV}$, are, respectively:

$$GI(\tilde{F}_j)_r = PEAR\left(\tilde{F}_j\right)_r^2 = \frac{\sigma_{F_j}^2}{(1+i_l)^{2j} \, \sigma_Y^2} = \frac{(F_{j_r} - F_{j_l})^2}{(NPV_r - NPV_l)^2 \, (1+i_l)^{2j}} \tag{6.7}$$

$$GI(\tilde{F}_j)_m = PEAR\left(\tilde{F}_j\right)_m^2 = \frac{\sigma_{F_j}^2}{(1+i_m)^{2j} \, \sigma_{NPV}^2} = \frac{(F_{j_r} - F_{j_l})^2}{(NPV_r - NPV_l)^2 \, (1+i_m)^{2j}} \tag{6.8}$$

$$GI(\tilde{F}_j)_l = PEAR\left(\tilde{F}_j\right)_l^2 = \frac{\sigma_{F_j}^2}{(1+i_r)^{2j} \, \sigma_{NPV}^2} = \frac{(F_{j_r} - F_{j_l})^2}{(NPV_r - NPV_l)^2 \, (1+i_{j_r})^{2j}}. \tag{6.9}$$

By using GI values of the project's cash flows, the decision maker knows which cash flows could change the worth of the project more than the others. The cash flow, which has the biggest global importance has higher influence on the changes of the $\widetilde{NPV}$ than the other cash flows. A decision maker may decide on the strategies as to the time when the most influencing cash flow takes place.

By using (6.9), the global importance values of the factors can also be calculated. The ranking of the global importance values of the factors gives the decision maker an idea as to which factor affects most the $\widetilde{NPV}$ of the project. For example, if the demand of the project is uncertain and more risky than its other parameters, decision maker could try to make demand more stable by making advertisements or establishing special campaigns to their customers.

7. Application

A company who has five customers and three suppliers wants to invest on a new project. The first supplier delivers materials for the production for Customers

1 and 2, the second supplier delivers materials for the production for Customer 3, and the third supplier delivers materials for the production for Customers 4 and 5. The cash flows from customers are given in Table 1. The project has five years of useful life and there will be no salvage value of the assets after five years. The interest rate is taken as $\tilde{\imath} = (8, 10, 12)\,\%$.

The cash flows due to suppliers are calculated by taking the net cash flows, linked with each supplier and given in Table 2. The present value of the sum of the cash flows due to customers is of course equal to the present value of the sum of the cash flows due to suppliers.

Global importance and preference values of the cash flows of each factor are given in Table 3.

Usually, the decision makers want to decrease the uncertainty of $\widetilde{NPV}$. Less uncertainty of the $\widetilde{NPV}$ means smaller variance values of the $\widetilde{NPV}$. For this purpose, the cash flow which has the biggest influence on the changes of the $\widetilde{NPV}$ should be treated as more important and the decision maker has to pay attention to this cash flow more than to the others.

When the total cash flows of the project is examined, the cash flows taking place in the 4$^{\text{th}}$ year has the greatest global importance on the changes of $\widetilde{NPV}$, so, if it is possible, the strategies which could decrease the variance of the cash flows taking place in the 4$^{\text{th}}$ year have to be applied. If it is not possible for some reasons, the strategies which could decrease the variance of the cash flows taking place in the 3$^{\text{rd}}$ year have to be applied to decrease uncertainty of $\widetilde{NPV}$.

If the cash flows are examined due to their sources, it is found that the cash flows from the 4$^{\text{th}}$ year of the project have the biggest global importance on $\widetilde{NPV}$ for both customers and suppliers. If it is possible, the strategies which could decrease the variance of the cash flows from the 4$^{\text{th}}$ year have to be applied, if not - the strategies which could decrease the variance of the cash flows from the 3$^{\text{rd}}$ year have to be applied for Customers 1, 2, 3 and 4, and Suppliers 1 and 2, and the strategies which could decrease the variance of the cash flows from the 5$^{\text{th}}$ year have to be applied for Customer 5 and Supplier 3.

Another interpretation for the cash flows could be obtained from global importance values of the factors (in this case suppliers and customers) for each year. Assume that the company has different strategies for each year, which could affect the cash flows in that year. For the first and second years, the decision maker should apply the strategies which could decrease the variance of the cash outflows of Supplier 3 or the variance of the cash inflows of Customer 3. If they are not applicable, the strategies which could decrease the variance of the cash outflows of Supplier 1 or the variance of the cash inflows of Customer 5 should be applied. For the third year, the decision maker should apply the strategies which could decrease the variance of the cash outflows of Supplier 3 or the variance of cash inflows of Customer 3. If they are not applicable, the strategies which could decrease the variance of the cash outflows of Supplier 2 or

Table 1. Cash flows of the project for each customer and total cash flows of the project

