

Optimization of urban MV multi-loop electric power  
distribution networks structure using Artificial  
Intelligence methods\*

by

Mirosław Parol<sup>1</sup>, Dariusz Baczyński<sup>1</sup> and Janusz Brożek<sup>2</sup>

<sup>1</sup>Warsaw University of Technology  
Institute of Electrical Power Engineering  
00-662 Warsaw, Koszykowa 75, Poland  
mirosław.parol@ien.pw.edu.pl  
dariusz.baczynski@ien.pw.edu.pl

<sup>2</sup>Cracow University of Mining and Metallurgy  
Faculty of Electrical Engineering, Automatics,  
Computer Science and Biomedical Engineering  
30-059 Cracow, al. Mickiewicza 30, Poland  
jbroz@agh.edu.pl

**Abstract:** Urban medium voltage (MV) electric power distribution networks are supplied with primary (HV/MV) substations. These networks supply secondary (MV/LV) transformer substations and are often built as closed structures - loop arrangements. The design problem of optimal urban MV distribution network structure consists of determining the number of primary substations, establishing the number of MV loops supplied with the primary substations, and assigning the secondary MV/LV transformer substations to the MV loops. The optimization task becomes especially complex when the number of the primary substations is greater than one. The minimum of total annual costs is sought. The total annual costs include: fixed (investment) costs, variable (operating) costs and supply-interruption costs. Typical constraints are also accounted for. The so defined optimization problem is a complicated mathematical problem in respect of computational effort. In order to resolve the mathematical model of the optimization problem, evolutionary algorithms and artificial neural networks have been used. Exemplary computational experiments have been executed on the model of urban MV multi-loop electric power distribution networks. The results from the evolutionary algorithm and the artificial neural network calculations have been compared.

**Keywords:** electric power distribution networks, optimization of network structure, evolutionary algorithms, artificial neural networks.

## 1. Introduction

Design of the optimal structure of Medium Voltage (MV) multi-loop electric power networks, typical for urban areas, where there are more than one supplying points (HV/MV primary substations, the so called Main Feeding Points (MFP)) is a complex problem, requiring the use of optimization algorithms.

Optimization is meant to find the proper number of supply points (HV/MV primary substations) and of MV loops, and to assign electric energy receiving points (MV/LV distribution substations) to the MV loops and supply points. This task is similar to the Multi Depot Vehicle Routing Problem (MDVRP). The MDVRP is a generalization of the Vehicle Routing Problem (VRP), where loops and supplying points have to be designed in such a way that their capacity is not exceeded. The MDVRP is a very complicated mathematical problem in terms of calculation effort (so called NP problem).

The classical approach to the design of multi-loop network structure consists in determining a set of loops with minimal total annual costs. MV open-loop distribution network planning problem has been described in Glamocanin and Filipovic (1993) and Levitin et al. (1995). Glamocanin and Filipovic (1993) presented an heuristic algorithm, which solves the problem of minimum sum of loop lengths. In turn, multi-criteria approach (based on a genetic algorithm) for distribution network structure optimisation has been presented in Levitin et al. (1995). Minimum sum of loop lengths, minimum power losses and minimum load imbalance between transformers have been taken into account. In both papers MV loops begin and end in different, strictly determined substations.

Approach based on artificial intelligence methods was also used in optimization problems with one supplying point (Brożek, 1999, 2004; Parol, 2003). In Brożek (1999 and 2004) the approach with artificial neural networks and a single criterion has been described. Use of evolutionary algorithms, fuzzy numbers and multi-criteria optimization was presented in Parol (2003).

Different aspects concerning MV and LV distribution network planning have been presented in many papers in recent years (Ramirez-Rosado et al., 2006; Mendoza et al., 2006; Nahman and Peric, 2008; Navarro and Rudnick, 2009A,B; Najafi et al., 2009; Lavorato et al., 2010; Wang et al., 2011). These papers concerned mainly distribution systems operating as tree-based radial networks.

A new tabu search algorithm to solve a multiobjective fuzzy model for optimal planning of distribution systems has been presented in Ramirez-Rosado et al. (2006). The algorithm allows to obtain multiobjective non-dominated solution for three objective functions.

Application of two optimization techniques: the non-dominated sorting genetic algorithm and the strength Pareto evolutionary algorithm, to the multiobjective design of power distribution systems has been shown, in turn, in Mendoza et al. (2006). The algorithms have been used to minimize the total annual cost and simultaneously to maximize the reliability of the distribution systems.

Optimal planning of radial distribution systems by means of combination

of two methods: steepest descent and simulated annealing, has been presented in Nahman and Peric (2008). The aim was to find the routes ensuring the minimum total annual cost.

Navarro and Rudnick (2009A,B) described planning methods for large low voltage distribution systems, showing combined optimization of transformers and networks associated with them. Voronoi diagram and tabu search were used in optimization.

Application of genetic algorithms for optimal planning of large distribution systems was considered in Najafi et al. (2009). The aim was to obtain optimal sizes and locations of HV and MV substations, as well as MV feeder routes. Fixed and variable costs were taken into account in the problem.

In Lavorato et al. (2010) distribution system planning with the use of a constructive heuristic algorithm has been described. Tree-based distribution networks supplied by one or greater number of substations have been analyzed.

A new balanced genetic algorithm and modified data envelopment analysis to solve multistage distribution system expansion planning problems considering future uncertainties have been presented in Wang et al. (2011). The multistage expansion of tree-based distribution network structure was considered.

When we take into account all annual costs components, the approach based on MDVRP usually cannot be used for problems of realistic sizes with multi-supply points. In these types of problems with large number of power receiving points it is worth using heuristic methods. Utilization of artificial neural networks to such a problem was presented in Brożek (2003).

## 2. Problem formulation

Multi-loop network structure design problem consists in determining a set of supply points and a set of loops with minimal total annual costs. Loops begin and end at one selected supplying point. The general aim of multi-loop network construction is to supply a set of receiving points with a fixed power (electric energy) demand and a fixed location, with assumption that each receiving point should be supplied from one and only one loop. The total annual costs include fixed (investment) costs, variable (operating) costs and supply-interruption costs:

$$C_{tot} = \sum_{k=1}^m \left\{ \sum_{j=1}^{n_k+1} (C_{f,j} + C_{v,j}) + \sum_{i=1}^{n_k} C_{ur,ik} \right\} + \sum_{l=1}^z (C_{f,l} + C_{v,l}) \quad (1)$$

where:  $C_{f,j}$ ;  $C_{v,j}$  – fixed and variable costs of  $j$ -th network loop section,  
 $C_{ur,ik}$  – unreliability (supply-interruption) costs of  $i$ -th receiving point in  $k$ -th network loop,

$C_{f,l}$ ;  $C_{v,l}$  – fixed and variable costs of  $l$ -th supply point (HV/MV substation),

$m$  – number of loops in the designed network,

$n_k$  – number of receiving points in  $k$ -th network loop,

$z$  – number of supply points.

