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An MST cluster analysis method under hesitant fuzzy environment*

by

Xiaolu Zhang, Zeshui Xu†

School of Economics and Management, Southeast University
Nanjing, Jiangsu 211189, China

Abstract: Hesitant fuzzy sets (HFSs) are useful means to describe and deal with uncertain data. In this article, a minimal spanning tree (MST) algorithm based clustering technique under hesitant fuzzy environment is proposed. We first introduce the concepts of graph, MST, HFS, and hesitant fuzzy distance. Then, we present a hesitant fuzzy MST clustering algorithm to perform clustering analysis of HFSs via some hesitant fuzzy distances, and finally illustrate the effectiveness of our algorithm through two numerical examples.

Keywords: hesitant fuzzy set, minimal spanning tree, graph theory-based clustering algorithm, hesitant fuzzy distance.

1. Introduction

Clustering is a process aiming at grouping a set of objects into classes or clusters according to the characteristics of data so that objects within a cluster have mutual high similarity and objects in different clusters are dissimilar (Dong et al., 2006). There have been many applications of clustering analysis to practical problems in engineering, computer sciences, life and medical sciences, astronomy and earth sciences, social sciences, economics, and so on (Anderberg, 1973; Hartigan, 1975; Everitt et al., 2001). The traditional (hard) clustering algorithms strictly allocate an object to exactly one cluster. However, due to the fuzzy nature of many practical problems, for objects so we cannot directly conclude which class they should belong to in real life. Fuzzy sets (Zadeh, 1965) give an idea of uncertainty of belonging, which is described by a membership function. Then, Ruspini (1969) proposed the concept of fuzzy division, which opened the door to research in fuzzy clustering. Subsequently, several extensions have been developed, such as intutionistic fuzzy sets (Atanassov, 1986), type-2 fuzzy sets (Dubois and Prade, 1980; Miyamoto, 2005), type-n fuzzy sets (Dubois and Prade, 1980), fuzzy multisets (Yager, 1986; Miyamoto, 2000) and hesitant fuzzy sets (Torra and Narukawa, 2009; Torra, 2010). So far, a lot of work has been

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†Corresponding author. E-mails: xuzeshui@263.net; xiaolu_jy@163.com

done about the first four types of fuzzy sets, for example, Torra and Miyamoto (2011) defined I-fuzzy partitions (or intuitionistic fuzzy partitions as they were called by Atanassov, or interval-valued fuzzy partitions) to solve the difficulty of comparing the results of fuzzy clustering methods and, in particular, the difficulty of finding the global optimum, and applied I-fuzzy partitions to represent sets of fuzzy partitions (Torra and Min, 2011a) as well as clustering uncertainties (Torra and Min, 2011b). Meanwhile, a lot of research has been performed on hesitant fuzzy sets. Torra and Narukawa (2009) and Torra (2010) discussed the relationship between hesitant fuzzy sets and other three kinds of fuzzy sets, Xia and Xu (2011) proposed some aggregation operators for hesitant fuzzy information, Xia et al. (2011) developed a series of confidence induced hesitant fuzzy aggregation operators. Xu and Xia (2011a, b) gave a detailed study on distance and similarity measures for hesitant fuzzy sets and hesitant fuzzy elements respectively. Graph theory (Harary, 1969) appears to be very convenient to describe clustering problems. The graph theory-based clustering techniques are an active research area. Zahn (1971) proposed clustering algorithm using the minimal spanning tree (MST). Jain and Dubes (1988) provided a detailed description and discussion of hierarchical clustering from the point of view of graph theory. Dong et al. (2006) gave a hierarchical clustering algorithm based on fuzzy graph connectedness. Chen et al. (2007) introduced the concept of maximum spanning tree of fuzzy graph by constructing the fuzzy similarity relation matrix, and used the threshold of fuzzy similarity relation matrix to cut the maximum spanning tree, and then obtained classification on the respective level. Zhao et al. (2012) developed two intuitionistic fuzzy minimal spanning tree clustering algorithms, and extended them to clustering interval-valued intuitionistic fuzzy sets. Yet, until now there has been no study on clustering of data represented by hesitant fuzzy information. In real life, due to uncertainty of information as well as vagueness of human feelings and recognition, the evaluation data on the objects are sometimes expressed as hesitant fuzzy sets, and in such cases, we need to develop clustering techniques to cluster such data. Based on graph theory, this article develops a hesitant fuzzy minimal spanning tree (HFMST) clustering algorithm to deal with hesitant fuzzy information. To do so, we organize the article as follows: Section 2 reviews some concepts related to graph theory, hesitant fuzzy sets and distance measures. Based on the minimal spanning tree (MST), Section 3 develops a novel intuitionistic fuzzy clustering technique, and then we illustrate its effectiveness via two numerical examples in Section 4. The article finishes with some concluding remarks in Section 5.

2. Preliminaries

In what follows, we introduce some basic concepts and terminology to be used in the next sections:

2.1. The basic concepts related to HFSs

Hesitant fuzzy sets (HFSs) were first introduced by Torra and Narukawa (2009) and Torra (2010), allowing the membership degree of an element to a set to be represented with several possible values between 0 and 1. HFSs are very useful in dealing with the situations where people hesitate when providing data information.

Definition 1 (Torra and Narukawa, 2009; Torra, 2010)**.** Let X be a fixed set, a hesitant fuzzy set (HFS) on X is represented by a function that when applied to X returns a subset of $[0,1]$, which can be expressed by a mathematical symbol:

$$Q = \{< x,\ h_Q(x) > | x \in X\} \tag{1}$$

where $h_Q(x)$ is a set of values in $[0,1]$, denoting the possible membership degrees of the element $x \in X$ to the set Q. For convenience, we call $h = h_Q(x)$ a hesitant fuzzy element (HFE) and S the set of all HFEs.

Given three HFEs represented by h, h_1 and h_2, Torra and Narukawa (2009) and Torra (2010) defined some operations on them, which can be described as:

1) $h^c = \bigcup\limits_{\gamma \in h} \{1-\gamma\}$;

2) $h_1 \cup h_2 = \bigcup\limits_{\gamma_1 \in h_1, \gamma_2 \in h_2} \max\{\gamma_1, \gamma_2\}$;

3) $h_1 \cap h_2 = \bigcup\limits_{\gamma_1 \in h_1, \gamma_2 \in h_2} \min\{\gamma_1, \gamma_2\}$.

Torra and Narukawa (2009) and Torra (2010) showed that the envelope of a HFE is an intuitionistic fuzzy value (IFV), expressed in the following definition:

Definition 2 (Torra and Narukawa, 2009; Torra, 2010)**.** Given a HFE h, we define the IFV $A_{env}(h)$ as the envelope of h, where $A_{env}(h)$ can be represented as $(h^-, 1-h^+)$, with $h^- = \min\{\gamma | \gamma \in h\}$ and $h^+ = \max\{\gamma | \gamma \in h\}$.

