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Price-based coordinability in hierarchical systems with information asymmetry: a comparative analysis of Nash equilibrium conditions*

by

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Abstract: The well-known problem of price-based coordinability is studied for the case of a multi-agent system in which information regarding the goals of the interacting subsystems is asymmetric. The paper illustrates how the uniform-price-based coordination rules may create incentives to anticipate the values of coordination signals and, thus, why the coordinability condition cannot be satisfied under asymmetric information. For this purpose a comparison is given of Nash equilibrium outcomes that are reachable individually by price-anticipating agents in two noncooperative games. These games are induced by the uniform-price-based coordination mechanism and are referred to as payment-bidding auction and demand-bidding auction. The analysis presented shows that in the games considered some of the agents may improve payoffs and allocations by applying the price-anticipating bidding strategies. However, the payment-bidding auction cannot be strictly dominated by the demand-bidding auction with respect to the resource allocation levels individually received by each agent. The derived results of theoretic considerations are illustrated by numerical examples.

Keywords: Nash equilibrium, coordination, asymmetric information, optimization.

1. Introduction

This paper deals with the problem of price-based coordinability of a multi-agent system in which individual goals of the agents, controlling their plants under a price-based coordination regime, remain privately known only to the agents themselves. The agents are assumed to be active, i.e. autonomous, intelligent

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and rational in a sense that is specified later. Selfishly competing with each other to reach their private goals, the agents are also assumed to control their plants by solving optimization problems that are parameterized by the rules of a coordination mechanism. This mechanism is introduced into the system by the coordinator that attempts to harmonize interactions of the plants.

A distributed system is called coordinable if there exists a signal that leads the interacting subsystems to a solution of the coordination problem (Mesarović et al., 1970; Findeisen et al., 1980). This multi-objective problem represents the coordinator's preferences defined with respect to the performance of the system at hand. A springboard for the study presented in this paper is the observation concerning a common engineering practice according to which the system's performance is evaluated by means of a scalarizing function aggregating individual performance indices of the component subsystems. Clearly, such a performance index can only be optimized when the goals of the subsystems are known to the coordinator. If this condition is not satisfied due to *asymmetry of information*, which is commonly observed in reality, then coordinability of the system can be questioned. Indeed, in a typical distributed environment the coordinator must rest his/her decisions on information revealed by the agents actively controlling their plants (subsystems). However, if the agents are autonomous in their decisions, then by acting in intelligent and rational manner they may find a profitable way to take advantage of the monopoly they have on the knowledge regarding their individual preferences. The coordinator may, therefore, receive a purposely modified information regarding the performance of the system, which in turn may also affect the choice of coordinating actions.

The aim of this paper is to illustrate the consequences that the anticipative control strategies, applied by the active agents interacting under asymmetric information in a distributed system, may have for the agents' *individual* outcomes and, thus, for the performance of a price-based coordination mechanism. To be more specific, the analysis presented is built upon the model studied in Kelly (1997), Johari and Tsitsiklis (2004), Johari et al. (2005), characterizing efficiency of the price-based coordination viewed from the *coordinator's* perspective. In the following sections this model is further developed in a complementary study of the properties of Nash equilibrium outcomes that are reachable individually by a price-anticipating agent. Our intention is to enrich the collection of results that deal with the problem of coordinability in distributed systems, thereby contributing to the theory of mechanism design (Green and Laffont, 1979; Groves et al., 1987; Mas-Colell et al., 1995; Krishna, 2002; Milgrom, 2004; Hurwicz and Reiter, 2008). Indeed, the notion of coordinability corresponds here to the concept of incentive compatibility, introduced in Hurwicz (1977) and extensively investigated in the literature on game theory.

Mechanisms analyzed in the following sections prescribe *uniform-prices* on the interaction variables in the system and permit the agents to derive their decisions in response to these prices. The interaction variables can be naturally interpreted as demand and supply signals communicated by the agents. These signals are assumed to be determined by the control rules optimizing performance indicators of the agents in response to the observed prices. The system becomes coordinated by the mechanism when equilibrium price is reached at which demand equates supply. Classic results on price-based coordination under perfect (or symmetric) information can be found in Arrow and Debreu (1954), Arrow and Hurwicz (1958,1959), Negishi (1960), Uzawa (1960). For an extensive discussion of mechanism design problems arising under asymmetric information see, e.g., Stiglitz (2000), Laffont and Martimort (2002). Recent engineering applications have been discussed in Low and Lapsley (1999), Malinowski (2002), Jin et al. (2005), Karpowicz (2011,2012b).

The paper is organized as follows. In Section 2 the base model is introduced and the addressed coordination problem is formulated. Next, the definition of mechanism is given and the strategies applied by the agents are described. The strategies are determined by the assumed *interpretations* of the coordinability condition which, in turn, is expressed by virtue of the locally applied definition of a mechanism. In Section 4 a solution to the coordination problem is described. Sections 5 and 6 present an analysis of the incentives that an active agent may have to apply the price-anticipating strategy. The outcomes attainable in two games are compared, referred to as *payment-bidding* auction and *demand-bidding* auction. A summary of the obtained results is presented in Section 7.