The fixed costs are directly independent of power flows in network loops and substations. In turn, variable costs (power and electric energy loss costs) depend on power flows in network elements. The supply-interruption costs are a complex function of distribution network structure, reliability parameters of network elements and undelivered energy in each receiving point.

The fixed costs are calculated from the commonly known equations (Kulczycki, 1990).

The variable costs in  $j$ -th network loop section can be calculated from the following equation

$$C_{v,j} = \Delta P_j (c_P + c_A \tau_{\max}) \quad (2)$$

where:  $\Delta P_j$  – maximal power losses in  $j$ -th loop section,

$c_P$  – unit power cost in the network,

$c_A$  – unit electric energy cost,

$\tau_{\max}$  – time of maximal load loss duration.

The variable costs in  $l$ -th supply point (with one HV/MV transformer) can be calculated from the following equation

$$C_{v,l} = \left[ \Delta P_{Fe,l} + P_{krT,l} \left( \frac{S_l}{S_{rT,l}} \right)^2 \right] (c_P + c_A \tau_{\max}) \quad (3)$$

where:  $\Delta P_{Fe,l}$ ;  $P_{krT,l}$  – iron and rated copper losses in  $l$ -th transformer,

$S_l$  – apparent power flow in  $l$ -th transformer,

$S_{rT,l}$  – rated apparent power of  $l$ -th transformer.

Usually two transformers are located in an MFP. Then, the apparent power flow  $S_l$  in  $l$ -th supply point is divided among these transformers and the variable costs of the substation are the sum of costs for particular transformers.

The supply-interruption costs can be defined as follows (Kulczycki, 1990):

$$C_{ur} = \sum_{k=1}^m \sum_{i=1}^{n_k} P_{ki} T_{p,ki} q^{ki} c_{ur,ki} \quad (4)$$

$$q^{ki} = q_l^{ki} q_r^{ki} \quad (5)$$

where:  $P_{ki}$ ;  $T_{p,ki}$ ;  $q^{ki}$  – peak load, utilization time and relative supply-interruption duration of  $i$ -th receiving point in  $k$ -th network loop,

$c_{ur,ki}$  – unit unreliability costs regarding  $i$ -th receiving point in  $k$ -th network loop, caused by random outages,

$q_l^{ki}$ ;  $q_r^{ki}$  – unreliability factor of the part of  $k$ -th network loop from  $i$ -th receiving point to the left (to supply point) or to the right, respectively.

In the here presented method assumption is made that line cross-sections in the MV loops of designed network are fixed and are able to endure the expected thermal short circuit flows.

Voltage level at the  $i$ -th point of service connection to the electric network should remain, conform to the obligatory legal regulations, in the interval

$\langle U_i^l, U_i^u \rangle$ , where  $U_i^l$  is the lower feasible limit of voltage level, and  $U_i^u$  is the upper feasible limit of voltage level at the  $i$ -th point of an electric network.

In order to design the optimal structure of MV multi-loop electric power network it is necessary to assume that the maximal loads, which can appear in the calendar year, are considered. Usually this takes place at the so called autumn-winter peak. In the method proposed it is assumed that these loads (apparent powers) are known and appear simultaneously at all receiving points.

The essential conditions, which the designed network must meet, are as follows:

- there are  $z$  supplying points with known locations,
- there are  $n$  receiving points with known apparent power demand and known location,
- there is a set of feasible line (loop) routes,
- the designed electric power network operates in open configuration,
- a set of technical constraints must be fulfilled in the designed network.

### 3. Mathematical model

Optimization of multi-loop electric power distribution network structure is a complex all integer mathematical problem.

Mathematical model of the optimization problem is formulated as follows (see Kulczycki, 1990): network  $S = \langle N, B, C_f, C_v, C_{ur}, P' \rangle$  in the sense of graph theory is given, where:  $N$  – set of electric power network nodes,  $B$  – set of power network branches,  $C_f$  – set of fixed costs in loop sections and substations,  $C_v$  – set of variable costs in loop sections and substations,  $C_{ur}$  – set of supply-interruption costs,  $P'$  – set of power demands at receiving points. Optimization should divide the graph  $G = \langle N, B \rangle$  of network  $S$  into  $m$  subgraphs  $G_{p,k}$  ( $k = 1, 2, \dots, m$ ). The set  $N$  contains two subsets:  $N_{sp}$  (supplying points) and  $N_{rp}$  (receiving points). We have to define  $m$  subsets  $N_k$  to minimize total annual costs in the electric power network. Each set  $N_k$  contains respective subset  $N_{rp,k}$  and one and only one node from the set  $N_{sp}$ .

The choice of the optimal multi-loop network structure is defined as follows: to find the optimal solution to the following problem:

$$\begin{aligned} \min C_{tot} = & \sum_{k=1}^m \left\{ \sum_{j=1}^{n_k+1} \left( C_{f,j} \left( G_{p,k}, P'_k \right) + C_{v,j} \left( G_{p,k}, P'_k \right) \right) + \sum_{i=1}^{n_k} C_{ur,i} \left( G_{p,k}, P'_k \right) \right\} + \\ & + \sum_{l=1}^z \left( C_{f,l} \left( G, P' \right) + C_{v,l} \left( G, P' \right) \right) \end{aligned} \quad (6)$$

subject to the following set of constraints:

- (1) I Kirchhoff law must be fulfilled,
- (2) II Kirchhoff law must be fulfilled,

(3) Power flows in branches (loops and substations) do not exceed the capacity of each designed branch,

$$P_j \leq c_k, \quad \forall j = 1, 2, \dots, n_k + 1; \forall k = 1, 2, \dots, m \quad (7)$$

$$S_l \leq c_l, \quad \forall l = 1, 2, \dots, z \quad (8)$$

(4) Voltage drops in selected subset of routes do not exceed feasible values (limits),

$$\sum_{j=1}^{n_k+1} b_{ij} \Delta U_j(P_j) \leq \Delta U_{f,i}, \quad \forall i = 1, 2, \dots, n_r \quad (9)$$

where:  $b_{ij}$  is the element of the route-branch incidence matrix,  
 $\Delta U_j(P_j)$  – voltage drop in the  $j$ -th branch (loop section),  
 $\Delta U_{f,i}$  – feasible voltage drop in the  $i$ -th route,

(5) Nodes from the subset  $N_{sp}$  are the only common nodes of all subsets  $N_k$ ,

$$N_{k1,l} \cap N_{k2,l} = N_{sp,l}, \quad \forall k1, k2 = 1, 2, \dots, m; \quad k1 \neq k2; \quad l = 1, 2, \dots, z \quad (10)$$

(6) Each node from the set  $N_{rp}$  should be assigned to one of the created subsets  $N_k$ ,

$$\cup N_{rp,k} = N \setminus N_{sp}, \quad k = 1, 2, \dots, m \quad (11)$$

Expressions (10) and (11) guarantee that each receiving point will be connected to one and only one network loop.

