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Soft set mappings and their properties*

by

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Abstract: In the present paper we introduce the concept of soft set mapping. Next, we present some basic properties of such mappings.

Keywords: soft set, soft set mapping, conical, super-additive, closed soft set mappings

1. Introduction

Many complicated problems in economics, engineering, social sciences, medical sciences, etc. involve data containing uncertainties. These uncertainties cannot be handled using traditional mathematical tools. Very often mathematical tools from probability theory, fuzzy set theory (Zadeh, 1965), rough set theory (Pawlak, 1982), intuitionistic fuzzy sets (Atanassov, 1986) and interval mathematics (Gorzałczany, 1987) are useful in describing uncertainty. As pointed out by Molodtsov (Molodtsov, 1999) each of these theories has its inherent difficulties. Consequently, he initiated the concept of soft set theory as a mathematical tool, free of the problems affecting the existing methods. In the paper referred to, Molodtsov successfully applied soft set theory in several directions, such as smoothness of functions, operations research, Riemann integration, game theory, theory of probability and so on. Maji, Biswas and Roy (2003) defined several basic notions of soft set theory and presented an application of soft set theory in combination with rough sets in a decision making problem. After the work of Molodtsov some different applications of soft set theory were studied (see, for example, Chen, Tsang, Yeung and Wang, 2005; Maji and Roy, 2002; Xiao, Li, Zhong and Yang, 2003). Other authors investigated the theory of soft sets. Thus, e.g., Feng et al. (2008) initiated the study of soft semirings, Sun et al. (2008) the study of soft module theory. Kong et al. (2008) defined a normal parameter reduction of soft set and proposed a heuristic algorithm for normal

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parameter reduction. Shabir and Naz (2011) introduced topological spaces and discussed some basic properties of such spaces.

Molodtsov defined the soft set as a pair $(F,\ E)$, where F is a mapping from E into the power set $P(U)$ and E is a set of parameters. This means that soft set is a parameterized family of subsets of the universe U. We can study relations between soft sets and fuzzy sets, topological space and rough sets. A fuzzy set is defined by its membership function whose values belong to the closed interval [0, 1]. So, the family of α-level sets for the membership function may be considered as a soft set (see Molodtsov, 1999). In the same paper Molodtsov shows that the family of open neighbourhoods $T(x)$ of a point x, where $T(x) = \{V \in \tau\colon\ x \in V\ \}$ and (X,τ) is topology, may be considered as a soft set. In 1982 Pawlak introduced rough set theory using equivalence classes to approximate crisp sets. This theory has been applied to many fields, such as machine learning, data mining, data analysis, medicine and expert systems (see, for example, Pawlak, 1981; Pawlak and Skowron, 1994; Skowron and Stepaniuk, 1996; Walczak and Massart, 1999). Aktas and Cağman (2007) presented a comparison of soft sets with rough sets. They proved that every rough set may be considered as a soft set. They considered a rough set $R(x)$ of X in the universe U, with respect to the equivalence relation R. The rough set of X is defined by an R-upper approximation $R^*(X)$, and R-lower approximation $R_*(X)$, and the equivalence relation R. Next, they considered the predicates $p_1\,(x)$, which stands for "$[x]_R \subseteq X$", and $p_2\,(x)$, which stands "$[x]_R \cap X \neq \emptyset$". The conditions $p_1\,(x)$ and $p_2\,(x)$ may be treated as elements of a parameter set; that is $E = \{p_1\,(x)\,,\,p_2\,(x)\}$. Then we can write the function $F\,:\,E \to P(U)$, $F\,(p_i\,(x)) = \{x \in U\,:\,p_i\,(x)\ is\,true\}$, $i = 1, 2$. Thus, every rough set $R\,(X)$ of X may also be considered as a soft set.