2. Problem formulation

Suppose there are $n \geq 2$ active agents competing for a single divisible resource. Let $x_i \in \mathbb{R}_+$ denote the amount of the resource allocated to agent $i = 1, ..., n$. The cost of supplying amount $y \geq 0$ of the resource to the agents is determined by a real-valued function C which satisfies the following condition:

Assumption 1. *There exists a continuous, convex and strictly increasing function p such that $p(0) = 0$ and such that:*

$$C(y) = \int_0^y p(s)\mathrm{d}s. \tag{1}$$

According to the above assumption function p determines a unit *price* or *marginal cost* of the resource at supply level y. As demonstrated later, properties of function p play an important role in the model studied.

Assumption 2. *Function p is convex, strictly increasing and differentiable. Furthermore, its elasticity:*

$$\varepsilon(y) = \frac{y}{p(y)} \frac{\partial p(y)}{\partial y} \tag{2}$$

is nondecreasing.

If p is not differentiable at y, then the corresponding right and left directional derivatives of p define:

$$\varepsilon^+(y) \equiv \frac{y}{p(y)} \frac{\partial^+ p(y)}{\partial y} \quad \text{and} \quad \varepsilon^-(y) \equiv \frac{y}{p(y)} \frac{\partial^- p(y)}{\partial y},$$

respectively. Notice that, by the assumption of continuity and convexity of p, directional derivatives exist (Rockafellar, 1970; Ekeland and Temam, 1999). In the following sections it is assumed that p is differentiable, unless explicitly stated otherwise.

Each agent $i = 1, ..., n$, is assumed to receive utility $U_i(x_i)$ from amount $x_i \geq 0$ of the resource. Utility function U_i is assumed to be known only to agent $i = 1, ..., n$. Furthermore, it is assumed to satisfy the following condition:

Assumption 3. *For every $i = 1, ..., n$, for $x_i \geq 0$ the utility function $U_i(x_i)$ is strictly concave, increasing and continuous, and for $x_i > 0$, $U_i(x_i)$ is continuously differentiable. Furthermore, its right directional derivative at $x_i = 0$ is finite.*

In the considered asymmetric information setting the coordinator is faced with the multi-objective problem of allocating to the agents the resources that are at his/her disposal. Our focus is on the following formulation of this problem:

Assumption 4. *The goal of the coordinator is to reach a solution to the problem*

$$\texttt{SYSTEM}(\mathbf{U}, C): \quad \left| \begin{array}{ll} \text{maximize} & \sum_{i=1}^{n} U_i(x_i) - C(\sum_{i=1}^{n} x_i) \\ \text{over} & \mathbf{x} \in \mathbb{R}^n_+. \end{array} \right.$$

The performance of the system is evaluated by the coordinator as above. It is thus assumed to be described by the *utilitarian* (multi-objective) preference indicator aggregating utility functions of the agents $U_i,\ i = 1, ..., n$, and the cost function C of resource allocation. Notice that by Assumptions 1 and 3 there exists a solution to $\texttt{SYSTEM}(\mathbf{U}, C)$ which is Pareto-optimal; see e.g. Ogryczak (2007), Branke et al. (2008).

Since utility functions $U_i,\ i = 1, ..., n$, are not known to the coordinator, the coordination problem, $\texttt{SYSTEM}(\mathbf{U}, C)$, can only be solved based on information revealed by the agents. For this reason it is further assumed that the coordinator introduces into the system a mechanism that requires the agents to submit messages $\theta_i,\ i = 1, ..., n$, providing some sort of information regarding their utility functions. The mechanism is defined as a tuple of allocation rules $\xi_i,\ i = 1, .., n$, and payment rules $\eta_i,\ i = 1, .., n$, processing vector of messages $\boldsymbol{\theta} = (\theta_1, ..., \theta_n)$ submitted by the agents. Based on the observed messages the mechanism determines allocations $x_i = \xi_i(\boldsymbol{\theta}),\ i = 1, ..., n$, and the corresponding payments $w_i = \eta_i(\boldsymbol{\theta}),\ i = 1, ..., n$.

Definition 1. *Let us denote by* $\mathbf{\Theta} = \Theta_1 \times \cdots \times \Theta_n$ *the product of sets of messages, by* X *the set of feasible allocations and by* W *the set of payments. Mechanism* $\mathbf{m}$ *is a product of functions* $m_i \colon \mathbf{\Theta} \to X \times W,\ i = 1, ..., n$*, such that:*

$$m_i(\boldsymbol{\theta}) = (\xi_i(\boldsymbol{\theta}), \eta_i(\boldsymbol{\theta})),\ i = 1, ..., n. \tag{3}$$

For each $i = 1, ..., n$ *function* $\xi_i \colon \mathbf{\Theta} \to X$ *is called allocation rule and* $\eta_i \colon \mathbf{\Theta} \to W$ *is called payment rule.*

Our considerations are limited to the class of mechanisms for which $\Theta_i = X = W = \mathbb{R}_+$. This implies that the analyzed mechanisms process point-wise characterization of the agents' preferences. It should be pointed out though that the definition of mechanism admits much more abstract domains as well. For example, it is often convenient to assume that Θ_i denotes space of functions representing preferences of the interacting agents, e.g. U_i. In such a case each agent is required to reveal to the mechanism a complete model of the preference relation. Discussion of classic designs of mechanisms can be found in Green and Laffont (1979), Groves et al. (1987), Mas-Colell et al. (1995), Krishna (2002), Milgrom (2004), Hurwicz and Reiter (2008). Furthermore, our focus is also limited to the class of mechanisms that communicate a uniform *equilibrium* or *market-clearing price* $\mu \geq 0$ to every agent $i = 1, ..., n$. Therefore, in the considered setting the individual payments w_i for receiving resource share x_i have the form of $w_i = x_i\mu$ for every $i = 1, ..., n$.