(7) Each node from the set  $N_{rp}$  should be assigned to one and only one supply point

$$N_{k1,l} \cap N_{k2,r} = \emptyset, \quad \forall k1, k2 = 1, 2, \dots, m; \quad k1 \neq k2 \quad \forall l, r = 1, 2, \dots, z; \quad l \neq r \quad (12)$$

(8) Each subset  $N_k$  should contain at least two nodes,

$$|N_k| \geq 2, \quad \forall k = 1, 2, \dots, m \quad (13)$$

(9) Number of incidence branches with receiving nodes is two,

$$\sum_{j=1}^{n_k+1} |a_{ij}| = 2, \quad \forall i = 1, 2, \dots, n_k \quad (14)$$

(10) Number of incidence branches with supplying point is at least two,

$$\sum_{j \in J_0} |a_{ij}| \geq 2, \quad \forall i = 1, 2, \dots, n \quad (15)$$

Formulae (14) and (15) guarantee that the designed electric power network has a loop structure, i.e. each receiving point is directly connected with only two other receiving points.

In formulae (6)÷(15) the following notations are introduced:

$c_k$  – capacity of the  $k$ -th loop, which supplies power to receiving points,

$c_l$  – capacity of the  $l$ -th substation,

$n_k$  – number of power receiving points in  $k$ -th network loop,

$n_r$  – number of routes in the network, for which the voltage drop requirements must be fulfilled,

$a_{ij}$  – an element of the node-branch incidence matrix,

$J_0$  – set of pointers of incidence branches (loop sections) with supplying points.

A feasible solution of this optimization problem is constituted by each network  $S^i \subset S$ , satisfying constraints (1)÷(11). The aim of optimization is to choose such feasible solution, which attains the best value of the objective function (6).

To calculate power flow  $P_j$  and a voltage drop  $\Delta U_j$  in each section of a loop, it is necessary to solve an additional problem. This additional problem is to find a set of splitting places in the loops, because the designed network must operate as open. We assume, for simplicity, that this problem will be solved locally, i. e. optimization calculations will be executed separately for each loop.

#### 4. Solution of the problem based on evolutionary technique

Working of the evolutionary (genetic) algorithms was presented in details in many publications, e.g., in Michalewicz (1996) and Cheng et al. (1997). The main idea consists in adaptation of some notions and phenomena, appearing in natural genetics and biological evolution processes.

The evolutionary algorithm for designing an urban multi-loop electric power network optimal structure is presented below. In construction of this algorithm some concepts and solutions from Cheng et al. (1995, 1997) and Cheng and Gen (1996) have been utilized.

In the further part of the paper main components of the designed evolutionary algorithm are presented.

##### 4.1. Problem representation

Each individual (chromosome) consists of genes. In turn, each gene is an ordered triple (index numbers of power receiving point, network loop, and supply point) corresponding to a power receiving point in the designed network. The chromosome structure is given in Table 1.

Table 1. Chromosome structure (the manner of coding)

	gene 1	gene 2	...	gene n
<b>data</b>	receiving point $rp_1$	receiving point $rp_2$	...	receiving point $rp_n$
	loop $l_{rp_1}$	loop $l_{rp_2}$	...	loop $l_{rp_n}$
	supply point $sp_{rp_1}$	supply point $sp_{rp_2}$	...	supply point $sp_{rp_n}$

#### 4.2. Population initialization

In the analysed problem, due to loop and supply point capacity constraints it is not possible to use a simple random procedure to create the initial population. Moreover, to create a promising initial population, power receiving points should be connected to supply points located in their neighbourhood. For this purpose a procedure of assigning receiving points to supply points was included. The assignment made is only a kind of “suggestion” (introductory solution) for the rest of population initialisation procedure.

```

procedure Initial_Population;
begin
  Assign receiving points to supply points
   $i := 0$ ;
  while ( $i \leq N$ ) do
    begin
      Create random permutation of receiving points;
      Assign receiving points to loops;
       $i := i + 1$ ;
    end
  end

```

A procedure dealing with loops, proposed in this paper, contains two steps: to create random permutation of power receiving points and to assign receiving points to power network loops.

*Receiving\_Points\_Assign\_to\_Loops* procedure is a sequential adding procedure. Its task is to assign receiving points to respective loops and supply points. When new receiving point is added to a current loop, it is necessary to check whether permissible loop capacity is not exceeded. If this condition is fulfilled, then receiving point is assigned to that current loop; otherwise the receiving point is assigned to a new loop. During assignment of each receiving point the capacity of the supply point is examined. If it is exceeded, assignment to a new supply point follows. The described procedure is repeated until all receiving points are assigned to loops. The formal description of this procedure looks as follows:

```

procedure Receiving_Points_Assign_to_Loops;
begin {description of actions for each chromosome}
   $s := 1$ ; {number of a successive supplying point}

```



```

 $k := 1$ ; {number of a successive loop}
 $i := 1$ ; {number of a successive receiving point}
while ( $i \leq n$ ) do
begin
    Check permissible capacity of current supplying point;
    if Supplying point capacity not exceeded then
        begin
            Check permissible capacity of current loop;
            if Capacity not exceeded
                then Add receiving point to current loop;
            else
                begin
                     $k := k + 1$ ;
                    Add receiving point to new loop;
                end;
            end
        end
    else
        begin
             $s := s + 1$ ;
             $k := k + 1$ ;
            Add receiving point to new loop connected to new
            supplying point;
        end;
         $i := i + 1$ ;
    end
end

```

#### 4.3. Genetic operations

In the presented method two genetic operators, crossover and mutation, were used. Crossover operator is based on the best insertion heuristics. The best insertion algorithm is regarded as the best one among all approaches in problems similar to here considered optimization.

The action of the specialized crossover operator can be presented in the following way (Cheng et al., 1995, 1997, and Cheng and Gen, 1996):

Step 1. Choose randomly as initial receiving point of chromosome\_offspring either the rightmost receiving points of chromosome\_parents, which were earlier selected for crossover operation.

Step 2. Exchange in the second (not selected) chromosome\_parent positions of two receiving points in such a way that randomly selected receiving point is in the last position in both chromosome\_parents.

Step 3. Remove the selected receiving point from both chromosome\_parents to chromosome\_offspring and assign it to loop no. 1 and to the nearest supply point (which now becomes the current one).

Step 4. Two adjacent receiving points, with respect to selected previously receiving point, become the candidates for insertion. Calculate the best insertion place (position) on the partial tour of chromosome\_offspring for each of them and select “the best” insertion receiving point.

Step 5. Insert “the best” receiving point into the current loop.

Step 6. If neither of the two candidates can be connected to the current supply point, one of them is randomly inserted in the last current position of chromosome\_offspring, to open a new loop connected to the nearest supply point (which now becomes the current one) that can supply it.

Step 7. If neither of the two candidates can be inserted into the current loop, one of them is randomly inserted in the last current position of chromosome\_offspring, to open a new loop.

Step 8. Repeat the above steps of calculating, comparing, exchanging, removing and inserting until a complete chromosome\_offspring is created.