The main purpose of this paper is to introduce soft set mappings. Set valued mappings (multifunctions) have many diverse and interesting applications in control problems and theory of contingent equations, in mathematical economics and in various branches of analysis. In the models of economic dynamics, theory of multifunctions plays the central role. Mutifunctions were interpreted as certain technological transformations assigning a set of commodities to a set of production factors. It is assumed that the producer follows a precise behaviour pattern, by this we mean that the producer has complete information concerning the condition of this productive activity and has perfect command over both the set of inputs and the set of outputs. He realizes the maximum profit allowed by the technological constraint, which limits possible actions and by the given price system. In practice, the result of a production process is by its nature imprecise. It follows that a technically possible production is more or less efficient. The above mentioned situation is difficult to describe but a soft set mapping seems to be a very useful tool in this respect.

This paper is organized as follows. Section 2 presents basic definitions of soft sets. In Section 3, we propose a definition of soft set mapping and present some basic properties of soft set mappings necessary for further considerations on economic systems.

2. Preliminaries and basic definitions

Let U be an initial universe set and let E be a set of parameters. Let $P(U)$ denote the power set of U.

Definition 2.1. (Molodtsov, 1999). A pair $(F,\ E)$ is called a soft set (over U) if and only if $F$is a mapping from E into $P(U)$.

In other words, a soft set is a parameterized family of subsets of the universe U. For $e \in E$, $F(e)$ may be considered as the set of $e-$approximate elements of the soft set $(F,\ E)$.

Definition 2.2. For two soft sets $(F,\ E_1)$ and $(G,\ E_2)$ over a common universe U, we say that $(F,\ E_1)$ is a soft subset of $(G,\ E_2)$ if

(i) $E_1 \subseteq E_2$ and

(ii) for any $e \in E_1$, $F(e) \subseteq G(e)$.

We write $(F,\ E_1)\tilde{\subset}(G,\ E_2)$.

Definition 2.3. Two soft sets $(F,\ E_1)$ and $(G,\ E_2)$ over a common universe U are said to be soft equal if $(F,\ E_1)\tilde{\subset}(G,\ E_2)$ and $(G,\ E_2)\tilde{\subset}(F,\ E_1)$.

We write $(F,\ E_1)\tilde{=}(G,\ E_2)$.

Definition 2.4. (Maji, Biswas, Roy, 2003). The union of two soft sets $(F,\ E_1)$ and $(G,\ E_2)$over the common universe U is the soft set $(H,\ E)$, where $E = E_1 \cup E_2$ and for any $e \in E$

$$H(e) = \begin{cases} F(e) & \text{if } e \in E_1 - E_2 \\ G(e) & \text{if } e \in E_2 - E_1 \\ F(e) \cup G(e) & \text{if } e \in E_1 \cap E_2. \end{cases}$$

We write $(F,\ E_1)\tilde{\cup}(G,\ E_2) = (H,\ E)$.

Definition 2.5. (Pei, Miao, 2005). The intersection of two soft sets $(F,\ E_1)$ and $(G,\ E_2)$ over a common universe U is the soft set $(H,\ E)$, where $E = E_1 \cap E_2$, and for any $e \in E$, $H(e) = F(e) \cap G(e)$.

We write $(F,\ E_1)\tilde{\cap}(G,\ E_2) = (H,\ E)$.

If for any $e \in E$, $F(e) = \{x(e)\}$ then such a soft set will be denoted $(x,\ E)$ and called soft point.

We say that a soft point $(x,\ E)$ belongs to the soft set $(F,\ E)$ if for any $e \in E$, $x(e) \in F(e)$. We write $(x,\ E)\tilde{\in}(F,\ E)$.

Now, assume that U is a linear space with scalar multiplication by real numbers.

Definition 2.6. The sum of two soft sets $(F,\ E_1)$ and $(G,\ E_2)$ over the common universe U is the soft set $(H,\ E)$, where $E = E_1 \cap E_2$ for all $e \in E$, $H(e) = F(e) + G(e)$.