The rules of mechanism $\mathbf{m}$ should be viewed as the coordination instruments applied in the system to harmonize interactions of the agents. Clearly, in practical engineering applications these interactions are usually realized in the course of an iterative process in which information is exchanged between the agents and the mechanism. In such a process the agents submit to the mechanism a sequence of bids and, in response, receive from the mechanism a sequence of outcomes of the resource allocation process. The dynamics of the process are not investigated in this paper though. Instead, the *static* properties of the emerging Nash equilibrium points are studied (Nash, 1950, 1951). For this purpose behavior of each agent is modeled by means of the following strategy:

Assumption 5. *Each agent submits to mechanism* $\mathbf{m} = (m_1, ..., m_n)$ *message* $\theta_i \in \mathbb{R}_+,\ i = 1, ..., n$*, that solves the problem*

$\texttt{AGENT}_i(m_i)\colon$

$$\left|\begin{array}{ll} \text{maximize} & Q_i(m_i(\boldsymbol{\theta})) = U_i(\xi_i(\boldsymbol{\theta})) - \eta_i(\boldsymbol{\theta}) \\ \text{over} & \theta_i \geq 0. \end{array}\right.$$

According to the above assumption, decisions of the agents represent their best response to the imposed rules of coordination $m_i,\ i = 1, ..., n$. Notice that the agents take into account the direct impact that their individual decisions,

$\theta_i,\ i = 1, ..., n$, have on the outcomes of the resource allocation process. Furthermore, each agent *may* exploit the knowledge of the fact that these outcomes also depend on decisions of other agents, $\boldsymbol{\theta}_{-i} = (\theta_1, ..., \theta_{i-1}, \theta_{i+1}, ..., \theta_n),\ i = 1, ..., n$. As a result, the agents becomes involved in a noncooperative game induced by the rules of mechanism $\mathbf{m}$.

As strong as the above assumption may seem from the descriptive point of view, it is necessary for further study in which the implications of a particular strategic reasoning pattern are investigated. Therefore, the assumption is in fact dictated by the former one, according to which the agents are active. Discussion of applicability conditions of the models based of Nash equilibrium concept can be found e.g. in Raiffa and Luce (1957), Myerson (1991), Camerer (2003), Milgrom (2004), Wierzbicki and Nakamori (2006), Maskin (2011).

3. Coordination game

The assumptions made so far bring us to the model of interaction between the coordinator and the agents known as a *game of mechanism design* or *coordination game*.

In order to solve $\texttt{SYSTEM}(\mathbf{U}, C)$, the coordinator introduces a uniform-price mechanism $\mathbf{m} = (m_1, ..., m_n)$ that determines resource allocation outcomes $(x_i, w_i, \mu),\ i = 1, ..., n$, based on messages $\theta_i,\ i = 1, ..., n$, submitted by the agents. These messages inform the coordinator, in the language of mechanism $\mathbf{m}$, about the agents' preferences. However, since in the environment with asymmetric information the agents' preferences are private, the coordinator cannot verify the accuracy or truthfulness of the observed claims. The central determinant of the studied model is, therefore, the strategy applied by the agents to calculate their bids. In what follows it is assumed that the choice of message $\theta_i,\ i = 1, ..., n$, depends on how the agents *interpret* the imposed rules of their mutual interactions. Namely, problem $\texttt{AGENT}_i(m_i^\star),\ i = 1, ..., n$, is viewed as being determined by interpretation $m_i^\star$ of the interaction rules m_i. The construction of $m_i^\star$ is assumed to reflect (or model) the agents' interpretation of the coordination rules in the system.

Under the above assumptions, two special interpretations of the uniform-price coordination mechanism are to be analyzed, called *payment-bidding auction* $\hat{\mathbf{m}}$, and *demand-bidding auction* $\tilde{\mathbf{m}}$. The key element of these constructions is the *interaction balancing* or *market-clearing* equation, according to which aggregate demand $D(\mu, \boldsymbol{\theta}) = \sum_{i=1}^n D_i(\mu, \theta_i)$, revealed by the agents through messages $\theta_i,\ i = 1, ..., n$, meets supply $S(\mu)$ at equilibrium price μ, i.e.:

$$D(\mu, \boldsymbol{\theta}) - S(\mu) = 0,\ \text{where } S(\mu) = \arg\max\{\mu y - C(y)\colon y \geq 0\}. \tag{4}$$

Notice that supply function S is assumed to be determined by the Legendre-Fenchel transform (conjugate) of C; see, e.g., Mordukhovich (2006), Rockafellar and Wets (2004). This assumption has an implication that is important from the viewpoint of our further analysis. Indeed, $y = S(\mu)$ is well-behaved if $C(y)$ is a convex function that satisfies Assumption 1 and p is differentiable. Suppose that $C(0) > 0$ and $p(0) > 0$, i.e. the required conditions are violated. It is easy to see that in such a case $S(\mu) = \{0 \text{ for } \mu \in [0, p(0)];\ p^{-1}(\mu) \text{ for } \mu \geq p(0)\}$. As a consequence, solution to the interaction balancing equation may be zero, which is not feasible in the setting considered here. Finally, it may also be noticed that, by Assumption 5, demand $D_i(\mu, \theta_i)$ can be viewed as defined by the conjugate of U_i as well.