2.2. The distance measures of HFSs

Let $X = \{x_1, x_2, \cdots, x_n\}$ be a discrete universe of discourse. Consider that the elements x_i $(i = 1, 2, \cdots, n)$ in the universe X may have different importance, let $w = \{w_1, w_2, \cdots, w_n\}^T$ be the weight vector of x_i $(i = 1, 2, \cdots, n)$, with $w_i \geq 0$, $i = 1, 2, \cdots, n$, $\sum\limits_{i=1}^{n} w_i = 1$, and let M and N be two HFSs on $X = \{x_1, x_2, \cdots, x_n\}$. Xu and Xia (2011a) defined the generalized hesitant weighted distance, the generalized hesitant weighted Hausdorff distance and the generalized hybrid hesitant weighted distance, respectively:

(1) The generalized hesitant weighted distance:

$$z_1(M,N) = \left[\sum_{i=1}^{n} w_i \left(\frac{1}{l_{x_i}} \sum_{j=1}^{l_{x_i}} \left| h_M^{\sigma(j)}(x_i) - h_N^{\sigma(j)}(x_i) \right|^{\lambda}\right)\right]^{1/\lambda}. \tag{2}$$

In particular, if $\lambda = 1, 2$, then Eq.(2) reduces to the hesitant weighted Hamming distance and the hesitant weighted Euclidean distance, respectively:

$$z_2(M,N) = \sum_{i=1}^{n} w_i \left[\frac{1}{l_{x_i}} \sum_{j=1}^{l_{x_i}} \left| h_M^{\sigma(j)}(x_i) - h_N^{\sigma(j)}(x_i) \right| \right] \tag{3}$$

$$z_3(M,N) = \left[\sum_{i=1}^{n} w_i \left(\frac{1}{l_{x_i}} \sum_{j=1}^{l_{x_i}} \left| h_M^{\sigma(j)}(x_i) - h_N^{\sigma(j)}(x_i) \right|^2 \right) \right]^{1/2} . \tag{4}$$

(2) The generalized hesitant weighted Hausdorff distance:

$$z_4(M,N) = \left[\sum_{i=1}^{n} w_i \max_j \left| h_M^{\sigma(j)}(x_i) - h_N^{\sigma(j)}(x_i) \right|^{\lambda} \right]^{1/\lambda} \tag{5}$$

where $\lambda > 0$.

Especially, if $\lambda = 1, 2$, then Eq.(5) reduces to the hesitant weighted Hamming-Hausdorff distance and the hesitant weighted Euclidean-Haudorff distance, respectively:

$$z_5(M,N) = \sum_{i=1}^{n} w_i \max_j \left| h_M^{\sigma(j)}(x_i) - h_N^{\sigma(j)}(x_i) \right| \tag{6}$$

$$z_6(M,N) = \left[\sum_{i=1}^{n} w_i \max_j \left| h_M^{\sigma(j)}(x_i) - h_N^{\sigma(j)}(x_i) \right|^2 \right]^{1/2} . \tag{7}$$

(3) The generalized hybrid hesitant weighted distance:

$$z_7(M,N) = \left[\sum_{i=1}^{n} w_i \left(\frac{1}{l_{x_i}} \sum_{j=1}^{l_{x_i}} \left| h_M^{\sigma(j)}(x_i) - h_N^{\sigma(j)}(x_i) \right|^{\lambda} + \max_j \left| h_M^{\sigma(j)}(x_i) - h_N^{\sigma(j)}(x_i) \right|^{\lambda} \right) \right]^{\lambda} \tag{8}$$

where $\lambda > 0$.

In particular, if $\lambda = 1, 2$, Eq.(8) reduces to a hybrid hesitant weighted Hamming distance and a hybrid hesitant weighted Euclidean distance, respectively:

$$z_8(M,N) = \sum_{i=1}^{n} w_i \left(\frac{1}{l_{x_i}} \sum_{j=1}^{l_{x_i}} \left| h_M^{\sigma(j)}(x_i) - h_N^{\sigma(j)}(x_i) \right| + \max_j \left| h_M^{\sigma(j)}(x_i) - h_N^{\sigma(j)}(x_i) \right| \right) \tag{9}$$

$$z_9(M,N) = \left[\sum_{i=1}^{n} w_i \left(\frac{1}{l_{x_i}} \sum_{j=1}^{l_{x_i}} \left| h_M^{\sigma(j)}(x_i) - h_N^{\sigma(j)}(x_i) \right|^2 + \max_j \left| h_M^{\sigma(j)}(x_i) - h_N^{\sigma(j)}(x_i) \right|^2 \right) \right]^{1/2} \tag{10}$$

In what follows, we define the concept of hesitant fuzzy distance matrix:

Definition 3 Let $A_j (j = 1, 2, \cdots, m)$ be m HFSs, then $Z = (z_{ij})_{m\times m}$ is called a hesitant fuzzy distance matrix, where $z_{ij} = z(A_i, A_j)$ is the distance between A_i and A_j, which has the following properties:

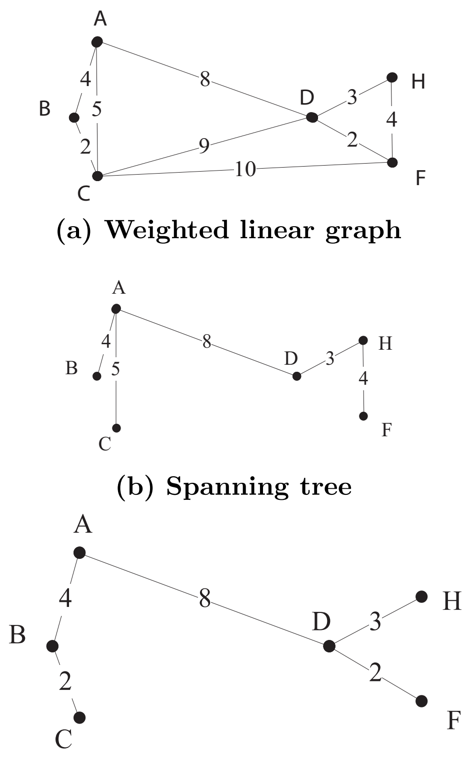
1) $0 \leq z_{ij} \leq 1$, for all $i, j = 1, 2, \cdots, m$;

2) $z_{ij} = 0$ if and only if $A_i = A_j$;

3) $z_{ij} = z_{ji}$, for all $i, j = 1, 2, \cdots, m$.

2.3. The graph and the minimal spanning trees

A graph G is a pair of sets $G = (V, E)$, where V is the set of nodes and E is the set of edges. In an undirected graph, each edge is an unordered pair $\{v_1, v_2\}$. In a directed graph (also called a digraph in some literature), edges are ordered pairs. The nodes v and w are called the endpoints of an edge. In a weighted graph, ω is defined as a weight on each edge (Schaeffer, 2007). The graphs considered in the rest of the article are undirected. Next, we introduce some other notions through Fig. 1, where Fig. 1(a) depicts a weighted graph with 6 nodes and 9 edges.