We write $(H,\ E)\tilde{=}(F, E_1\ )\tilde{+}(G,\ E_2)$.

Definition 2.7. For any soft set $(F,\ E)$ and any real number λ we define a multiplication $\lambda\tilde{\cdot}(F,\ E)$ as the soft set $(H,\ E)$ such that for any $e \in E$, $H(e) = \lambda \cdot F(e)$.

Definition 2.8. A soft set $(F,\ E)$ is called convex iff for any $e \in E$, $F(e)$ is a convex set.

3. Soft set mappings

Let $U_i (i = 1,\ 2,\ 3)$ denote arbitrary, but for further considerations fixed reference spaces and let E_i denote the sets of parameters and $P(E_i)$ the family of subsets of E_i. Let T_i be the collection of soft sets over U_i with the set of parameters from $P(E_i)$.

Definition 3.1. A soft set mapping, say $a\ :\ U_1 \times E_1 \to \mathrm{T}_2$, is a mapping from $U_1 \times E_1$ to T_2.

This means that for any $(x,\ e_1) \in U_1 \times E_1$, $a\,((x,\ e_1))$ is a soft set from T_2.

For any soft point $(x,\ E_1)\tilde{\in}(F_1,\ E_1) \in \mathrm{T}_1$ we put

$$a\,((x,\ E_1)) = \tilde{\bigcap_{e_1 \in E_1}}\ a\,((x(e_1),\ e_1))$$

and

$$a\,((F_1,\ E_1)) = \tilde{\bigcup_{(x,\ E_1)\tilde{\in}(F_1,\ E_1)}}\ a\,((x,\ E_1)).$$

From the above definition it follows that: if for any $e \in E_1 \quad G_1(e) \subseteq F_1(e)$ then $a\,((G_1,\ E_1))\,\tilde{\subset} a\,((F_1,\ E_1))$.

Definition 3.2. The graph of a soft set mapping $a\ :\ U_1 \times E_1 \to \mathrm{T}_2$ is the set

$$W_a = \{((x,\ e_1),\ (y,\ e_2))\ :\ e_1 \in E_1,\ e_2 \in E_2^{'}\ \in P(E_2)\ y \in F(e_2),\ a\,((x,\ e_1)) = (F,\ E_2^{'})\Big\}.$$

Let $W_a^{e_1,\,e_2} = \ \{(x,\ y)\ :\ ((x,\ e_1),\ (y,\ e_2)) \in W_a\}$. Because $a((x,\ e_1) = (F,\ E_2)$ is the soft set, which we can interpret as a parameterized family of subsets of the universe U_2, so we will write that $F(e_2) \in a\,((x,\ e_1))$. This notification indeed simplify the notations in the proofs of the next theorems. Now, let us assume that U_1, U_2 and U_3 denote linear spaces with scalar multiplication by real numbers.

Definition 3.3. A soft set mapping, $a\ :\ U_1 \times E_1 \to \mathrm{T}_2$ say, is called conical iff for any $(x,\ e_1) \in U_1 \times E_1$ and for any $\alpha \geqslant 0$, $a\,((\alpha\, x,\ e_1))\,\tilde{=}\alpha\tilde{\cdot} a\,((x,\ e_1))$.

Theorem 3.1. *If $a\ :\ U_1 \times E_1 \to \mathrm{T}_2$ is a conical soft set mapping then for any $e_1 \in E_1$ and $e_2 \in E_2$ the set $W_a^{e_1,\,e_2}$ is a cone.*

Proof. Let $(x,\ y) \in W_a^{e_1,\,e_2}$ and $\alpha > 0$. This means that $y \in F(e_2) \in a\,((x,\ e_1))$. Then $\alpha\, y \in \alpha\, F(e_2) \in \alpha\tilde{\cdot} a\,((x,\ e_1))$. Because a is a conical soft set mapping, for any $\alpha \geqslant 0$ we have $\alpha\, y \in \alpha\, F(e_2) \in a\,((\alpha\, x,\ e_1))$. This means that $((\alpha\, x,\ e_1),\ (\alpha\, y,\ e_2) \in W_a$ and finally $(\alpha\, x,\ \alpha\, y) \in W_a^{e_1,\,e_2}$.