4. Coordinability with price-taking agents

To specify the reference solution for further study, a setting is first considered in which the agents do not inquire into how the observed value of price is generated and simply take it as given. In order to model this bidding pattern definitions of two price-based coordination mechanisms are introduced below. Since the relation between the messages and the price is assumed to be ignored by the agents, the notation introduced in Definition 1 is slightly abused in this section. Namely, it is assumed that each agent responds to the coordination rules of the form $\bar{m}_i(\theta_i, \mu)$, even though $\mu = \mu(\boldsymbol{\theta})$ and it would be legitimate to apply the form $\bar{m}_i(\boldsymbol{\theta})$ under the assumptions that the agents are price-takers.

The first price-based coordination mechanism assigns to agent i amount $x_i = \bar{\xi}_i(\theta_i, \mu) \geq 0$ of the resource based on bid $\theta_i = x_i\mu \geq 0$ denoting the *willingness to pay* for amount $x_i \geq 0$ of the resource at price $\mu \geq 0$.

Definition 2 (Payment-based coordination mechanism). *The payment-based coordination mechanism* $\bar{\mathbf{m}} = (\bar{\boldsymbol{\xi}}, \bar{\boldsymbol{\eta}})$ *is defined by the set of messages* $\Theta_i = \mathbb{R}_+$ *and the coordination rules:*

$$\bar{\xi}_i(\theta_i, \mu) = \begin{cases} \theta_i/\mu, & \text{if } \theta_i > 0; \\ 0, & \text{if } \theta_i = 0, \end{cases} \tag{5}$$

$$\bar{\eta}_i(\theta_i, \mu) = \theta_i \tag{6}$$

for every $i = 1, .., n$.

The second uniform-price-based coordination mechanism is defined under the requirement that message θ_i, communicated by each agent $i = 1, ..., n$, provides information regarding the *demand* at price μ. This requirement is expressed by the following definition:

Definition 3 (Demand-based coordination mechanism). *The demand-based coordination mechanism* $\bar{\mathbf{m}} = (\bar{\boldsymbol{\xi}}, \bar{\boldsymbol{\eta}})$ *is defined by the set of messages* $\Theta_i = \mathbb{R}_+$ *and the coordination rules*

$$\bar{\xi}_i(\theta_i, \mu) = \theta_i, \tag{7}$$

$$\bar{\eta}_i(\theta_i, \mu) = \theta_i\mu \tag{8}$$

for every $i = 1, .., n$.

As can be seen, in each case the described above price μ is a parameter of the coordination rules. This construction is intended to describe the behavioral pattern according to which the agents, responding to the rules of coordination, do not explore the way the value of price is generated. Nevertheless, it is still necessary to define how the price is in fact calculated (by the coordinator) based on the observed messages $\boldsymbol{\theta} = (\theta_1, ..., \theta_n)$. This brings us to the complementary definitions of the balancing equations corresponding to the introduced mechanisms.

Definition 4 (Payment-based balancing equation). *For every vector of messages, $\boldsymbol{\theta} \geq 0$, the equilibrium price is equal to $\mu = p(\bar{y})$, where $\bar{y} \geq 0$ is a unique solution to the payment-based balancing equation:*

$$\bar{y}p(\bar{y}) = \sum_{i=1}^{n} \bar{\eta}_i(\theta_i, \mu). \tag{9}$$

Definition 5 (Demand-based balancing equation). *For every vector of messages, $\boldsymbol{\theta} \geq 0$, the equilibrium price is equal to $\mu = p(\bar{y})$, where $\bar{y} \geq 0$ is a unique solution to the demand-based balancing equation:*

$$\bar{y} = \sum_{i=1}^{n} \bar{\xi}_i(\theta_i, \mu). \tag{10}$$

By Assumption 5, the price-taking strategy amounts to solving the problem

$\texttt{AGENT}_i(\bar{m}_i)$:

$$\left|\begin{array}{ll} \text{maximize} & Q_i(\bar{m}_i(\theta_i, \mu)) = U_i(\bar{\xi}_i(\theta_i, \mu)) - \bar{\eta}_i(\theta_i, \mu) \\ \text{over} & \theta_i \geq 0. \end{array}\right.$$