YEAR	CASH FLOWS FOR C1			CASH FLOWS FOR C2			CASH FLOWS FOR C3		
	left	**medium**	**right**	**left**	**medium**	**right**	**left**	**medium**	**right**
0	0	0	0						
1	-243667	-161333	-90667	-86667	-53333	-20000	-496000	-395000	-294000
2	90167	209167	313167	102167	138833	175500	-81667	63333	208333
3	484167	604833	742167	301000	357167	413333	526667	746667	966667
4	821000	972000	1123000	530167	598500	666833	1043000	1300000	1557000
5	840000	962000	1084000	532500	600000	657500	938000	1190000	1442000
$\widetilde{PV}$	**1197340**	**1741834**	**2336886**	**857397**	**1115934**	**1397688**	**1062001**	**1881046**	**2799604**
YEAR	**CASH FLOWS FOR C4**			**CASH FLOWS FOR C5**					
	left	**medium**	**right**	**left**	**medium**	**right**			
0	0	0	0	0	0	0			
1	-60000	4000	68000	-260000	-176333	-92667			
2	140000	210000	280000	216667	303667	390667			
3	452500	590000	737500	608000	778333	948667			
4	772500	930000	1087500	954333	1253333	1552333			
5	785000	950000	1115000	965333	1227667	1490000			
$\widetilde{PV}$	**1316484**	**1845544**	**2446664**	**1527597**	**2293762**	**3157294**			

Table 2. Cash flows of the project for each supplier

YEAR	**CASH FLOWS FOR S1**			**CASH FLOWS FOR S2**			**CASH FLOWS FOR S3**		
	left	**medium**	**right**	**left**	**medium**	**right**	**left**	**Medium**	**right**
0	0	0	0	0	0	0	0	0	0
1	-330333	-214667	-110667	-496000	-395000	-294000	-320000	-172333	-24667
2	192333	348000	488667	-81667	63333	208333	356667	513667	670667
3	785167	962000	1155500	526667	746667	966667	1060500	1368333	1686167
4	1351167	1570500	1789833	1043000	1300000	1557000	1726833	2183333	2639833
5	1372500	1562000	1741500	938000	1190000	1442000	1750333	2177667	2605000
$\tilde{PV}$	**2054737**	**2857768**	**3734573**	**1062001**	**1881046**	**2799604**	**2844081**	**4139306**	**5603958**

Table 3. GI and preference values of the cash flows for the different factors

Year	GI **of total cash flows for NPV (x10^{-2})**					GI **of Customer 1 for NPV (x10^{-2})**				
	left	**mean**	**right**	**Pre. Val.**	**Importance Order**	**left**	**mean**	**right**	**Pre. Val.**	**Importance Order**
0	0.025	0.025	0.025	0.0325	6	0	0	0	0	
1	1.198	1.243	1.288	1.6294	5	0.046	0.048	0.0499	0.0630	5
2	1.339	1.439	1.549	1.9070	4	0.078	0.084	0.0910	0.1116	3
3	2.673	2.978	3.324	3.9889	2	0.084	0.093	0.1044	0.1251	2
4	3.498	4.040	4.679	5.4760	1	0.092	0.106	0.1226	0.1437	1
5	2.391	2.864	3.440	3.9303	3	0.048	0.057	0.0686	0.0785	4
Year	GI **of Customer 2 for NPV (x10^{-2})**					GI **of Customer 3 for NPV (x10^{-2})**				
	left	**mean**	**right**	**Pre. Val.**	**Importance Order**	**left**	**mean**	**right**	**Pre. Val.**	**Importance Order**
0	0	0	0	0		0	0	0	0	
1	0.009	0.009	0.009	0.0117	5	0.081	0.084	0.087	0.1101	5
2	0.008	0.009	0.010	0.0120	4	0.133	0.143	0.154	0.1895	4
3	0.016	0.018	0.020	0.0240	2	0.244	0.272	0.304	0.3643	2
4	0.019	0.022	0.025	0.0295	1	0.265	0.307	0.355	0.4155	1
5	0.012	0.015	0.018	0.0204	3	0.203	0.244	0.293	0.3345	3
Year	GI **of Customer 4 for NPV (x10^{-2})**					GI **of Customer 5 for NPV (x10^{-2})**				
	left	**mean**	**right**	**Pre. Val.**	**Importance Order**	**left**	**mean**	**right**	**Pre. Val.**	**Importance Order**
0	0	0	0	0		0	0	0	0	
1	0.032	0.034	0.035	0.0442	4	0.055	0.058	0.059	0.0750	5
2	0.031	0.033	0.036	0.0441	5	0.048	0.051	0.055	0.0678	4
3	0.102	0.114	0.127	0.1524	2	0.146	0.163	0.182	0.2183	3
4	0.099	0.115	0.133	0.1556	1	0.359	0.415	0.481	0.5626	1
5	0.087	0.104	0.125	0.1428	3	0.220	0.264	0.317	0.3621	2

Table 4. Table 3. continued

Year	GI of Supplier 1 for NPV ($\mathrm{x}10^{-2}$)					GI of Supplier 2 for NPV ($\mathrm{x}10^{-2}$)				
	left	**mean**	**right**	**Pre. Val.**	**Importance Order**	**left**	**mean**	**right**	**Pre. Val.**	**Importance Order**
0	0	0	0	0		0	0	0	0	
1	0.096	0.099	0.103	0.1302	5	0.081	0.084	0.087	0.1101	5
2	0.139	0.149	0.161	0.1980	3	0.133	0.143	0.154	0.1895	4
3	0.173	0.193	0.215	0.2582	2	0.244	0.272	0.304	0.3645	2
4	0.193	0.223	0.259	0.3027	1	0.265	0.307	0.355	0.4155	1
5	0.109	0.131	0.157	0.1794	4	0.203	0.244	0.293	0.3346	3