(a) Weighted linear graph

(b) Spanning tree

(c) Minimal spanning tree

Figure 1. A graph and the minimal spanning tree

A sequence of edges and nodes that can be traveled between two different nodes is called a path. For instance, it might be the case that two different paths exist from node A to node H, such as the one denoted ($ABCFH$) and the one denoted ($ABCDH$). A path where the start node and destination node is the same is called a circuit, like ($ABCA$) or ($ACFHDA$). A connected graph has paths between any pair of nodes. A connected acyclic graph that contains all nodes of G is called a spanning tree of the graph. Obviously, Fig. 1(b) shows such a graph. If we define the weight of a tree to be the sum of the weights of its constituent edges then a minimal spanning tree of the graph G is a spanning tree whose weight is minimal among all spanning trees of G, like in Fig. 1(c) (Zahn, 1971).

The set E in a normal graph is a crisp relation over $V \times V$. That is to say, if there exists an edge between the nodes v_1 and v_2 then the membership degree equals 1, i.e., $\mu_E(v_1, v_2) = 1$; otherwise $\mu_E(v_1, v_2) = 0$, where $(v_1, v_2) \in (V \times V)$. If a fuzzy relation R over $V \times V$ is defined, then the membership function $\mu_R(v_1, v_2)$ takes values from 0 to 1 and such graph is called fuzzy graph. If R is a hesitant fuzzy relation over $V \times V$, then $G = (V, R)$ is called hesitant fuzzy graph.

Based on the hesitant fuzzy distance matrix given in Definition 3, we shall use the idea of Zahn (1971) to develop a novel and handy hesitant fuzzy clustering technique, a hesitant fuzzy minimal spanning tree (HFMST) clustering algorithm, presented in the next section.

3. The HFMST clustering algorithm

Let $X = \{x_1, x_2, \cdots, x_n\}$ be an attribution space and $w = \{w_1, w_2, \cdots, w_n\}^T$ be the weight vector of the elements x_i $(i = 1, 2, \cdots, n)$, with $w_i \geq 0$, $i = 1, 2, \cdots, n$, and $\sum_{i=1}^{n} w_i = 1$. Let $A_j (j = 1, 2, \cdots, m)$ be a collection of m HFSs expressing m samples to be clustered, having the following forms:

$$A_j = \{< x_j,\ h_A(x_j) > | x_j \in X\}, \quad j = 1, 2, \cdots, m. \tag{11}$$

Then we propose a hesitant fuzzy minimal spanning tree (HFMST) clustering algorithm, whose steps are as follows:

Step 1. Compute the hesitant fuzzy distance matrix and the fuzzy graph:

1) Calculate the distance $z_{ij} = z(A_i, A_j)$ by Eqs.(2)(10) and get the hesitant fuzzy distance matrix $Z = (z_{ij})_{m \times m}$.

2) Build the hesitant fuzzy graph $G = (V, E)$ where every edge between A_i and A_j has the weight z_{ij} represented by HFSs as an element of the hesitant fuzzy distance matrix $Z = (z_{ij})_{m \times m}$, which shows the dissimilarity degree between the samples A_i and A_j.

Step 2. Compute the MST of the hesitant fuzzy graph $G = (V, E)$ by Kruskal's method (Kruskal, 1956) or Prim's method (Prim, 1957)):

1) Sort the edges of G in increasing order by weight.

2) Keep a sub-graph D of G, which is initially empty, and choose at each step the edge e with the smallest weight to add to D, where the endpoints of e are disconnected.

3) Repeat the process 2) until the sub-graph D spans all nodes. Thus, we get the MST of the hesitant fuzzy graph $G = (V, E)$.

Step 3. Perform clustering by using the minimal hesitant fuzzy spanning tree. We can get a certain number of sub-trees (clusters) by disconnecting all the edges of the MST with weights greater than a threshold λ. The clustering results induced by the sub-trees do not depend on some particular MST (Gaertler, 2002).

For convenience, we express the clustering process of our algorithm above by the flow chart of Fig. 2.

4. Numerical examples

In this section, two illustrative examples will be given in order to demonstrate the practical usage and the effectiveness of our approach

Example 1. Jiangxi province is located in southeast of China and in the middle reaches the Changjiang (Yangtze) River. The favorable physical conditions, with a diversity of natural resources leading to the suitability for growing various crops there. However, there are also some restrictive factors for developing agriculture such as tight man–land relations, constant degradation of natural resources and growing population pressure on land resource reserves. Based on the distinctness and differences in environment and natural resources, Jiangxi Province can be roughly divided into ten cities: A_1—Fuzhou, A_2—Nanchang, A_3—Shangrao, A_4—Jiujiang, A_5—Pingxiang, A_6—Yingtan, A_7—Ganzhou, A_8—Yichun, A_9—Jingdezhen, A_{10}—Ji'an. Hence, in order to coordinate the development and improve people's living standards, local government intends to classify these cities into different regions. Suppose that several decision makers are invited to evaluate the ten alternatives (cities) based on two attributes: x_1—Ecological benefit and x_2—Economic benefit. Some attribute values for an alternative, provided by different decision makers may be repeated. However, a value repeated more times does not necessarily have higher importance than other values repeated less times. For example, the value one time may be provided by a decision maker who is an expert in this area, and the value repeated twice may be provided by two decision makers who are not familiar with this area. In such cases, the value given one time may be more important than the one repeated twice. To get a more reasonable result, it is better that the decision makers give their evaluations anonymously. We only collect all the possible values for an alternative and an attribute, and each value provided only means that it is a possible value, but its importance is unknown. As the number of times that the values are repeated is unimportant, it is reasonable to allow values repeated many times to appear only once. The HFS is just a tool to deal with such cases, and all possible evaluations for an alternative can be considered as an HFS. The results evaluated by the decision makers are contained in