The best  $k$ -th receiving point for insertion between  $i$ -th and  $j$ -th receiving points is determined by the following formula (Cheng et al., 1997):

$$c(i, k, j) = 1 - \frac{d_{ij}}{d_{ik} + d_{kj}} \quad (16)$$

where:  $d_{ij}$  – distance between  $i$ -th and  $j$ -th receiving points.

Insertion measure function  $c(i, k, j)$  is a quasi normalized detour. The best  $k$ -th receiving point for insertion is the one that minimizes this function.

*Remarks:*

1. A receiving point can be inserted between any two receiving points already assigned to the loop, also between supply point and the first (last) receiving point in the loop.
2. Each time during the insertion it is necessary to check:
  - (a) is it possible to insert a given receiving point into the current loop in view of its capacity?
  - (b) where is the best insertion place on the already existing tour (according to the optimal insertion rule)?

The first problem oriented mutation operator acts as follows:

Step 1. Choose randomly a receiving point of a chromosome\_parent, which was earlier selected for mutation operation.

Step 2. Find the beginning and the end of the loop to which the chosen receiving point belongs.

Step 3. Search for the best insertion position of the chosen receiving point on the tour from the beginning to the end of the defined loop. The best insertion position is a place, which minimises the summarised length of the loop.

Step 4. Insert randomly chosen receiving point in the best insertion position.

Action of the second problem oriented mutation operator can be formulated in the following way (see Fig. 1):

Step 1. Choose randomly a receiving point A of a chromosome\_parent, which was earlier selected for the mutation operation.

Step 2. Find the beginning and the end of the loop (the first and the last receiving point in the loop) to which the chosen receiving point belongs.

Step 3. Find a receiving point B which is the closest one to the receiving point A and does not belong to the loop where point A is located.

Step 4. Create set R of receiving points, located in those loops to which points A and B belong.

Step 5. Find two receiving points C and D from the set R, which are the most distant to each other and to the supply points of those loops.

Step 6. Determine which of points C and D is closer to which supply point.

Step 7. Start creating new loops by adding as first to these loops points C and D, respectively.

Step 8. Add to loops successively the remaining receiving points which are the closest to both the supply point and the previously added receiving point.

The primary aim of the second mutation operator is to split the loops, which can cross each other. This results in shortening of the total length of all loops (see Fig. 2).

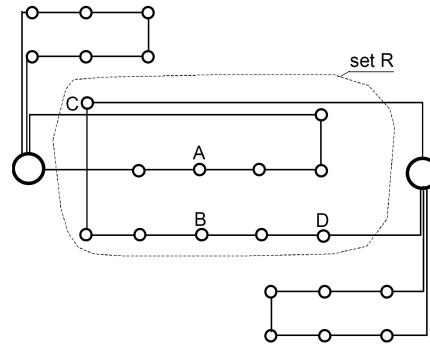


Figure 1. Example of an action of the second mutation operator – initial state

#### 4.4. Evaluation function

The fitness function for each individual in the population is calculated in the following manner (Cheng et al., 1997):

$$eval(\mathbf{v}_k) = \frac{g(\mathbf{v}_{\max}^0) - g(\mathbf{v}_k)}{g(\mathbf{v}_{\max}^0) - g(\mathbf{v}_{\min}^0)} \quad (17)$$

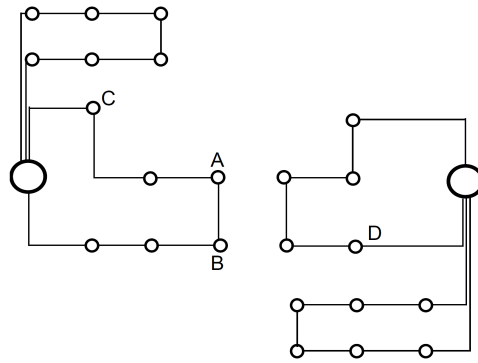
$$g(\mathbf{v}_k) = \frac{C_{tot}(\mathbf{v}_k)}{C_{tot\max}^0} \quad (18)$$

where:  $\mathbf{v}_k$  –  $k$ -th individual in current population,

$C_{tot}(\mathbf{v}_k)$  – total annual network costs for individual  $\mathbf{v}_k$ ,

$C_{tot\max}^0$  – maximum total annual network costs among all individuals of the initial population.

Figure 2. Example of an action of the second mutation operator – final state



## 5. Solution of the problem based on artificial neural networks

There are  $n$  receiving points – MV/LV transformer substations (set  $N_{rp}$ ) with their locations given. These locations compose the vector  $\mathbf{X}_{\mathbf{Koh}}$  of receiving point coordinates. The components of this vector can be defined as:  $X_{rp,i}, Y_{rp,i}$ ,  $i = 1, 2, \dots, n$ . There are also  $z$  supplying points – HV/MV substations (set  $N_{sp}$  – MFPs) together with their locations:  $X_{sp,j}, Y_{sp,j}$ ,  $j = 1, 2, \dots, z$ .

To solve the presented optimization problem an unsupervised self-organizing neural network using the WTM learning rule (the Winner Takes Most) was used. The WTM (Anil et al., 1996; Hertz et al., 1991) is a modified WTA (the Winner Takes All) algorithm. In the WTA algorithm in each iteration-step only one neuron (i.e., the winning one) is adapted. In the WTM algorithm the winning neuron and the neighboring neurons have their weights adapted. The longer the distance between the winning and the neighboring neuron the smaller the change of the neighboring neuron's weight.

The algorithm has the following three stages (Brożek, 2004):

### 5.1. STAGE A

In STAGE A the input set  $N_{rp}$  is divided into  $z$  subsets  $N_{rp}^l$  in such way that  $\bigcup N_{rp}^l = N_{rp}$ ,  $l = 1, 2, \dots, z$ . The input set division is realized with the use of a competitive neural network that is based on the WTM learning rule. In order to calculate the neighborhood-function the neural-gas algorithm is used (Martinetz et al., 1993). This stage of the algorithm is realized in the following steps:

1. Normalize the input vector  $\mathbf{X}_{\mathbf{Koh}}$ , initialize weights of neurons and set the initial learning rate.
2. Present normalized vector  $\overline{\mathbf{X}}_{\mathbf{Koh}}$  and evaluate the network's outputs.

3. Calculate:

$$\|\bar{\mathbf{X}}_{\text{Koh}} - \mathbf{w}_k\| \leq \|\bar{\mathbf{X}}_{\text{Koh}} - \mathbf{w}_i\| \quad \forall 1 \leq i \leq N_e \quad (19)$$

where  $k$  is the best-match neuron,  $N_e$  is the number of neurons.