Definition 3.4. A soft set mapping, $a\ :\ U_1 \times E_1 \to \mathrm{T}_2$ say, is called superadditive iff for any $(x_1,\ e_1),\ (x_2,\ e_1) \in U_1 \times E_1$

$$a\,((x_1 + x_2,\ e_1))\,\tilde{\supset} a\,((x_1,\ e_1))\,\tilde{+} a\,((x_2,\ e_1))\,.$$

Theorem 3.2. *If **a** is conical and superadditive soft set mapping then for any $e_1 \in E_1$ and $e_2 \in E_2$ the set $W_a^{e_1,\, e_2}$ is convex.*

Proof. Let $(x_1,\ y_1),\ (x_2,\ y_2) \in W_a^{e_1,\, e_2}$. This means that $y_1 \in F(e_2) \in a\,((x_1,\ e_1))$ and $y_2 \in G(e_2) \in a\,((x_2,\ e_1))$. A soft set mapping **a** is conical and superadditive so for any $\alpha,\ \beta \geqslant 0,\ \alpha + \beta = 1$ we have

$$\alpha\, y_1 + \beta\, y_2 \in \alpha\, F(e_2) + \beta\, G(e_2) \in \alpha\tilde{\cdot} a\,((x_1,\ e_1))\,\tilde{+}\beta\tilde{\cdot} a\,((x_2,\ e_1)) \tilde{=}$$

$$\tilde{=} a\,((\alpha\, x_1,\ e_1))\,\tilde{+} a\,((\beta\, x_2,\ e_1))\,\tilde{\subset} a\,((\alpha\, x_1 + \beta\, x_2,\ e_1))\,.$$

This means that $(\alpha\, x_1 + \beta\, x_2,\ \alpha\, y_1 + \beta\, y_2) \in W_a^{e_1,\, e_2}$, i.e. a set $W_a^{e_1,\, e_2}$ is convex.

Theorem 3.3. *If **a** is conical and superadditive soft set mapping then for any $(x,\ e_1) \in U_1 \times E_1$ $\quad a\,((x,\ e_1))$ is convex soft set.*

Proof. Let $a\,((x,\ e_1)) = (F,\ E_2^{'})$and let $e_2 \in E_2^{'}$ and $y_1,\ y_2 \in F(e_2) \in a\,((x,\ e_1))$.

This means that for any $\alpha,\ \beta \geqslant 0,\ \alpha + \beta = 1$ $\alpha\, y_1 \in \alpha\, F(e_2) \in \alpha\tilde{\cdot} a\,((x,\ e_1))$ and $\beta\, y_2 \in \beta\, \cdot F(e_2) \in \beta\tilde{\cdot} a\,((x,\ e_1))$. Because a soft set mapping is conical and superadditive so we have

$$\alpha\, y_1 + \beta\, y_2 \in \alpha\, F(e_2) + \beta\, F(e_2) \in \alpha\tilde{\cdot} a\,((x,\ e_1))\,\tilde{+}\beta\tilde{\cdot} a\,((x,\ e_1)) \tilde{=}$$

$$\tilde{=} a\,((\alpha\, x,\ e_1))\,\tilde{+} a\,((\beta\, x,\ e_1))\,\tilde{\subset} a\,((x,\ e_1))\,.$$

This means that if $\alpha\tilde{\cdot} a\,((x,\ e_1))\,\tilde{+}\beta\tilde{\cdot} a\,((x,\ e_1))\,\tilde{\subset} a\,((x,\ e_1))$ then $\alpha\, F(e_2) + \beta\, F(e_2) \subset F(e_2)$ and $\alpha\, y_1 + \beta\, y_2 \in F(e_2)$, i.e. $F(e_2)$ is a convex set.