If each agent solves the above problem, then equilibrium point can be reached at which total demand revealed by the agents, $\sum_{i=1}^{n} \bar{\xi}_i(\theta_i, \bar{\mu})$, equals supply, $\bar{y}$, at equilibrium price $\bar{\mu}$ determined by an appropriate balancing equation. To show that this indeed is the case, notice that the first-order necessary and sufficient optimality conditions for $\texttt{SYSTEM}(\mathbf{U}, C)$, given by:

$$\begin{cases} \bar{x}_i \Big[U_i'(\bar{x}_i) - p(\sum_{i=1}^{n} \bar{x}_i) \Big] = 0, \\ U_i'(\bar{x}_i) - p(\sum_{i=1}^{n} \bar{x}_i) \leq 0, \\ \bar{x}_i \geq 0, \ i = 1, ..., n, \end{cases} \tag{11}$$

are compatible (or harmonized) with the first-order necessary and sufficient optimality conditions for system $\texttt{AGENT}_i(\bar{m}_i)$, $i = 1, ..., n$, according to which each agent $i = 1, ..., n$ maximizes Q_i at $\bar{\theta}_i$ such that:

$$\begin{cases} \bar{x}_i \Big[U_i'(\bar{x}_i) - \bar{\mu} \Big] = 0, \\ U_i'(\bar{x}_i) - \bar{\mu} \leq 0, \\ \bar{x}_i = \bar{\xi}_i(\bar{\theta}_i, \bar{\mu}) \geq 0, \ \bar{\mu} = p(\sum_{i=1}^{n} \bar{x}_i). \end{cases} \tag{12}$$

Therefore, allocations $\bar{x}_i = \bar{\xi}_i(\bar{\theta}_i, \bar{\mu}),\ i = 1, ..., n$, with payments $\bar{w}_i = \bar{x}_i\bar{\mu}$, yield an optimal solution to the coordination problem $\texttt{SYSTEM}(\mathbf{U}, C)$.

Separating the coordination rules from the balancing rules, as above, seems to be necessary here in order to model the price-taking bidding strategy of the agents. The agents must not see, or must ignore, the relation between their messages and the coordinating feedback. However, this is precisely the relation that defines the equilibrium conditions in which a solution to the coordination problem is reached. Indeed, the balancing equations formulated above, combined with the adequate rules of coordination, give rise to equation (4) efficiently balancing interactions of the agents. It should also be noted that the interaction balancing equation is defined by demand $D_i(\mu, \bar{\theta}_i) = U_i'^{-1}(\mu),\ i = 1, ..., n$, determined by the Legendre-Fenchel transform (conjugate) of U_i, i.e. we have $D_i(\mu, \bar{\theta}_i) = \arg\max\{\mu x_i - U_i(x_i)\colon x_i \geq 0\}$. Consequently, in equilibrium corresponding to supply $S(\mu) = p^{-1}(\mu)$ each price-taking agent truthfully reveals to the coordinator his/her demand $D_i(\mu, \bar{\theta}_i) = \bar{x}_i = \bar{\theta}_i/\mu$.

5. Coordinability with price-anticipating agents

This section presents an analysis of outcomes of the agents' bidding strategies, defined by system $\texttt{AGENT}_i,\ i = 1, ..., n$, exploiting a particular formulation (interpretation) of the interaction balancing equation, expressed by the rules of the mechanisms. More precisely, the *price-anticipating* strategies are studied that result from the interpretations of the following equation:

$$D(\mu(\boldsymbol{\theta}), \boldsymbol{\theta}) - S(\mu(\boldsymbol{\theta})) = 0, \tag{13}$$

where $S(\mu(\boldsymbol{\theta})) = \arg\max\{\mu(\boldsymbol{\theta})y - C(y)\colon y \geq 0\}$. To formalize the above concept two mechanisms are defined below, referred to as *payment-bidding* auction and *demand-bidding* auction. By construction, both mechanisms take into account the fact that the agents apply the *price-anticipating* strategies in order to exploit their informational advantage.

The above assumption does not influence the structure of the iterative coordination process in which the agents submit bids in response to the observed price. In each considered case the price provides information that is required by the agents to verify the first-order equilibrium conditions. What changes is only the way that the agents calculate their messages. Namely, the agents exploit the fact the observed feedback can be viewed as an implicit solution, $\mu = \mu(\boldsymbol{\theta})$, to the interaction balancing equation.

5.1. Payment-bidding auction

Suppose that the agents' actions are coordinated by the rules of the mechanism described by Definition 2. However, let us also assume that every agent $i = 1, ..., n$ actively exploits the knowledge of the fact that the messages submitted to the mechanism are used by the coordinator to solve the balancing equation described in Definition 4. It follows that each agent applies the price-anticipating strategy by solving the problem

$\texttt{AGENT}_i(\hat{m}_i):$

$$\left|\begin{array}{ll}\text{maximize} & Q_i(\hat{m}_i(\boldsymbol{\theta})) = U_i(\hat{\xi}_i(\boldsymbol{\theta})) - \hat{\eta}_i(\boldsymbol{\theta}) \\ \text{over} & \theta_i \geq 0,\end{array}\right.$$

where $\hat{m}_i$ is defined for every $i = 1, ..., n$ by the rules of the payment-bidding auction.