4. Rank the neurons according to their distance from the input vector:

$$\|\bar{\mathbf{X}}_{\text{Koh}} - \mathbf{w}_{k_0}\| \leq \|\bar{\mathbf{X}}_{\text{Koh}} - \mathbf{w}_{k_1}\| \leq \dots \leq \|\bar{\mathbf{X}}_{\text{Koh}} - \mathbf{w}_{k_{N_e-1}}\| \quad (20)$$

where:

$\mathbf{w}_{k_0}$  – is the weight vector of the closest neuron to  $\bar{\mathbf{X}}_{\text{Koh}}$ ,

$\mathbf{w}_{k_1}$  – is the weight vector of the second - closest neuron to  $\bar{\mathbf{X}}_{\text{Koh}}$ ,

$\mathbf{w}_{k_j}$  – is the weight vector of the  $(j+1)$ -th closest neuron to  $\bar{\mathbf{X}}_{\text{Koh}}$ ,

$\mathbf{w}_{k_{N_e-1}}$  – is the weight vector of the last neuron in the rank, and

$$1 \leq j \leq N_e - 1.$$

5. Update all weights according to the learning rule:

$$\mathbf{w}_i(t+1) = \mathbf{w}_i(t) + \eta(t) \Lambda(j) [\bar{\mathbf{X}}_{\text{Koh}}(t) - \mathbf{w}_i(t)] \quad (21)$$

where  $\eta(t)$  is the learning rate, and  $\Lambda(j)$  is the neighbourhood function in the  $t^{\text{th}}$  iteration step.

6. Calculate the values of  $\eta(t)$  and  $\Lambda(j)$ :

$$\eta(t) = \eta_{\max} \left( \frac{\eta_{\min}}{\eta_{\max}} \right)^{\frac{t}{itrr}} \quad (22)$$

where  $\eta_{\min}$  and  $\eta_{\max}$  are the minimum and the maximum values of the learning rate  $\eta$ , respectively, and  $itrr$  is the maximum number of iteration steps in STAGE A of the algorithm;

$$\Lambda(j) = e^{-\frac{j}{\lambda(t)}} \quad (23)$$

$$\lambda(t) = \lambda_{\max} \left( \frac{\lambda_{\min}}{\lambda_{\max}} \right)^{\frac{t}{itrr}} \quad (24)$$

is the decay constant.  $\lambda_{\min}$  and  $\lambda_{\max}$  stand for the minimum and the maximum values of the decay constant, respectively.

7. Repeat steps 2 through 6 until  $t = itrr$ , where  $itrr$  is the given number of iteration steps.

8. Define all subsets  $N_{rp}^l$ .

The division of the set  $N_{rp}$  results in creation of separate sets  $N_{rp}^l$  “concentrated” around their “geographical” centers.

## 5.2. STAGE B

In STAGE B the subsets  $N_{rp}^l$  are divided into  $m_l$  subsets  $N_{rp,k}^l$  such that  $\bigcup N_{rp,k}^l = N_{rp}^l$ ,  $k = 1, 2, \dots, m_l$ , where  $m_l$  is the given number of loops in the set  $N_{rp}^l$ . Every subset  $N_{rp}^l$  is taught according to the principles described in STAGE A. The process is repeated  $z$  times.

### 5.3. STAGE C

During STAGE C the supply point (MFP) is added to each subset  $N_{rp,k}^l$ , yielding  $m_k$  learning sequences ( $N_k^l$ ). In each subset  $N_k^l$  a Hamilton cycle (Hertz et al., 1991) is defined with use of Kohonen's SOM (Kohonen, 1995; and Osowski, 2006). In this stage the number of neurons in the Kohonen's layer equals to the size of the input vector multiplied by  $nn$ . The value of  $nn = 3$  has been established experimentally. The steps of this stage of the algorithm are the following:

1. Creation of the subsets  $N_k^l = N_{rp,k}^l \cup N_{sp,k}^l$ ,  $k = 1, 2, \dots, m_l$  (where  $N_{sp,k}^l$  is one-element set representing MFP for the set  $N_{rp}^l$ ). Coordinates of the respective receiving and supply point locations form the vector  $\mathbf{X}_{\mathbf{Koh}}^{\mathbf{kl}}$ .
2. Normalize the input vector  $\mathbf{X}_{\mathbf{Koh}}^{\mathbf{kl}}$ , initialize weights of neurons, set the initial learning rate and neighbourhood.
3. Present the normalized vector  $\bar{\mathbf{X}}_{\mathbf{Koh}}^{\mathbf{kl}}$  and evaluate the network's outputs.
4. Select the winning neuron  $k$  according to the formula:

$$\|\bar{\mathbf{X}}_{\mathbf{Koh}}^{\mathbf{kl}} - \mathbf{w}_k\| = \min_i \|\bar{\mathbf{X}}_{\mathbf{Koh}}^{\mathbf{kl}} - \mathbf{w}_i\| \quad (25)$$

5. Update all weights according to the learning rule:

$$\mathbf{w}_i(t+1) = \mathbf{w}_i(t) + \eta(t) \Lambda(k, i) [\bar{\mathbf{X}}_{\mathbf{Koh}}^{\mathbf{kl}}(t) - \mathbf{w}_i(t)] \quad (26)$$

where  $\eta(t)$  is the learning rate, and  $\Lambda(k, i)$  is the neighbourhood function in the  $t^{th}$  iteration step.

6. Decrease the value of  $\eta(t)$  according to the formula (22) substituting  $itr$  with  $itr$ , where  $itr$  is the maximum number of iteration steps in STAGE C.
7. Calculate  $\Lambda(k, i)$  according to the formula:

$$\Lambda(k, i) = e^{-\frac{l(k, i)}{\lambda(t)}} \quad (27)$$

where  $l(k, i)$  is the distance (i.e., the number of neurons) between the winning neuron  $k$  and the neuron  $i$ ; and  $\lambda(t)$  is calculated according to the formula (24) with the substitution  $itr \rightarrow itr$ .

8. Repeat steps 3 through 7 until  $t = itr$ .
9. Calculate the annual cost for the  $k$ -th loop  $C_{tot,kl}$ .
10. Repeat all steps for each  $N_k^l$ ,  $k = 1, 2, \dots, m_l$ .
11. Calculate the annual cost of network  $C_{tot,l} = \sum_{k=1}^m C_{tot,kl}$ .
12. Repeat the calculations for every set  $N_{rp}^l$ ,  $l = 1, 2, \dots, z$ .
13. Calculate the total annual cost of the network  $C_{tot} = \sum_{l=1}^z C_{tot,l}$ .

## 6. Calculation experiments

To test the effectiveness of the proposed evolutionary algorithm and the artificial neural network appropriate calculations were conducted. Two cases of the problem with two and three supply points (MFPs) and 114 receiving points were investigated. The rectangular coordinates of each receiving point are presented in Table 2.

*Remarks:*

1. Active power for each receiving point is 300 kW and power factor is 0.8945.
2. Coordinates of the supplying points are as follows:
  - (a) case with two MFPs:  $x_1 = 0.200$ ;  $y_1 = 1.400$ ,  $x_2 = 3.300$ ;  $y_2 = 1.400$ ;
  - (b) case with three MFPs:  $x_1 = 1.200$ ;  $y_1 = 0.350$ ,  $x_2 = 1.350$ ;  $y_2 = 2.250$ ,  $x_3 = 2.900$ ;  $y_3 = 1.400$ .