Definition 3.5. A composite, $b \circ a\ :\ U_1 \times E_1 \to \mathrm{T}_3$ say, of two soft set mappings $a\ :\ U_1 \times E_1 \to \mathrm{T}_2$ and $b\ :\ U_2 \times E_2 \to \mathrm{T}_3$ is a soft set mapping such that for any $(x,\ e_1) \in U_1 \times E_1$ $\quad (b \circ a)\,((x,\ e_1)) \tilde{=} b\,(a\,((x, e_1)))$.

Theorem 3.4. *If a and b are conical soft set mappings then $b \circ a$ is a conical soft set mapping too.*

Proof. Let $(x,\ e_1) \in U_1 \times E_1$ and $\lambda \geqslant 0$. So, taking into account Definition 3.3 and Definition 3.5 we have

$$(b \circ a)\,((\lambda x,\ e_1)) \tilde{=} b\,(a\,((\lambda x, e_1))) \tilde{=} b\,(\lambda\tilde{\cdot} a\,((x,\ e_1))) \tilde{=}$$

$$\tilde{=} \tilde{\bigcup_{\lambda\tilde{\cdot}(y,\ E_2)\tilde{\in}\lambda\tilde{\cdot} a((x,\ e_1))}} b\,(\lambda\tilde{\cdot}(y,\ E_2)) \tilde{=} \tilde{\bigcup_{(y,\ E_2)\tilde{\in} a((x,\ e_1))}}\ \tilde{\bigcap_{e_2 \in E_2}} b\,((\lambda \cdot y(e_2),\ e_2))$$

$$\tilde{=} \tilde{\bigcup_{(y,\ E_2)\tilde{\in} a((x,\ e_1))}}\ \tilde{\bigcap_{e_2 \in E_2}} \lambda\tilde{\cdot} b\,((y(e_2),\ e_2)) \tilde{=} \lambda\tilde{\cdot}(b \circ a)\,((x,\ e_1))\,.$$

This means that $b \circ a$ is a conical soft set mapping.

Theorem 3.5. *If a and b are superadditive soft set mappings then $b \circ a$ is a superadditive soft set mapping too.*

Proof. Let $(x_1,\ e_1),\ (x_2,\ e_1) \in U_1 \times E_1$. Taking into account Definition 3.4 and Definition 3.5 we have

$$(b \circ a)\,((x_1 + x_2,\ e_1)) \tilde{=} b\,(a\,((x_1 + x_2,\ e_1)))\,\tilde{\supset} b\,\big(a\,((x_1,\ e_1))\,\tilde{+} a\,((x_2,\ e_1))\big) \tilde{=}$$

$$\tilde{=} \tilde{\bigcup}_{\substack{(y_1,\ E_2)\tilde{+}(y_2,\ E_2)\ :\\ (y_1,\ E_2)\tilde{\in} a((x_1,\ e_1))\\ (y_2,\ E_2)\tilde{\in} a((x_2,\ e_1))}} b\left((y_1,\ E_2)\tilde{+}(y_2,\ E_2)\right)\tilde{=}$$

$$\tilde{=} \tilde{\bigcup}_{\substack{(y_1,\ E_2)\tilde{+}(y_2,\ E_2)\ :\\ (y_1,\ E_2)\tilde{\in} a((x_1,\ e_1))\\ (y_2,\ E_2)\tilde{\in} a((x_2,\ e_1))}} \tilde{\bigcap}_{e_2\in E_2} b\left((y_1(e_2)+y_2(e_2),\ e_2)\right)\tilde{\supset}$$