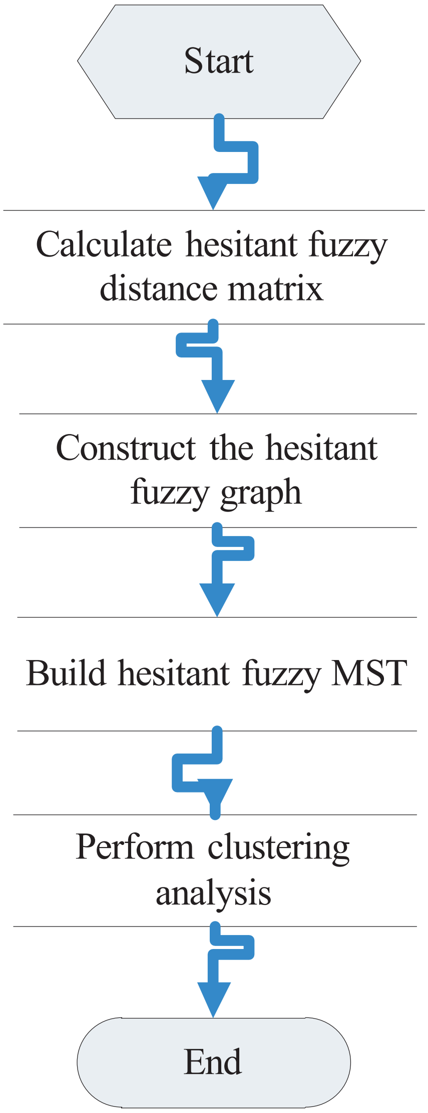


Figure 2. The flow chart of the HFMST clustering algorithm

$$Z = \begin{pmatrix} 0 & 0.3502 & 0.4399 & 0.2272 & 0.2502 & 0.469 & 0.2107 & 0.134 & 0.3512 & 0.2035 \\ 0.3502 & 0 & 0.0952 & 0.2814 & 0.2272 & 0.131 & 0.2117 & 0.3983 & 0.1155 & 0.463 \\ 0.4399 & 0.0952 & 0 & 0.3766 & 0.2327 & 0.1366 & 0.3014 & 0.4916 & 0.1617 & 0.5417 \\ 0.2272 & 0.2814 & 0.3766 & 0 & 0.3722 & 0.276 & 0.4217 & 0.2541 & 0.3255 & 0.1824 \\ 0.2502 & 0.2272 & 0.2327 & 0.3722 & 0 & 0.3176 & 0.2501 & 0.3263 & 0.2449 & 0.3505 \\ 0.469 & 0.131 & 0.1366 & 0.276 & 0.3176 & 0 & 0.2651 & 0.5081 & 0.1291 & 0.5689 \\ 0.2107 & 0.2117 & 0.3014 & 0.4217 & 0.2501 & 0.2651 & 0 & 0.3176 & 0.1405 & 0.4047 \\ 0.134 & 0.3983 & 0.4916 & 0.2541 & 0.3263 & 0.5081 & 0.3176 & 0 & 0.3969 & 0.2517 \\ 0.3512 & 0.1155 & 0.1617 & 0.3255 & 0.2449 & 0.1291 & 0.1405 & 0.3969 & 0 & 0.45 \\ 0.2035 & 0.463 & 0.5417 & 0.1824 & 0.3505 & 0.5689 & 0.4047 & 0.2517 & 0.45 & 0 \end{pmatrix}$$

a hesitant fuzzy decision matrix as follows:

$$A_1 = \{< x_1, \{0.8, 0.7, 0.6\} >,\ < x_2, \{0.8, 0.7, 0.3\} >\},$$

$$A_2 = \{< x_1, \{0.9, 0.8, 0.3\} >,\ < x_2, \{0.8, 0.7, 0.6\} >\},$$

$$A_3 = \{< x_1, \{0.9, 0.7, 0.1\} >,\ < x_2, \{0.8, 0.7, 0.6\} >\},$$

$$A_4 = \{< x_1, \{0.9, 0.8, 0.3\} >,\ < x_2, \{0.9, 0.8, 0.2\} >\}$$

$$A_5 = \{< x_1, \{0.8, 0.5, 0.4\} >,\ < x_2, \{0.7, 0.6, 0.5\} >\},$$

$$A_6 = \{< x_1, \{0.9, 0.8, 0.2\} >,\ < x_2, \{0.9, 0.8, 0.7\} >\},$$

$$A_7 = \{< x_1, \{0.8, 0.7, 0.6\} >,\ < x_2, \{0.9, 0.7, 0.6\} >\},$$

$$A_8 = \{< x_1, \{0.9, 0.8, 0.7\} >,\ < x_2, \{0.9, 0.8, 0.3\} >\},$$

$$A_9 = \{< x_1, \{0.9, 0.7, 0.3\} >,\ < x_2, \{0.9, 0.7, 0.6\} >\},$$

$$A_{10} = \{< x_1, \{0.7, 0.6, 0.5\} >,\ < x_2, \{0.9, 0.8, 0.1\} >\}.$$

Let the weight vector of the attributes x_j $(j = 1, 2)$ be $w = (0.45, 0.55)^T$. We utilize the HFMST clustering algorithm to group these operational plans $A_j (j = 1, 2, \cdots, 10)$:

Step 1. Construct the hesitant fuzzy distance matrix and the fuzzy graph where each node is associated to a city to be clustered as expressed by a HFS:

1) Calculate the distance $z_{ij} = z_9(A_i, A_j)$ by Eq. (10), and then get the hesitant fuzzy distance matrix $Z = (z_{ij})_{10\times 10}$ (above).

2) Construct the fuzzy graph $G = (V, E)$ where every edge between A_i and A_j has the weight z_{ij} represented by a HFS as an element of the hesitant fuzzy distance matrix $Z = (z_{ij})_{10\times 10}$, showing the dissimilarity degree between the samples A_i and A_j (see Fig. 3).

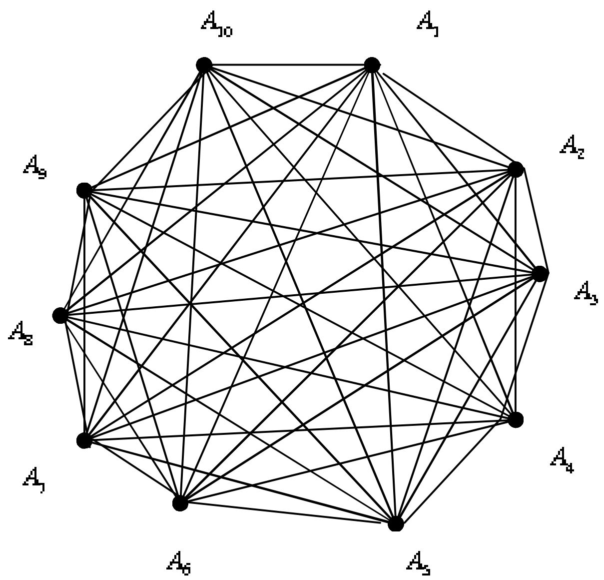


Figure 3. The hesitant fuzzy graph $G = (V, E)$

Step 2. Compute the MST of the hesitant fuzzy graph $G = (V, E)$ by Kruskal's method (Kruskal, 1956):

1) Sort the edges of G in increasing order by weights:

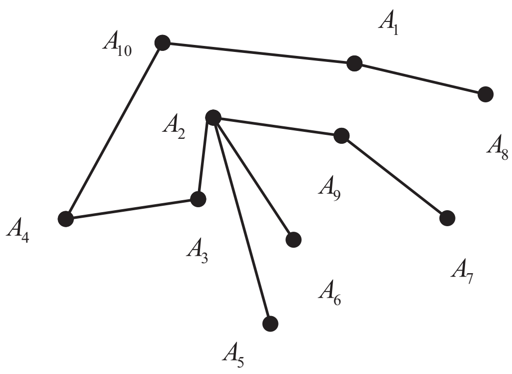
$$z_{23} < z_{29} < z_{26} < z_{18} < z_{36} < z_{79} < z_{39} < z_{4,10} < z_{1,10} < z_{27} < z_{25} = z_{14} < z_{34} < z_{59} < z_{57} < z_{15} < z_{8,10} < z_{4,8} < z_{67} < z_{45} < z_{24} < z_{37} < z_{56} = z_{78} < z_{49} < z_{58} < z_{12} < z_{5,10} < z_{19} < z_{45} < z_{34} < z_{89} < z_{7,10} < z_{47} < z_{13} < z_{9,10} < z_{2,10} < z_{16} < z_{38} < z_{68} < z_{3,10} < z_{6,10}$$

2) Keep an empty sub-graph D of G, and choose the edge e with the smallest weight to add to D, in which the endpoints of e are disconnected, so we can choose the edge e_{23} between A_2 and A_3.