The distances between receiving points and between the supply and receiving points are calculated from the equation:

$$d_{ij} = |x_i - x_j| + |y_i - y_j| \quad (28)$$

where:  $x_i, y_i, x_j, y_j$  – coordinates of the  $i$ -th and of the  $j$ -th points, respectively.

Electric and economic parameters of the underground cable, used in the analysis, are shown in Table 3. The electric and economic parameters of the MFPs are given in Table 4.

*Remark:* PLN – Polish currency, 1PLN  $\approx$  0.3 EUR.

Values of other electrical and economic parameters were selected as follows:

- nominal voltage: 15 kV,
- unit power cost: 80.64 PLN/kW,
- unit energy cost: 0.05828 PLN/kWh,
- unit unreliability costs caused by random outages: 4.0796 PLN/kWh,
- utilization time of peak load: 4500 h/year,
- time of maximal load losses duration: 3000 h/year,
- fixed cost rate for cable line: 0.15,
- fixed cost rate for substation: 0.18.

As mentioned, two cases have been considered. In Case 1 there are two supply points (MFPs) and in Case 2 - three MFPs. The optimal solutions obtained for Case 1 using evolutionary algorithms and artificial neural network technique are presented in Figs. 3 and 4, respectively. The optimal solutions obtained for Case 2 are shown in Figs. 5 and 6.

In both here presented optimal solutions for Case 1 (Figs. 3 and 4) 114 receiving points (MV/LV substations) are assigned to 8 MV loops. Four MV loops are supplied from MFP1 and other four MV loops from MFP2. Numbers shown in Figs. 3 and 4 correspond to particular MV/LV substations placed in each loop. Routes from one MV/LV substation to other one or from MFP to MV/LV substation are conducted in rectangular arrangement.

Table 2. Coordinates of receiving points in the test problem

No.	X[km]	Y[km]		No.	X[km]	Y[km]		No.	X[km]	Y[km]
1	0.300	0.0		39	0.900	0.900		77	3.000	1.800
2	0.600	0.0		40	1.200	0.900		78	3.300	1.800
3	0.900	0.0		41	1.500	0.900		79	3.600	1.800
4	1.200	0.0		42	1.800	0.900		80	0.300	2.100
5	1.500	0.0		43	2.100	0.900		81	0.600	2.100
6	1.800	0.0		44	2.400	0.900		82	0.900	2.100
7	2.100	0.0		45	2.700	0.900		83	1.200	2.100
8	2.400	0.0		46	3.000	0.900		84	1.500	2.100
9	2.700	0.0		47	3.300	0.900		85	1.800	2.100
10	3.000	0.0		48	3.600	0.900		86	2.100	2.100
11	3.300	0.0		49	0.900	1.200		87	2.400	2.100
12	3.600	0.0		50	1.200	1.200		88	2.700	2.100
13	0.000	0.300		51	1.500	1.200		89	3.000	2.100
14	0.300	0.300		52	1.800	1.200		90	3.300	2.100
15	0.600	0.300		53	2.100	1.200		91	3.600	2.100
16	0.900	0.300		54	2.400	1.200		92	0.300	2.400
17	1.200	0.300		55	2.700	1.200		93	0.600	2.400
18	1.500	0.300		56	3.000	1.200		94	0.900	2.400
19	1.800	0.300		57	3.300	1.200		95	1.200	2.400
20	2.100	0.300		58	3.600	1.200		96	1.500	2.400
21	2.400	0.300		59	0.900	1.500		97	1.800	2.400
22	2.700	0.300		60	1.200	1.500		98	2.100	2.400
23	3.000	0.300		61	1.500	1.500		99	2.400	2.400
24	3.300	0.300		62	1.800	1.500		100	2.700	2.400
25	3.600	0.300		63	2.100	1.500		101	3.000	2.400
26	0.300	0.600		64	2.400	1.500		102	3.300	2.400
27	0.600	0.600		65	2.700	1.500		103	3.600	2.400
28	0.900	0.600		66	3.000	1.500		104	0.300	2.700
29	1.200	0.600		67	3.300	1.500		105	0.600	2.700
30	1.500	0.600		68	3.600	1.500		106	0.900	2.700
31	1.800	0.600		69	0.600	1.800		107	1.200	2.700
32	2.100	0.600		70	0.900	1.800		108	1.500	2.700
33	2.400	0.600		71	1.200	1.800		109	1.800	2.700
34	2.700	0.600		72	1.500	1.800		110	2.100	2.700
35	3.000	0.600		73	1.800	1.800		111	2.400	2.700
36	3.300	0.600		74	2.100	1.800		112	2.700	2.700
37	3.600	0.600		75	2.400	1.800		113	3.000	2.700
38	0.300	0.900		76	2.700	1.800		114	3.300	2.700



Table 3. Electric and economic parameters of the underground cable

Parameters of line	Values
cross-section [ $\text{mm}^2$ ]	120
unit resistance [ $\Omega/\text{km}$ ]	0.253
unit reactance [ $\Omega/\text{km}$ ]	0.096
capacity [kW]	4996
unit investment cost [PLN/km]	190 000
unreliability factor [1/km]	0.0003

Table 4. Electric and economic parameters of the HV/MV substations – MFPs

Parameters of substation	Two MFPs	Three MFPs
apparent rated power [kVA]	2x16000	2x10000
capacity [kW]	19200	14000
investment cost of 110/MV transformer [PLN]	1.400.000	1.260.000
iron losses in transformer [kW]	16.5	11.0
rated copper losses in transformer [kW]	91.5	65.0
investment cost of MV bay [PLN]	112.500	112.500
investment cost of 110 kV switchgear [PLN]	3.360.000	3.360.000

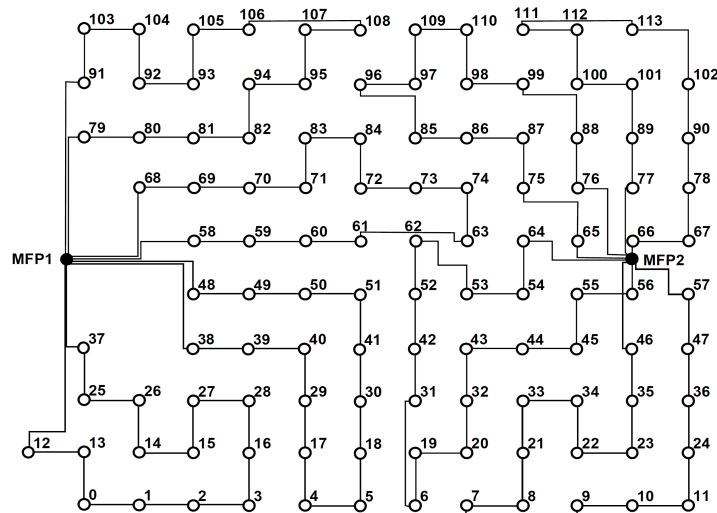


Figure 3. Optimal solution for the test problem (Case 1) obtained with the use of the evolutionary algorithm. Total annual costs of the designed network is  $C_{tot} = 3\,960\,440$  PLN