$$\tilde{\supset} \tilde{\bigcup}_{\substack{(y_1,\ E_2)\tilde{+}(y_2,\ E_2)\ :\\ (y_1,\ E_2)\tilde{\in} a((x_1,\ e_1))\\ (y_2,\ E_2)\tilde{\in} a((x_2,\ e_1))}} \tilde{\bigcap}_{e_2\in E_2} \left(b\left((y_1(e_2),\ e_2)\right)\tilde{+}b\left((y_2(e_2),\ e_2)\right)\right)\tilde{\supset}$$

$$\tilde{\supset} \tilde{\bigcup}_{(y_1,\ E_2)\tilde{\in} a((x_1,\ e_1))} \tilde{\bigcap}_{e_2\in E_2} b\left((y_1(e_2),\ e_2)\right)\tilde{+}$$

$$\tilde{+} \tilde{\bigcup}_{(y_2,\ E_2)\tilde{\in} a((x_2,\ e_1))} \tilde{\bigcap}_{e_2\in E_2} b\left((y_2(e_2),\ e_2)\right)\tilde{=}$$

$$\tilde{=}(b\circ a)\left((x_1,\ e_1)\right)\tilde{+}(b\circ a)\left((x_2,\ e_1)\right).$$

This means that $b\circ a$ is a superadditive soft set mapping.

Definition 3.6. A converse soft set mapping to a soft set mapping $a\ :\ U_1\times E_1\to \mathrm{T}_2$ is a mapping $a^{-1}\ :\ U_2\times E_2\to \mathrm{T}_1$ such that for $(y,\ e_2)\in U_2\times E_2$ $a^{-1}\ ((y,\ E_2))\ =(F,\ E_1^{'})$ iff for any $e_1\in E_1^{'}\in P(E_1)$, $F(e_1)=\{x\in U_1\ :\ y\in G(e_2)\in a(x,\ e_1)\}$.

From the above definition it follows that $((y,\ e_2),\ (x,\ e_1))\in W_{a^{-1}}$ iff $((x,\ e_1),\ (y,\ e_2))\in W_a$.

Theorem 3.6. *If a soft set mapping is conical, then its converse soft set mapping is conical too.*

Proof. As a matter of fact, let a be a conical soft set mapping. So, taking into account Definition 3.6 we have:

$a^{-1}\ ((\lambda y,\ e_2))\ =(F',\ E_1^{'})$ iff for any $e_1\in E_1^{'}\in P(E_1)$,

$F'(e_1)=\{x'\in U_1\ :\ \lambda y\in G'(e_2)\in a\left((x',\ e_1)\right)\}$.

Moreover

$\lambda\tilde{\cdot}a^{-1}\ ((y,\ e_2))\ =\lambda\tilde{\cdot}(F,\ E_1^{'})$ iff for any $e_1\in E_1^{'}\in P(E_1)$,

$$\lambda F(e_1)=\lambda\cdot\{x\ :\ y\in G(e_2)\in a\left((x,\ e_1)\right)\}=$$

$$=\{\lambda x\ :\ y\in G(e_2)\in a\left((x,\ e_1)\right)\}=$$

$$=\{\lambda x\ :\ \lambda y\in \lambda G(e_2)\in \lambda\tilde{\cdot}a\left((x,\ e_1)\right)\}=$$

$$= \{\lambda x \ : \ \lambda y \in \lambda G(e_2) \in a((\lambda x,\ e_1))\} =$$
$$= \{x' \ : \ \lambda y \in \lambda G(e_2) \in a((x',\ e_1))\} =$$
$$= \{x' \ : \ \lambda y \in G'(e_2) \in a((x',\ e_1))\}.$$

This means that a soft set mapping a^{-1} is conical.