Definition 6 (Payment-bidding auction). *Payment-bidding auction* $\hat{\mathbf{m}} = (\hat{\boldsymbol{\xi}}, \hat{\boldsymbol{\eta}})$ *is defined by the set of messages* $\Theta_i = \mathbb{R}_+$ *and the following coordination rules:*

$$\hat{\xi}_i(\boldsymbol{\theta}) = \begin{cases} \theta_i / p(\hat{y}(\boldsymbol{\theta})), & \text{if } \theta_i > 0; \\ 0, & \text{if } \theta_i = 0, \end{cases} \tag{14}$$

$$\hat{\eta}_i(\boldsymbol{\theta}) = \theta_i \tag{15}$$

for every $i = 1, .., n$. *For every vector of messages,* $\boldsymbol{\theta} \geq 0$, $\hat{y}(\boldsymbol{\theta})$ *is a unique solution to the balancing equation:*

$$\hat{y}(\boldsymbol{\theta}) p(\hat{y}(\boldsymbol{\theta})) = \sum_{i=1}^{n} \hat{\eta}_i(\boldsymbol{\theta}). \tag{16}$$

By the above definition, for an arbitrary vector of bids $\boldsymbol{\theta} \geq 0$ the mechanism determines solution $\hat{y}(\boldsymbol{\theta})$ to the balancing equation (16). Next, allocations $x_i = \hat{\xi}_i(\boldsymbol{\theta}),\ i = 1, ..., n$, are established based on (14). Notice that $\hat{y}(\boldsymbol{\theta}) = \sum_{i=1}^{n} \hat{\xi}_i(\boldsymbol{\theta})$. Signal $\theta_i = \hat{\xi}_i(\boldsymbol{\theta}) p(\hat{y}(\boldsymbol{\theta}))$ denotes the total payment that agent $i = 1, ..., n$ is willing to make for amount $x_i = \hat{\xi}_i(\boldsymbol{\theta}) \geq 0$ of the resource.

The construction of strategy $\texttt{AGENT}_i(\hat{m}_i)$ is now studied; see Johari et al. (2005). By Assumption 1 function p is continuous, convex and strictly increasing, which implies that $g(y) = yp(y)$ is strictly increasing, strictly convex, continuous and invertible. As a consequence, $\hat{y}(\boldsymbol{\theta}) = g^{-1}(\sum_{i=1}^{n} \theta_i)$ is strictly increasing and strictly concave function of $\sum_{i=1}^{n} \theta_i$, which implies that it is also directionally differentiable (Rockafellar, 1970). By (16) the marginal changes in supply $\hat{y}(\boldsymbol{\theta})$, caused by the agent i's unilateral deviation from the consumption level determined by θ_i are, therefore, given by:

$$\begin{cases} \dfrac{\partial^+ \hat{y}(\boldsymbol{\theta})}{\partial \theta_i} = \left(p(\hat{y}(\boldsymbol{\theta})) + \hat{y}(\boldsymbol{\theta}) \dfrac{\partial^+ p(\hat{y}(\boldsymbol{\theta}))}{\partial \theta_i} \right)^{-1}, \\ \dfrac{\partial^- \hat{y}(\boldsymbol{\theta})}{\partial \theta_i} = \left(p(\hat{y}(\boldsymbol{\theta})) + \hat{y}(\boldsymbol{\theta}) \dfrac{\partial^- p(\hat{y}(\boldsymbol{\theta}))}{\partial \theta_i} \right)^{-1}. \end{cases} \tag{17}$$

Since $\hat{y}(\boldsymbol{\theta})$ determines price $p(\hat{y}(\boldsymbol{\theta}))$ of the resource, any agent anticipating his/her individual impact on $\hat{y}(\boldsymbol{\theta})$ may also anticipate the marginal price changes. It is also important to observe that no assumption on differentiability of $p(y)$ has been made here, so right and left directional derivatives are not necessarily equal.

The following result shows that vector $\hat{\boldsymbol{\theta}}$ of solutions to system $\texttt{AGENT}_i(\hat{m}_i)$, $i = 1, ..., n$, is reached in the Nash equilibrium of the game induced by the payment-bidding auction $\hat{\mathbf{m}} = (\hat{\boldsymbol{\xi}}, \hat{\boldsymbol{\eta}})$. In Nash equilibrium each agent efficiently approximates his/her influence on the supply of the resource, $\hat{y}(\boldsymbol{\theta})$, and its price $p(\hat{y}(\boldsymbol{\theta}))$.

Theorem 1 (Nash equilibrium of payment-bidding auction, Johari, 2004). *Suppose that Assumptions 1, 3 and 5 are satisfied. A vector $\hat{\boldsymbol{\theta}}$ is a Nash equilibrium point of the game defined by payment-bidding auction $\hat{\mathbf{m}}$, if and only if $\sum_{i=1}^{n} \hat{\theta}_i > 0$ and the following conditions hold for every $\hat{x}_i > 0$:*

$$\begin{cases} U_i'(\hat{x}_i)\left(1 - \beta^+(\hat{y})\dfrac{\hat{x}_i}{\hat{y}}\right) \le p(\hat{y}), \\ U_i'(\hat{x}_i)\left(1 - \beta^-(\hat{y})\dfrac{\hat{x}_i}{\hat{y}}\right) \ge p(\hat{y}), \end{cases} \tag{18}$$

where $i = 1, ..., n$, $\hat{y} \equiv \hat{y}(\hat{\boldsymbol{\theta}})$, $\hat{x}_i \equiv \hat{\xi}_i(\hat{\boldsymbol{\theta}})$ and $\beta(y) = \varepsilon(y)/1 + \varepsilon(y)$. If, additionally, Assumption 2 holds, then there exists a unique Nash equilibrium for the considered game.