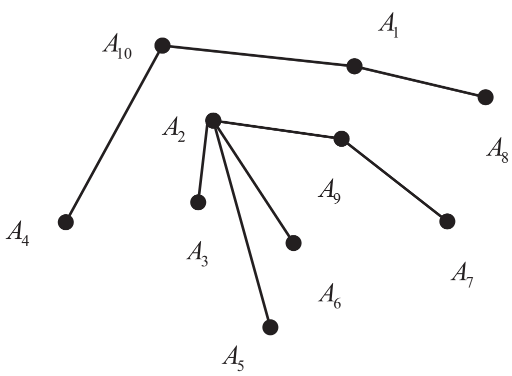
3) Repeat the process 2) until the sub-graph D spans ten nodes. Thus, we get the MST of the hesitant fuzzy graph $G = (V, E)$ (see Fig. 4(a-j)).

Step 3. Select a threshold λ and disconnect all the edges of the MST with weights greater than λ to get a certain number of sub-trees (clusters) automatically, as listed in Table 1.

Obviously, based on Table 1, the Jiangxi Provincial Government can divide its ten cities into different agroecological regions (clusters) in order to improve its overall development. For instance, if the government intends to classify these ten cities into four agroecological regions (clusters), thus, it can easily obtain the results from Table 1, derived by utilizing our algorithm to compute the assessment values of alternatives (cities) provided by decision makers (experts), as follows:


A_{10}
A_1
A_8
A_2
A_9
A_4
A_3
A_6
A_7
A_5

(a)


A_{10}
A_1
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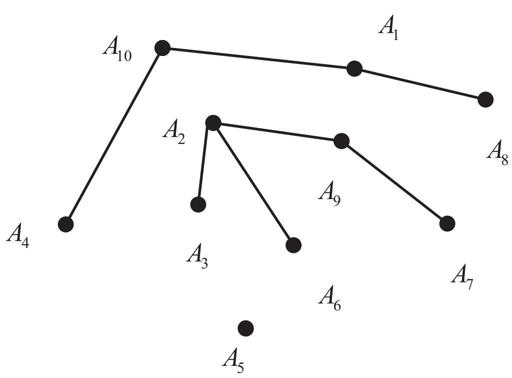
A_5

(b)


A_{10}
A_1
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A_4
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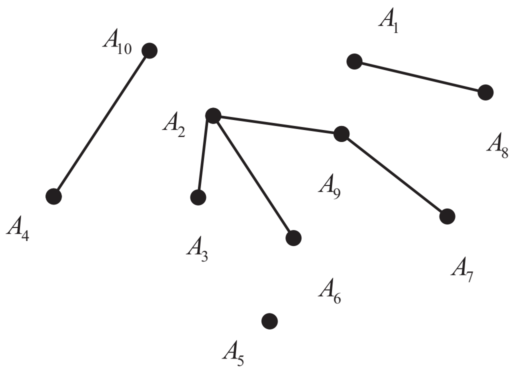
A_6
A_7
A_5

(c)


A_1
A_{10}
A_2
A_8
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A_4
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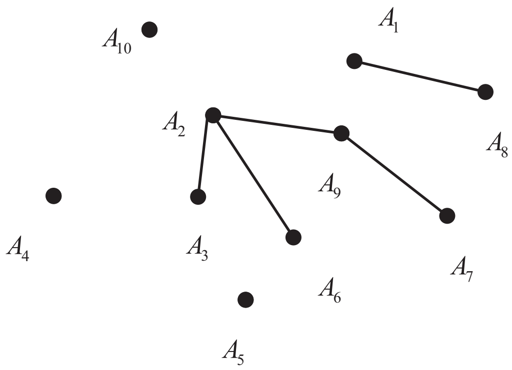
A_7
A_6
A_5

(d)


A_1
A_{10}
A_2
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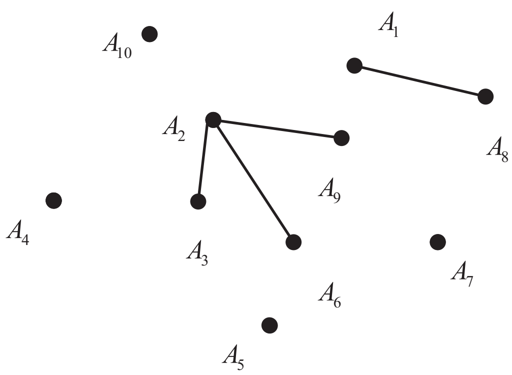
(e)


A_1
A_{10}
A_2
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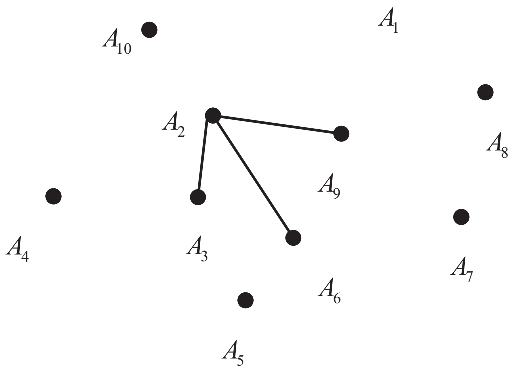
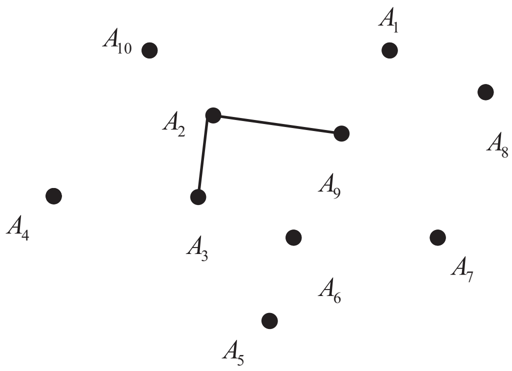
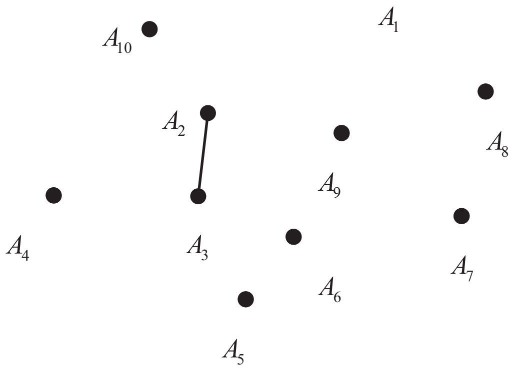
(f)


A_{10}
A_1
A_2
A_8
A_9
A_4
A_3
A_7
A_6
A_5

(g)


A_{10}
A_1
A_2
A_8
A_9
A_4
A_3
A_7
A_6
A_5

(h)


A_{10}
A_1
A_2
A_8
A_9
A_4
A_3
A_7
A_6
A_5

(i)