Theorem 3.7. *If a soft set mapping is superadditive then its converse soft set mapping is superadditive as well.*

Proof. In point of fact, let a soft set mapping $a \ : \ U_1 \times E_1 \to \mathrm{T}_2$ say, satisfy the assumption of the theorem. Let $(y_1,\ e_2),\ (y_2,\ e_2) \in U_2 \times E_2$. Then from Definition 3.6 it follows that:

- $a^{-1}\ ((y_1,\ e_2))\ = (F_1,\ E_1^{'})$ iff for any $e_1 \in E_1^{'} \in P(E_1)$

  $$F_1(e_1) = \{x_1 \ : \ y_1 \in G_1(e_2) \in a((x_1,\ e_1))\};$$
- $a^{-1}\ ((y_2,\ e_2))\ = (F_2,\ E_1^{"})$ iff for any $e_1 \in E_1^{"} \in P(E_1)$

  $$F_2(e_1) = \{x_2 \ : \ y_2 \in G_2(e_2) \in a((x_2,\ e_1))\};$$
- $a^{-1}\ ((y_1 + y_2,\ e_2))\ = (F_3,\ E_1^{'"})$ iff for any $e_1 \in E_1^{'"} \in P(E_1)$

  $$F_3(e_1) = \{x_3 \ : \ y_1 + y_2 \in G_3(e_2) \in a((x_3,\ e_1))\}.$$

We have to prove that for any

$$e_1 \in E_1^{'} \cap E_1^{"},\ F_1(e_1) + F_2(e_1) \subset F_3(e_1).$$

$$F_1(e_1) + F_2(e_1) =$$
$$= \{x_1 + x_2 \ : \ y_1 \in G_1(e_2) \in a((x_1,\ e_1));\ y_2 \in G_2(e_2) \in a((x_2,\ e_1))\} =$$
$$= \left\{x_1 + x_2 \ : \ y_1 + y_2 \in G_1(e_2) + G_2(e_2) \in a((x_1,\ e_1))\,\tilde{+}a((x_2,\ e_1))\right\} \subseteq$$
$$\subseteq \{x_1 + x_2 \ : \ y_1 + y_2 \in G_3(e_2) \in a((x_1 + x_2,\ e_1))\} \subseteq$$
$$\subseteq \{x_3 \ : \ y_1 + y_2 \in G_3(e_2) \in a((x_3,\ e_1))\}.$$

So, a^{-1} is a superadditive soft set mapping.

Now, let us that U_1, U_2 and U_3 are finite dimensional Euclidean spaces.

Definition 3.7. A soft set mapping, $a \ : \ U_1 \times E_1 \to \mathrm{T}_2$ say, is called closed iff for any $e_1 \in E_1$ and $e_2 \in E_2$ $\ W_a^{e_1,\,e_2}$ is a closed set.

Corollary. *For any closed soft set mapping its converse soft set mapping is closed.*

Theorem 3.8. *If* $a \ : \ U_1 \times E_1 \to \mathrm{T}_2$ *be a closed soft mapping and* ξ *compact subset of* U_1*. Then for any* $e_1 \in E_1$ *and* $e_2 \in E_2^{'} \in P(E_2)$ *the set* $G(e_2) \in a((\xi,\ e_1))$ *is closed, where* $a((\xi,\ e_1)) = (G,\ E_2^{'})$ *and* $a((\xi,\ e_1)) = \tilde{\bigcup\limits_{x \in \xi}} a((x,\ e_1))$.

Proof. Let $y_n \in G(e_2) \in a((\xi,\ e_1))$, $y_n \to y$, $x_n \in \xi$ and $y_n \in G_n(e_2) \in a((x_n,\ e_1))$ where $a((x_n,\ e_1)) = (G_n,\ E_2^n)$. Without losing generality we may assume that $x_n \to x$ as $n \to \infty$. Because $\boldsymbol{a}$ is a closed soft set mapping we observe that $(x,\ y) = W_{\underline{a}}^{e_1,\ e_2}$ what means that $y \in \bar{G}(e_2) \in a\left((x,\ e_1)\right) \tilde{\subset} a\left((\xi,\ e_1)\right)$, where $a\left((x,\ e_1)\right) = (\bar{G},\ \bar{E})$. From the definition of union of soft sets it follows that $\bar{G}(e_2) \subseteq G(e_2)$, what means that $G(e_2)$is a closed set.