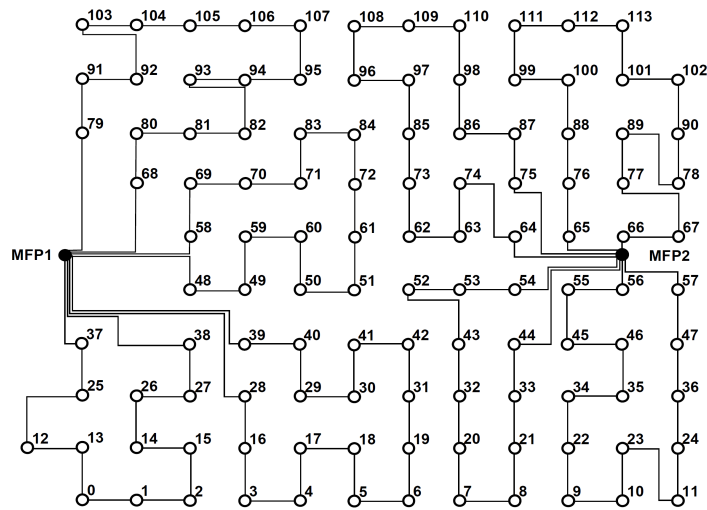


Figure 4. Optimal solution for the test problem (Case 1) obtained by means of the artificial neural network. Total annual costs of the designed network is  $C_{tot} = 3\,980\,027$  PLN

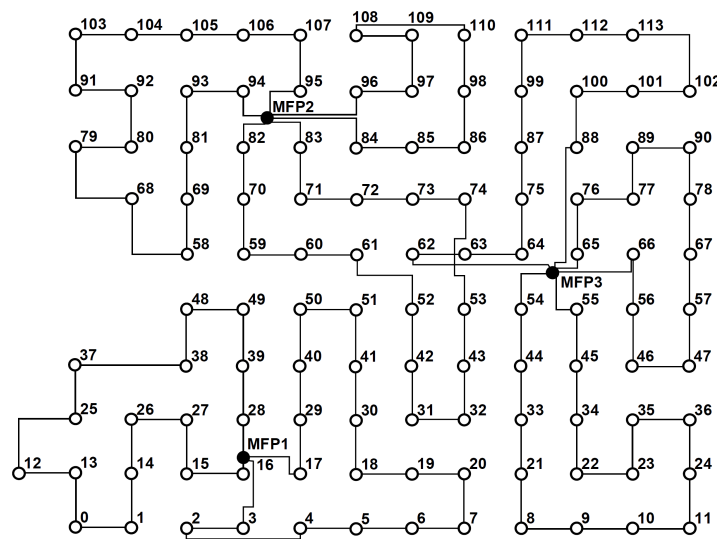


Figure 5. Optimal solution for the test problem (Case 2) obtained with the use of the evolutionary algorithm. Total annual costs of the designed network is  $C_{tot} = 4\,781\,697$  PLN

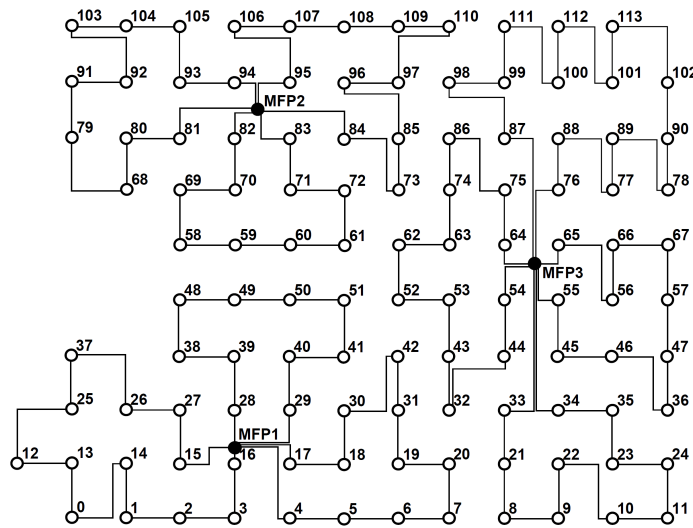


Figure 6. Optimal solution for the test problem (Case 2) attained by means of the artificial neural network. Total annual costs of the designed network is  $C_{tot} = 5\,001\,548$  PLN

In the optimal solution presented in Fig. 5 all receiving points are assigned to 8 loops. Two MV loops are supplied from MFP1, three other MV loops from MFP2, and yet other three MV loops from MFP3.

In turn, in the optimal solution shown in Fig. 6 the set of receiving points is divided into ten loops. Three MV loops are supplied from MFP1, three MV loops from MFP2, and four MV loops from MFP3.

Similarly as in Figs. 3 and 4, routes from one MV/LV substation to another one or from MFP to MV/LV substation, shown in Figs. 5 and 6, are also conducted in rectangular arrangement.

Results obtained by means of the evolutionary algorithm and the artificial neural network for two considered cases are presented in Table 6, where mainly the cost aspect is taken into account.

The settings of genetic parameters are given in Table.

Values of ANN parameters used in the test calculations are as follows:

- the number of iteration steps in STAGES A and B,  $itr_{rr} = 5\,000$ ;
- the number of iteration steps in STAGE C  $itr = 10\,000$ ;
- the minimum value of the learning rate  $\eta_{min} = 0.005$ ;
- the maximum value of the learning rate  $\eta_{max} = 0.5$ ;
- the minimum value of the decay constant  $\lambda_{min} = 0.01$ ;
- the maximum value of the decay constant  $\lambda_{max} = 10$ .

As shown in Table 5, in both Case 1 and Case 2 the evolutionary algorithm allowed for obtaining better solution of the optimization problem than

Table 5. Comparison of operational effectiveness of the evolutionary algorithm (EA) and artificial neural network (ANN)

Parameter name	Case 1		Case 2	
	EA	ANN	EA	ANN
Number of loops	8	8	8	10
Total loops length, km	45.8	46.8	41.0	46.6
Investment (fixed) costs, PLN	3846900	3875400	4667700	4908500
Exploitation (variable) costs, PLN	86 437	88 925	86 594	80 769
Supply-interruption costs, PLN	27 102	18 511	27 403	12 477
Total annual costs, PLN	3960440	3980027	4781697	5001548

Table 6. Values of genetic parameters used in the test calculations

Genetic parameters	Case 1	Case 2
Population size $N$	200	100
Maximum generation number	60000	30000
Crossover probability $p_c$	0.7	0.4
Mutation probability $p_m$	0.07	0.04
Elitist strategy	on	on
Linear scaling option	on	on
Generation with optimal solution	16947	18413

the artificial neural network, i.e. total annual costs for it were lower. Similarly, investment costs in both cases were lower, what is directly connected with lower total loops length. In Case 1, when loop number is the same and equals 8, variable costs for the solution obtained by EA were lower than for the one obtained with ANN. In turn, in Case 2, mainly because of greater number of loops, variable costs for the solution obtained by ANN were lower than for the one obtained with EA. In both considered cases supply-interruption costs for solutions obtained by ANN are lower than for ones obtained by EA. As it has been presented, fixed (investment) costs were the decisive factor for the total annual costs in both analyzed cases.