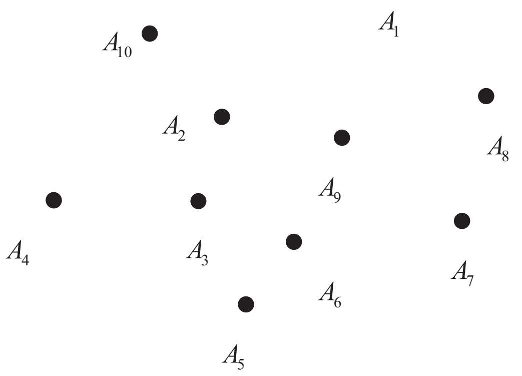


Figure 4. The sub-trees of the hesitant fuzzy graph $G=(V,E)$

Table 1. Clustering results with various thresholds λ

λ	Corresponding clustering results	Corresponding MST
$\lambda = z_{34} = 0.3766$	$\{A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}\}$	Figure 4(a)
$\lambda = z_{14} = z_{25} = 0.2272$	$\{A_1, A_4, A_8, A_{10}\}$, $\{A_2, A_3, A_5, A_6, A_7, A_9\}$	Figure 4(b)
$\lambda = z_{1,10} = 0.2305$	$\{A_5\}, \{A_1, A_4, A_8, A_{10}\}$, $\{A_2, A_3, A_6, A_7, A_9\}$	Figure 4(c)
$\lambda = z_{4,10} = 0.1824$	$\{A_5\}, \{A_1, A_8\}, \{A_4, A_{10}\}$, $\{A_2, A_3, A_6, A_7, A_9\}$	Figure 4(d)
$\lambda = z_{79} = 0.1405$	$\{A_5\}, \{A_1, A_8\}, \{A_4\}, \{A_{10}\}$, $\{A_2, A_3, A_6, A_7, A_9\}$	Figure 4(e)
$\lambda = z_{18} = 0.134$	$\{A_1, A_8\}, \{A_4\}, \{A_5\}, \{A_7\}$, $\{A_{10}\}, \{A_2, A_3, A_6, A_9\}$	Figure 4(f)
$\lambda = z_{26} = 0.131$	$\{A_1\}, \{A_4\}, \{A_5\}, \{A_7\}, \{A_8\}$, $\{A_{10}\}, \{A_2, A_3, A_6, A_9\}$	Figure 4(g)
$\lambda = z_{29} = 0.1155$	$\{A_1\}, \{A_4\}, \{A_5\}, \{A_6\}, \{A_7\}$, $\{A_8\}, \{A_{10}\}, \{A_2, A_3, A_9\}$	Figure 4(h)
$\lambda = z_{23} = 0.0952$	$\{A_1\}, \{A_2, A_3\}, \{A_4\}, \{A_5\}, \{A_6\}$, $\{A_7\}, \{A_8\}, \{A_9\}, \{A_{10}\}$	Figure 4(i)
$\lambda = 0$	$\{A_1\}, \{A_2\}, \{A_3\}, \{A_4\}, \{A_5\}$, $\{A_6\}, \{A_7\}, \{A_8\}, \{A_9\}, \{A_{10}\}$	Figure 4(j)

The first agroecological region includes: A_5—Pingxiang; the second: A_1—Fuzhou, and A_8—Yichun; the third: A_4—Jiujiang and A_{10}—Ji'an; and the fourth:A_2—Nanchang, A_3—Shangrao, A_6—Yingtan, A_7—Ganzhou, and A_9—Jingdezhen.

It is noted that the numbers of values in different HFEs of HFSs are the same in Example 1. However, in most cases, the numbers of values in different HFEs of HFSs may be different. In Example 2, we will make further discussion in detail.

To compare with the intuitionistic fuzzy MST (IFMST) clustering algorithm and the fuzzy MST (FMST) clustering algorithm, we give another example with six nodes for convenience. In Example 2, we will first make clustering analysis under hesitant fuzzy environment, and then consider the HFSs envelopes, i.e., intuitionistic fuzzy data, and make an IFMST clustering analysis. Finally, we will perform FMST clustering analysis when the considered IFSs reduce to the FSs by considering only the membership degrees of the data.

Example 2. In order to complete an operational mission, six sets of operational plans are made (adapted from Zhang et al., 2009 and 2012). To group these operational plans with respect to their comprehensive functions, a military committee has been set up to provide assessment information on them. The attributes which are considered here in assessment of the six sets of operational plans are: 1) x_1 is the effectiveness of operational organization; and 2) x_2 is the effectiveness of operational command. The military committee evaluates the performance of the six operational plans according to the attributes $x_j (j = 1, 2)$, and gives the hesitant fuzzy data as:

$$A_1 = \{< x_1, \{0.85, 0.70\} >, < x_2, \{0.80, 0.75, 0.60\} >\},$$

$$A_2 = \{< x_1, \{0.65, 0.5, 0.4\} >, < x_2, \{0.9, 0.8\} >\},$$

$$A_3 = \{< x_1, \{0.75, 0.6, 0.55\} >, < x_2, \{0.85, 0.8, 0.7\} >\},$$

$$A_4 = \{< x_1, \{0.65, 0.44\} >, < x_2, \{0.8, 0.7, 0.6\} >\},$$

$$A_5 = \{< x_1, \{0.65, 0.6, 0.5\} >, < x_2, \{0.8, 0.75\} >\},$$

$$A_6 = \{< x_1, \{0.75, 0.6, 0.55\} >, < x_2, \{0.85, 0.7, 0.57\} >\}.$$

Apparently the numbers of values in different HFEs of HFSs are different. To operate correctly, Xu and Xia (2011a) gave the following rules:
let $l(h(x))$ be the number of values in $h(x)$. If $l(h_1) < l(h_2)$, then h_1 should be extended by adding the minimal value in it until it has the same length as h_2; If $l(h_1) > l(h_2)$, then h_2 should be extended by adding the minimal value in it until it has the same length as h_1. At the same time, we can extend the shorter one by adding any value in it which mainly depends on the decision makers' risk preferences. Optimists anticipate desirable outcomes and may add the maximum value, while pessimists expect unfavorable outcomes and may add the minimal value. Then we consider that the decision makers are pessimistic

in Example 2, so we change the hesitant fuzzy data by adding the minimal values as below (for convenience of description, here we also list them in the corresponding sets):

$$A_1 = \{< x_1, \{0.85, 0.7, 0.7\} >, < x_2, \{0.8, 0.75, 0.6\} >\},$$

$$A_2 = \{< x_1, \{0.65, 0.5, 0.4\} >, < x_2, \{0.9, 0.8, 0.8\} >\},$$

$$A_3 = \{< x_1, \{0.75, 0.6, 0.55\} >, < x_2, \{0.85, 0.8, 0.7\} >\},$$

$$A_4 = \{< x_1, \{0.65, 0.44, 0.44\} >, < x_2, \{0.8, 0.7, 0.6\} >\},$$

$$A_5 = \{< x_1, \{0.65, 0.6, 0.5\} >, < x_2, \{0.8, 0.75, 0.75\} >\},$$

$$A_6 = \{< x_1, \{0.75, 0.6, 0.55\} >, < x_2, \{0.85, 0.7, 0.57\} >\}.$$

Then we proceed to utilize the HFMST clustering algorithm to group these operational plans $A_j (j = 1, 2, \cdots, 6)$:

Step 1. Construct the hesitant fuzzy distance matrix and the hesitant fuzzy graph:

1) Calculate the distance $z_{ij} = z_9(A_i, A_j)$ by Eq.(10), and then we get the hesitant fuzzy distance matrix $Z = (z_{ij})_{6\times 6}$ as:

$$Z = \begin{pmatrix} 0.0000 & 0.3264 & 0.1474 & 0.1733 & 0.2052 & 0.1140 \\ 0.3264 & 0.0000 & 0.1480 & 0.1761 & 0.1899 & 0.2406 \\ 0.1474 & 0.1480 & 0.0000 & 0.1609 & 0.090 & 0.0965 \\ 0.1733 & 0.1761 & 0.6090 & 0.0000 & 0.1540 & 0.1216 \\ 0.2052 & 0.1899 & 0.0900 & 0.1540 & 0.0000 & 0.1735 \\ 0.1140 & 0.2406 & 0.0965 & 0.1216 & 0.1735 & 0.0000 \end{pmatrix}.$$

2) Construct the fuzzy graph $G = (V, E)$ where every edge between A_i and A_j has the weight z_{ij} represented by HFSs as an element of the hesitant fuzzy distance matrix $Z = (z_{ij})_{6\times 6}$, which shows the dissimilarity degree between the samples A_i and A_j (see Fig. 5):

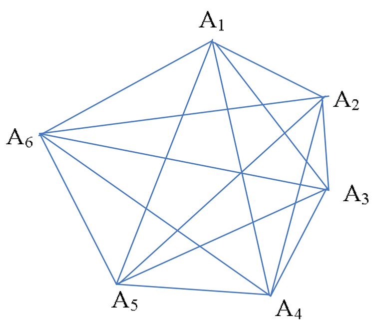


Figure 5. The hesitant fuzzy graph $G = (V, E)$

Step 2. Compute the hesitant fuzzy MST of the hesitant fuzzy graph $G = (V, E)$. See Step 2 in the HFMST clustering algorithm.

Step 3. Group the nodes (the operational plans) into clusters. See Step 3 in the HFMST clustering algorithm.

Hence, after the above steps, we obtain the clustering results listed in Table 2.

Table 2. The HFMST clustering results

λ	Corresponding clustering results
$\lambda = z_{23} = 0.1474$	$\{A_1, A_2, A_3, A_4, A_5, A_6\}$
$\lambda = z_{46} = 0.1216$	$\{A_2\}, \{A_1, A_3, A_4, A_5, A_6\}$
$\lambda = z_{16} = 0.114$	$\{A_2\}, \{A_4\}, \{A_1, A_3, A_5, A_6\}$
$\lambda = z_{36} = 0.0965$	$\{A_1\}, \{A_2\}, \{A_4\}, \{A_3, A_5, A_6\}$
$\lambda = z_{35} = 0.09$	$\{A_1\}, \{A_2\}, \{A_4\}, \{A_6\}, \{A_3, A_5\}$
$\lambda = 0$	$\{A_1\}, \{A_2\}, \{A_3\}, \{A_4\}, \{A_5\}, \{A_6\}$

According to Definition 2, the IFV $A_{env}(h)$ is the envelope of the HFE h, then we can transform the hesitant fuzzy data of Example 2 into intuitionistic fuzzy data:

$$A_1 = \{< x_1, 0.70, 0.15 >,\ < x_2, 0.60, 0.20 >\},$$
$$A_2 = \{< x_1, 0.40, 0.35 >,\ < x_2, 0.80, 0.10 >\},$$

$$A_3 = \{< x_1, 0.55, 0.25 >,\ < x_2, 0.70, 0.15 >\},$$
$$A_4 = \{< x_1, 0.44, 0.35 >,\ < x_2, 0.60, 0.20 >\},$$

$$A_5 = \{< x_1, 0.50, 0.35 >,\ < x_2, 0.75, 0.20 >\},$$
$$A_6 = \{< x_1, 0.55, 0.25 >,\ < x_2, 0.57, 0.15 >\}.$$

and then the operational plans x_i $(i = 1, 2, \cdots, 6)$ can be clustered with the following IFMST clustering algorithm (Zhao, 2012):

Step 1. Compute the intuitionistic fuzzy distance matrix and the fuzzy graph:

1) Calculate $z_{ij} = z(A_i, A_j)$ by the following distance measure:

$$z_{10}(A, B) = \sqrt{\frac{1}{2}\sum_{i=1}^{n} w_i((\mu_A(x_i) - \mu_B(x_i))^2 + (v_A(x_i) - v_B(x_i))^2 + (\pi_A(x_i) - \pi_B(x_i))^2)} \quad (12)$$

where the weight vector of the attributes $x_j (j = 1, 2)$ is $w = (0.45, 0.55)^T$, and we get the fuzzy distance matrix:

$$Z = \begin{pmatrix} 0.0000 & 0.2450 & 0.1225 & 0.1170 & 0.1725 & 0.1115 \\ 0.2450 & 0.0000 & 0.12250 & 0.1280 & 0.1000 & 0.1940 \\ 0.1225 & 0.1225 & 0.0000 & 0.1045 & 0.1000 & 0.0715 \\ 0.1170 & 0.1280 & 0.1045 & 0.0000 & 0.1095 & 0.0935 \\ 0.1725 & 0.1000 & 0.1000 & 0.1095 & 0.0000 & 0.1715 \\ 0.1115 & 0.1940 & 0.0715 & 0.0935 & 0.1715 & 0.0000 \end{pmatrix}.$$

2) Construct the fuzzy graph $G = (V, E)$ where every edge between A_i and A_j has the weight z_{ij} represented by an HFS as an element of the hesitant fuzzy distance matrix $Z = (z_{ij})_{6\times 6}$, which shows the dissimilarity degree between the samples A_i and A_j(see Fig. 5)

Step 2. Compute the MST of the intuitionistic fuzzy graph $G = (V, E)$. See also Step 2 in the HFMST clustering algorithm.

Step 3. Group the nodes (the operational plans) into clusters. See also Step 3 in the HFMST clustering algorithm.

Obviously, after the above steps, we can obtain the clustering results listed in Table 3.