Corollary. *If $a\ :\ U_1 \times E_1 \to \mathrm{T}_2$ is a closed soft set mapping then for any $e_1 \in E_1$, $e_2 \in E_2$ and $x \in U_1$ the set $G(e_2) \in a\left((x,\ e_1)\right)$ is closed.*

A soft set $(F,\ E)$ we will called bounded if for any $e \in E$, $F(e)$ is bounded set.

Definition 3.8. A soft set mapping, $a\ :\ U_1 \times E_1 \to \mathrm{T}_2$ say, is called bounded iff for any $(x,\ e_1) \in U_1 \times E_1$, $a\left((x,\ e_1)\right)$ is a bounded soft set.

Theorem 3.9. *Let $a\ :\ U_1 \times E_1 \to \mathrm{T}_2$ be a closed and bounded soft set mapping and f a function continuous on U_2. Then for any $e_1 \in E_1$ and $e_2 \in E_2$ the function*

$$u_f(x) = \max_{y \in G(e_2) \in a((x,\ e_1))} f(y), \quad x \in U_1$$

is upper semicontinuous.

Proof. Since a is closed and bounded soft set mapping, so for any $(x,\ e_1) \in U_1 \times E_1$ and any $e_2 \in E_2$ $G(e_2) \in a\left((x,\ e_1)\right)$ is compact set and f achieves its maximum on this set. Now, let (x_n) be an arbitrary sequence converging to x and $y_n \in G_n(e_2) \in a\left((x_n,\ e_1)\right)$ such that $f(y_n) = u_f(x_n)$. Because a is bounded soft set mapping, we can choose a convergent subsequence (y_{n_k}). Let $\lim y_{n_k} = y$. Then $y \in G(e_2) \in a\left((x,\ e_1)\right)$ since a is closed set mapping. Therefore

$$\lim u_f(x_{n_k}) = f(y) \leqslant \max_{y \in G(e_2) \in a((x,\ e_1))} f(y) = u_f(x)$$

and finally $\overline{\lim} u_f(x_n) \leqslant u_f(x)$.

Theorem 3.10. *Let*

- *U_1 be a compact and convex subset of finite dimensional Euclidean space,*
- *$a\ :\ U_1 \times E_1 \to \mathrm{T}_1$ be a closed, conical and superadditive soft set mapping,*
- *there exist e', $e'' \in E_1$ such that for any $x \in U_1$ the set $F(e'') \in a\left((x,\ e')\right)$ is nonempty.*

Then there exists $\bar{x} \in U_1$ such that $\bar{x} \in F(e'') \in a\left((\bar{x},\ e')\right)$.

Proof. Let us note that our soft set mapping generate a multifunction $b_{e''}\ :\ U_1 \to P(U_1)$ such that for any $x \in U_1$, $b_{e''}(x) = F(e'') \in a\left((x,\ e')\right)$. Since a is closed, conical and superadditive soft set mapping so from the Theorem 3.3 and Theorem 3.8 it follows that for any x, $b_{e''}(x)$ is convex and closed set. This means that for the multifunction $b_{e''}$ the assumptions of Fixed-point theorem are fulfilled. So, there exists $\bar{x} \in U_1$ such that $\bar{x} \in b_{e''}(\bar{x}) = F(e'') \in a\left((\bar{x},\ e')\right)$.

4. Conclusions

Multifunctions (set-valued mappings) have many diverse and interesting applications in control problems and theory of contingent equations, in mathematical economics, and in various branches of analysis. Multifunctions are interpreted, for example as certain technological transformations assigning a set of commodities to a set of production factors. In practice, the result of a production process is by its very nature imprecise. Such a situation is difficult to describe but a soft set mapping seems to be a very useful tool in this respect.

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