Year	GI of Supplier 3 for NPV ($\mathrm{x}10^{-2}$)				
	left	**mean**	**right**	**Pre. Val.**	**Importance Order**
0	0	0	0	0	
1	0.173	0.179	0.186	0.2351	4
2	0.156	0.168	0.,180	0.2220	5
3	0.493	0.550	0.614	0.7365	3
4	0.838	0.968	1.121	1.3120	1
5	0.585	0.701	0.842	0.9620	2

the variance of the cash inflows of Customer 5 should be applied to decrease the uncertainty of $\widetilde{NPV}$ of the project. For the fourth and fifth years, the decision maker should apply the strategies which could decrease the variance of the cash outflows of Supplier 3 or the variance of the cash inflows of Customer 5. If they are not applicable, the strategies which could decrease the variance of the cash outflows of Supplier 2 or the variance of the cash inflows of Customer 3 should be applied.

8. Discussion

In this paper, the global importance values of fuzzy cash flows for fuzzy net present value of a project are formulated by defining cash flows and interest rates as fuzzy numbers and the variance of the fuzzy numbers by their crisp equivalences. With this formulation it is easy to understand the effects of individual cash flows on the worth of a project and to determine the prior cash flow factors of the project. The global importance method gives a prioritization of the cash flows to improve the worth of the investment projects.

For a deeper study of information on uncertainty, the global importance values of fuzzy cash flows for fuzzy net present value of a project could be generated by using fuzzy variances of fuzzy numbers.

References

Borgonovo, E., Gatti, S., Peccati, L. (2010) What drives value creation in investment projects? An application of sensitivity analysis to project finance transactions. *European Journal of Operational Research*, **205**,1, 227-236.

Borgonovo, E., Peccati, L. (2004) Sensitivity analysis in investment project evaluation. *International Journal of Production Economics*, **90**,1, 17-25.

Borgonovo, E., Peccati, L. (2006) Uncertainty and global sensitivity analysis in the evaluation of investment projects. *International Journal of Production Economics*, **104**,1, 62-73.

Carlsson, C. and Fuller, R. (2001) On possibilistic mean value and variance of fuzzy numbers. *Fuzzy Sets and Systems*, **122**, 315-326.

Chen, S.J., Hwang, C.L., Hwang, F.P. (1992) *Fuzzy Multiple Attribute Decision Making: Methods and Applications*. Springer, Berlin.

Dubois, D., Prade, H. (1978) Operations on fuzzy numbers. *International Journal of Systems Science*, **9**, 613-626.

Haahtela, T.J. (2010) Regression sensitivity analysis for cash flow simulation based real option valuation. *Procedia - Social and Behavioral Sciences*, **2**,6, 7670-7671.

Haaker, M.P.R., Verheijen, P.J.T. (2004) Local and Global Sensitivity Analysis for a Reactor Design with Parameter Uncertainty. *Chemical Engineering Research and Design*, **82**,5, 591-598.

HANNS, M. (2005) *Applied Fuzzy Arithmetics. An Introduction with Engineering Applications*. Springer.

JOVANOVIC, P. (1999) Application of sensitivity analysis in investment project evaluation under uncertainty and risk. *International Journal of Project Management*, **17**,4, 217-222.

KLINGELHÖFER, H.E. (2009) Investments in EOP-technologies and emissions trading – Results from a linear programming approach and sensitivity analysis. *European Journal of Operational Research*, **196**,1, 370-383.

ROSS, T.J. (1995) *Fuzzy Logic With Engineering Applications*. Mc-Graw Hill.

SHEEN, J.N. (2005) Fuzzy evaluation of cogeneration alternatives in a petrochemical industry. *Computers & Mathematics with Applications*, **49**, 741-755.

TAYLOR, R.B., WEST, T.M. (1992) Development of a spreadsheet-based capital investment model with sensitivity analysis. *Computers & Industrial Engineering*, **23**,1-4, 431-433.

UÇAL SARI, İ. and KUCHTA, D. (2011) Multicriteria Evaluation of Fuzzy Net Present Value. In: T. Trzaskalik, ed., *Multiple Criteria Decision Making'12*. Publishing House of the University of Economics in Katowice, Katowice.

VAN GROENENDAAL, W.J.H. and KLEIJNEN, J.P.C. (2002) Deterministic versus stochastic sensitivity analysis in investment problems: an environmental case study. *European Journal of Operational Research* **141**(1), 8-20.

XU, N., YU, C.W., WEN, F.S. (2007) Valuation of reactive power support services based on sensitivity and risk analysis. *Electric Power Systems Research*, **77**,5-6, 646-651.

ZADEH, L.A. (1965) Fuzzy sets. *Information and Control*, **8**, 338-353.