By the first-order optimality conditions for $\texttt{AGENT}_i(\hat{m}_i)$, agent i's best response $\hat{\theta}_i$ to any fixed $\hat{\boldsymbol{\theta}}_{-i}$ satisfies conditions:

$$\begin{cases} \dfrac{\partial^+ Q_i(\hat{m}_i(\hat{\boldsymbol{\theta}}))}{\partial \theta_i} = U_i'(\hat{\xi}_i(\hat{\boldsymbol{\theta}}))\dfrac{\partial^+ \hat{\xi}_i(\hat{\boldsymbol{\theta}})}{\partial \theta_i} - 1 \le 0, \\ \dfrac{\partial^- Q_i(\hat{m}_i(\hat{\boldsymbol{\theta}}))}{\partial \theta_i} = U_i'(\hat{\xi}_i(\hat{\boldsymbol{\theta}}))\dfrac{\partial^- \hat{\xi}_i(\hat{\boldsymbol{\theta}})}{\partial \theta_i} - 1 \ge 0. \end{cases} \tag{19}$$

These conditions, satisfied for every $i = 1, ..., n$, constitute a Nash equilibrium $\hat{\boldsymbol{\theta}} \neq 0$ of the resource allocation game defined by mechanism $\hat{\mathbf{m}}$. There is no profitable deviation from allocation $\hat{x}_i = \hat{\xi}_i(\hat{\boldsymbol{\theta}})$ individually available for any agent $i = 1, ..., n$. Notice that differentiating $\hat{\xi}_i$ with respect to θ_i and substituting the directional derivatives of $\hat{y}(\hat{\boldsymbol{\theta}})$ yields (18). Furthermore, $\hat{\boldsymbol{\theta}}$ may be situated at a nondifferentiable point of $p(y)$.

Demand reduction strategy

It is now illustrated that allocations $\hat{x}_i = \hat{\xi}_i(\hat{\boldsymbol{\theta}})$, $i = 1, ..., n$, cannot be viewed as a solution to $\texttt{SYSTEM}(\mathbf{U}, C)$. Furthermore, it is also demonstrated that in Nash equilibrium each agent reveals to the coordinator a modified demand profile.

By the first-order necessary and sufficient conditions for $\texttt{SYSTEM}(\mathbf{U}, C)$, if agent i is assigned a positive amount $\bar{x}_i$ of the resource, then the corresponding marginal increase in the received utility $U_i'(\bar{x}_i)$ must be equal to the equilibrium price $p(\bar{y}) > 0$, where $\bar{y} = \sum_{i=1}^n \bar{x}_i$. If, on the other hand, no amount of the resource is allocated to the agent, then the equilibrium price must be greater than or equal to the agent i's marginal utility at zero. Theorem 2 proved below shows that these conditions cannot be satisfied by a solution to system $\mathtt{AGENT}_i(\hat{m}_i)$, $i = 1, ..., n$.

Theorem 2. *If Assumptions 1, 2, 3 and 5 hold, and vector* $\hat{\boldsymbol{\theta}}$ *is a Nash equilibrium of the game defined by the system* $\mathtt{AGENT}_i(\hat{m}_i)$, $i = 1, ..., n$, *then for each* $i = 1, ..., n$:

$$p(\hat{y}) < U_i'(\hat{x}_i) \leq p(\hat{y})(1 + \varepsilon(\hat{y})), \text{ when } \hat{x}_i > 0. \tag{20}$$

Proof. By differentiability of p and Theorem 1, if $\hat{\boldsymbol{\theta}}$ is a Nash equilibrium point, then for every $i = 1, ..., n$:

$$U_i'(\hat{x}_i)\left(1 - \beta(\hat{y})\frac{\hat{x}_i}{\hat{y}}\right) - p(\hat{y}) = 0, \text{ when } \hat{\theta}_i > 0.$$

Fix $\hat{\boldsymbol{\theta}}_{-i}$ and consider $\hat{\theta}_i > 0$. Since U_i is strictly increasing, from the fact that $\beta(\hat{y}) > 0$ for $\hat{y} > 0$ (observe that $\beta'(y) = \varepsilon'(y)/(1 + \varepsilon(y))^2 \geq 0$) it follows, that:

$$U_i'(\hat{x}_i) - p(\hat{y}) = U_i'(\hat{x}_i)\beta(\hat{y})\frac{\hat{x}_i}{\hat{y}} > 0.$$

Suppose next that $U_i'(\hat{x}_i) > p(\hat{y})(1 + \varepsilon(\hat{y}))$. Then, for every $i = 1, ..., n$ such that $\hat{\theta}_i > 0$:

$$p(\hat{y})(1 + \varepsilon(\hat{y}))\left(1 - \beta(\hat{y})\frac{\hat{x}_i}{\hat{y}}\right) < p(\hat{y}).$$

This, however, implies that $\varepsilon(\hat{y})(1 - \hat{x}_i/\hat{y}) < 0$, which is a contradiction. Indeed, in equilibrium $\varepsilon(\hat{y}) > 0$ and $\hat{y} = \sum_{k=1}^n \hat{\xi}_k(\hat{\boldsymbol{\theta}}) \geq \hat{x}_i$. In fact, for $z_i = \hat{y} - \hat{x}_i$ we have:

$$U_i'(\hat{x}_i) = p(\hat{y})\left(1 + \frac{\varepsilon(\hat{y})\hat{x}_i}{\hat{x}_i + z_i(1 + \varepsilon(\hat{y}))}\right) \leq p(\hat{y})(1 + \varepsilon(\hat{y})).$$