## 7. Conclusions

A new approach for designing MV urban multi-loop electric power network optimal structure has been presented. This approach has been based on minimization of total annual network costs. Investment costs, power and energy loss costs and supply-interruption costs have been included in the total annual network costs.

In order to solve the described optimization task, the problem oriented algorithms have been built. First algorithm is based on evolutionary technique. Special initial population procedure, crossover procedure and mutation procedures have been designed to improve the effectiveness of the evolutionary algorithm. Second algorithm is based on artificial neural network. The Kohonen's neural network has been built to attain this aim.

Computational experiments have been executed on the test problem. The comparison between the effects of evolutionary algorithm action and the artificial neural network operation has been made for the test task. For two analyzed cases both methods gave similar results. The obtained results show that both evolutionary algorithm and artificial neural network can be useful tools for designing MV urban multi-loop electric power network optimal structure.

Although both cases of the test problem are similar to the problems met in practice (considering problem size) the proposed methods should be tested with larger cases. This can show more clearly the drawbacks of these methods. Some of the problems were already observed during testing of the method based on the evolutionary algorithm. In authors' opinion the part of the algorithm which most needs changes is the crossover operator. Its deficiency in action results in slowing of the optimization process. There are also other subproblems which should be addressed when developing the evolutionary method.

It is planned to develop a hybrid algorithm for improving the results obtained with the presented algorithm based on the Artificial Neural Networks. The envisaged algorithm will take advantage of the Simulated Annealing method and Genetic Algorithms. It is also planned to adopt the presented algorithm for designing other types of electric power networks.

## References

- JAIN A.K., MAO J., MOHIUDDIN K. M. (1996) Artificial Neural Networks: A Tutorial, Neural Computing. *IEEE Computational Science & Engineering*. March, 31-44.
- BROŻEK J. (1999) Designing of distribution urban electric power networks using artificial neural networks (in Polish). *Proceedings of the 9<sup>th</sup> International Conference on "Present-day Problems of Power Engineering" APE'99*, Poland, Gdańsk-Jurata, June 9-11, 1999, **4**, 93-100.
- BROŻEK J. (2003) Optimization of urban electric power distribution networks (in Polish). *Proceedings of the 11<sup>th</sup> International Conference on "Present-day Problems of Power Engineering" APE'03*, Poland, Gdańsk-Jurata, June 11-13, 2003, **4**, 107-114.
- BROŻEK J. (2004) Hybrid algorithm for optimisation of m-loop electric power distribution network. *IEE Proceedings, Generation, Transmission & Distribution*, **151**, 2, 246-251.
- CHENG R. and GEN M. (1996) Fuzzy Vehicle Routing and Scheduling Problem Using Genetic Algorithms. In: F. Herrera and J. Verdegay, eds., *Genetic Algorithms and Soft Computing*. Springer-Verlag, 683-709.
- CHENG R. and GEN M. (1997) *Genetic Algorithms and Engineering Design*. John Wiley & Sons, New York.
- CHENG R., GEN M., TOZAWA T. (1995) Vehicle Routing Problem With Fuzzy Due-Time Using Genetic Algorithms. *Japanese Journal of Fuzzy Theory and Systems*, **7**, 5, 665-679.
- GLAMOCANIN V. and FILIPOVIC V. (1993) Open loop distribution system design. *IEEE Transactions on Power Delivery*, **8**, 4, October, 1900-1906.
- HERTZ J., KROGH A., PALMER R. G. (1991) *Introduction to the Theory of Neural Computation*. Addison Wesley Publishing Company.
- KOHONEN T. (1995) *Self-organizing Maps*. Springer-Verlag, Berlin.
- KULCZYCKI J. (1990) *Optimisation of Electric Power Network Structure* (in Polish). WNT.
- LAVORATO M., RIDER M. J., GARCIA A. V., ROMERO R. (2010) A Constructive Heuristic Algorithm for Distribution System Planning. *IEEE Transactions on Power Systems*, **25**, 3, August, 1734-1742.
- LEVITIN G., MAZAL-TOV S., ELMAKIS D. (1995) Genetic algorithm for open-loop distribution system design. *Electric Power System Research*, **32**, 81-87.
- MARTINETZ M., BERKOVICH S., SCHULTEN K. (1993) Neural-gas network for vector quantisation and its application to time series prediction. *IEEE Transactions on Neural Networks*, **4**, 558-569.
- MENDOZA F., BERNAL-AUGUSTIN J. L., DOMINGUEZ-NAVARRO J. A. (2006) NSGA and SPEA Applied to Multiobjective Design of Power Distribution Systems. *IEEE Transactions on Power Systems*, **21**, 4, November, 1938-1945.
- MICHALEWICZ Z. (1996) *Genetic Algorithms + Data Structure = Evolution*

- Programs*, 3<sup>rd</sup> ed. Springer-Verlag, New York.
- NAHMAN J. M., PERIC D. M. (2008) Optimal Planning of Radial Distribution Networks by Simulated Annealing Technique. *IEEE Transactions on Power Systems*, **23**, 2, May, 790-795.
- NAJAFI S., HOSSEINIAN S. H., ABEDI M., VAHIDNIA A., ABACHEZADEH S. (2009) A Framework for Optimal Planning in Large Distribution Networks. *IEEE Transactions on Power Systems*, **24**, 2, May, 1019-1028.
- NAVARRO A., RUDNICK H. (2009A) Large Scale Distribution Planning – Part I: Simultaneous Network and Transformer Optimization. *IEEE Transactions on Power Systems*, **24**, 2, May, 744-751.
- NAVARRO A., RUDNICK H. (2009B) Large Scale Distribution Planning – Part II: Macro-Optimization with Voronoi's Diagram and Tabu Search. *IEEE Transactions on Power Systems*, **24**, 2, May, 752-758.
- OSOWSKI S. (2006) *Neural Networks for Information Processing* (in Polish) Publishing House of the Warsaw University of Technology, 2<sup>nd</sup> ed., Warsaw.
- PAROL M. (2003) Designing of multi loop electric power network structures with the use of evolutionary algorithms. *Archives of Energetics*, **XXXII**, 1-2, 63-84.
- RAMIREZ-ROSADO I. J., DOMINGUEZ-NAVARRO J. A. (2006) New Multiobjective Tabu Search Algorithm for Fuzzy Optimal Planning of Power Distribution Systems. *IEEE Transactions on Power Systems*, **21**, 1, February, 224-231.
- WANG D. T.-C., OCHOA L. F., HARRISON G. P. (2011) Modified GA and Data Envelopment Analysis for Multistage Distribution Network Expansion Planning Under Uncertainty. *IEEE Transactions on Power Systems*, **26**, 2, May, 897-904.