Table 3. The IFMST clustering results

λ	Corresponding clustering results
$\lambda = z_{16} = 0.1115$	$\{A_1, A_2, A_3, A_4, A_5, A_6\}$
$\lambda = z_{25} = z_{35} = 0.1$	$\{A_1\}, \{A_2, A_3, A_4, A_5, A_6\}$
$\lambda = z_{46} = 0.088$	$\{A_1\}, \{A_2\}, \{A_5\}, \{A_3, A_4, A_6\}$
$\lambda = z_{36} = 0.0715$	$\{A_1\}, \{A_2\}, \{A_4\}, \{A_5\}, \{A_3, A_6\}$
$\lambda = 0$	$\{A_1\}, \{A_2\}, \{A_3\}, \{A_4\}, \{A_5\}, \{A_6\}$

It is well known that both the IFS and the HFS are the extensions of the traditional fuzzy sets. Compared with the IFS, which is composed of a membership degree, a non-membership degree and a hesitancy degree, the fuzzy set is only composed of the membership degree. So the intuitionistic fuzzy data are reduced to the fuzzy data when we only consider the membership degrees of the intuitionistic data, and the operational plan information given by the military committee become:

$$A_1 = \{< x_1, 0.70 >, < x_2, 0.60 >\}, \quad A_2 = \{< x_1, 0.40 >, < x_2, 0.80 >\}$$

$$A_3 = \{< x_1, 0.55 >, < x_2, 0.70 >\}, \quad A_4 = \{< x_1, 0.44 >, < x_2, 0.60 >\}$$

$$A_5 = \{< x_1, 0.50 >, < x_2, 0.75 >\}, \quad A_6 = \{< x_1, 0.55 >, < x_2, 0.57 >\}$$

and then the operational plans $x_i(i = 1, 2, \cdots, 6)$ can be clustered with the following FMST clustering algorithm:

Step 1. Compute the fuzzy distance matrix and the fuzzy graph:

1) Calculate $z_{ij} = z(A_i, A_j)$ by the following distance measure:

$$z_{11}(A, B) = \sum_{i=1}^{2} w_i(|\mu_A(x_i) - \mu_B(x_i)|) \tag{13}$$

where the weight vector of the attributes $x_j (j = 1, 2)$ is $w = (0.45, 0.55)^T$, and we get the fuzzy distance matrix:

$$D = \begin{pmatrix} 0.0000 & 0.2450 & 0.1225 & 0.1170 & 0.1725 & 0.0840 \\ 0.2450 & 0.0000 & 0.1225 & 0.1280 & 0.0725 & 0.1940 \\ 0.1225 & 0.1225 & 0.0000 & 0.1045 & 0.0500 & 0.0715 \\ 0.1170 & 0.1280 & 0.1045 & 0.0000 & 0.1095 & 0.0660 \\ 0.1725 & 0.0725 & 0.0500 & 0.1095 & 0.0000 & 0.1215 \\ 0.0840 & 0.1940 & 0.0715 & 0.0660 & 0.1215 & 0.0000 \end{pmatrix}.$$

2) Construct the fuzzy graph $G = (V, E)$ where every edge between A_i and A_j has the weight z_{ij} represented by an HFS as an element of the hesitant fuzzy distance matrix $Z = (z_{ij})_{6\times 6}$, which shows the dissimilarity degree between the samples A_i and A_j(see Fig. 5)

Step 2. Compute the MST of the fuzzy graph $G = (V, E)$. See also Step 2 in the HFMST clustering algorithm.

Step 3. Group the nodes (the operational plans) into clusters. See also Step 3 in the HFMST clustering algorithm.

Analogously, after the above steps, we get the clustering results listed in Table 4.

Table 4. The FMST clustering results

λ	Corresponding clustering results
$\lambda = z_{16} = 0.0840$	$\{A_1, A_2, A_3, A_4, A_5, A_6\}$
$\lambda = z_{25} = 0.0725$	$\{A_1\}, \{A_2, A_3, A_4, A_5, A_6\}$
$\lambda = z_{36} = 0.0715$	$\{A_1\}, \{A_2\}, \{A_3, A_4, A_5, A_6\}$
$\lambda = z_{46} = 0.0660$	$\{A_1\}, \{A_2\}, \{A_3, A_5\}, \{A_4, A_6\}$
$\lambda = z_{35} = 0.0500$	$\{A_1\}, \{A_2\}, \{A_3, A_5\}, \{A_4\}, \{A_6\}$
$\lambda = 0$	$\{A_1\}, \{A_2\}, \{A_3\}, \{A_4\}, \{A_5\}, \{A_6\}$

In order to provide a better view of the comparison, we put the clustering results of those three algorithms into Table 5.

After calculations, we find that the clustering results of those three clustering algorithms are quite different. The main reason is that the HFMST clustering algorithm clusters the fuzzy information which is represented by several possible values, not by a margin of error (as in IFSs), while the FMST clustering algorithm clusters the fuzzy information which only considers the membership degrees and thus loses too much information. Obviously, compared with the clustering results of the FMST clustering algorithm, both the HFMST clustering results and the IFMST clustering results are more reasonable. Moreover,

Table 5. Clustering results

Classes	The HFMST clustering Algorithm	The IFMST clustering algorithm	The FMST clustering algorithm
6	$\{A_1\},\{A_2\},\{A_3\},$ $\{A_4\},\{A_5\},\{A_6\}$	$\{A_1\},\{A_2\},\{A_3\},$ $\{A_4\},\{A_5\},\{A_6\}$	$\{A_1\},\{A_2\},\{A_3\},$ $\{A_4\},\{A_5\},\{A_6\}$
5	$\{A_1\},\{A_2\},\{A_4\},$ $\{A_6\},\{A_3,A_5\}$	$\{A_1\},\{A_2\},\{A_4\},$ $\{A_5\},\{A_3,A_6\}$	$\{A_1\},\{A_2\},\{A_4\},$ $\{A_6\},\{A_3,A_5\}$
4	$\{A_1\},\{A_2\},\{A_4\},$ $\{A_3,A_5,A_6\}$	$\{A_1\},\{A_2\},\{A_5\},$ $\{A_3,A_4,A_6\}$	$\{A_1\},\{A_3,A_5\},$ $\{A_2\},\{A_4,A_6\}$
3	$\{A_2\},\{A_4\},$ $\{A_1,A_3,A_5,A_6\}$		$\{A_1\},\{A_2\},$ $\{A_3,A_4,A_5,A_6\}$
2	$\{A_2\},$ $\{A_1,A_3,A_4,A_5,A_6\}$	$\{A_1\},$ $\{A_2,A_3,A_4,A_5,A_6\}$	$\{A_1\},$ $\{A_2,A_3,A_4,A_5,A_6\}$
1	$\{A_1,A_2,A_3,A_4,A_5,A_6\}$	$\{A_1,A_2,A_3,A_4,A_5,A_6\}$	$\{A_1,A_2,A_3,A_4,A_5,A_6\}$

when we encounter some situations where information is represented by several possible values, the HFMST clustering algorithm demonstrates its great superiority in clustering those hesitant fuzzy data.

5. Concluding remarks

The well-known MST clustering algorithm is an intuitively simple and effective clustering technique, which has been extensively applied to various fields. However, the traditional MST clustering algorithm cannot cluster data represented by hesitant fuzzy information. In this article, we have proposed the HFMST clustering algorithm, which can be used for clustering hesitant fuzzy information successfully. Furthermore, the computational tests on the HFMST clustering algorithm, the IFMST clustering algorithm and the FMST clustering algorithm have shown that the clustering results of the HFMST clustering algorithm and the IFMST clustering algorithm are more reasonable than those of the FMST clustering algorithm. At the same time, in situations where information is represented by several possible values, the HFMST clustering algorithm shows its great superiority in clustering those hesitant fuzzy data and provides many detailed clustering results. Because the HFSs are a powerful tool to deal with vagueness and uncertainty, in the future, the developed algorithm can be used in many applications including information retrieval, equipment evaluation, investment decision making, data mining, etc.

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