As a result, for all $i = 1, ..., n$ such that $\hat{x}_i > 0$ we have: $p(\hat{y}) < U_i'(\hat{x}_i) \leq p(\hat{y})(1 + \varepsilon(\hat{y}))$. ∎

Theorem 2 describes lower and upper bounds for marginal utility in Nash equilibrium $\hat{\boldsymbol{\theta}}$ of the game induced by the payment-bidding auction. In particular, it shows that for every $i = 1, ..., n$ marginal gains U_i' from allocation $\hat{\xi}_i(\hat{\boldsymbol{\theta}}) > 0$ exceed the equilibrium price $p(\hat{y}(\hat{\boldsymbol{\theta}}))$ of the resource. As a result, solution to $\mathtt{SYSTEM}(\mathbf{U}, C)$ is not reached in Nash equilibrium of the payment-bidding game.

By Theorems 1 and 2, the agents' best response to $\hat{\mathbf{m}}$ is to reveal to the system a modified profile of preferences. Precisely, agent i capable of applying the price-anticipating strategy receives incentives to act as if his/her preference indicator were given by $\check{U}_i(x_i)$ such that:

$$\check{U}_i'(x_i) = U_i'(x_i)\left(1 - \beta(y)\frac{x_i}{y}\right) < U_i'(x_i). \tag{21}$$

It follows that in Nash equilibrium of the game defined by $\hat{\mathbf{m}}$ each agent reveals a *reduced level of demand*.

The last conclusion suggests that the outcomes of the price-anticipation game can be studied through the properties of optimal *demand reduction* strategies. This concept is expressed by the following definition.

Definition 7. *Let p be a function that satisfies Assumption 2. For each agent $i = 1, ..., n$, the demand reducing factor $\hat{\chi}_i$ corresponding to aggregate allocation level $y \geq 0$ is defined for mechanism $\hat{\mathbf{m}}$ and function p as follows:*

$$\hat{\chi}_i(x_i, z_i) = \frac{p(x_i + z_i) + z_i p'(x_i + z_i)}{p(x_i + z_i) + (x_i + z_i)p'(x_i + z_i)}, \tag{22}$$

where $z_i = y - x_i,\ i = 1, ..., n$.

It should be noticed that $\hat{\chi}_i,\ i = 1, ..., n$, is completely specified by function p. The following result describes properties of the demand reducing factor.

Proposition 1. *Demand reducing factor $\hat{\chi}_i$ corresponding to supply $y = x_i + z_i$ is equal to:*

$$\hat{\chi}_i(x_i, z_i) = 1 - \beta(y)\frac{x_i}{y}. \tag{23}$$

Furthermore, it satisfies the following conditions:

$$x_i \geq 0 \wedge z_i > 0 \Rightarrow \hat{\chi}_i(x_i, z_i) \in (0, 1], \tag{24}$$

$$x_i > 0 \wedge z_i \geq 0 \Rightarrow \hat{\chi}_i(x_i, z_i) \in (0, 1). \tag{25}$$

Proof. Elementary manipulations of (22) show that:

$$\hat{\chi}_i(x_i, z_i) = \frac{p(y) + z_i p'(y)}{p(y) + y p'(y)} = 1 - \beta(y)\frac{x_i}{y}.$$

It also follows that:

$$z_i > 0 \Rightarrow \lim_{x_i \downarrow 0} \hat{\chi}_i(x_i, z_i) = \frac{p(z_i) + z_i p'(z_i)}{p(z_i) + z_i p'(z_i)} = 1,$$

$$x_i > 0 \Rightarrow \lim_{z_i \downarrow 0} \hat{\chi}_i(x_i, z_i) = \frac{p(x_i)}{p(x_i) + x_i p'(x_i)} < 1.$$

Furthermore, $p(x_i + z_i) + z_i p'(x_i + z_i) > 0$ and $\hat{\chi}_i(x_i, z_i) = 1$ only if $x_i = 0$. ■

Theorems 1 and 2 show that at Nash equilibrium point of the analyzed price-anticipation game a positive amount of the resource allocated to agent i satisfies demand that is purposefully reduced by the agent by the factor determined by function $\hat{\chi}_i$. Furthermore, the level of demand reduction determined $\hat{\chi}_i$ is optimal under the rules of the payment-bidding auction. Truthful revelation of demand is optimal only if no amount of the resource is allocated to the agent. Hence, in equilibrium the message $\hat{\theta}_i$ communicated to the coordinator by agent i almost always reveals a modified preference profile of the agent.

5.2. Demand-bidding auction

Suppose now that the agents interact with each other under the coordination regime of mechanism described by Definition 3. As before, let us drop the assumption that the price-taking strategy is applied and suppose that every agent $i = 1, ..., n$ applies the following *price-anticipating* strategy

$$\texttt{AGENT}_i(\tilde{m}_i):$$

$$\left| \begin{array}{ll} \text{maximize} & Q_i(\tilde{m}_i(\boldsymbol{\theta})) = U_i(\tilde{\xi}_i(\boldsymbol{\theta})) - \tilde{\eta}_i(\boldsymbol{\theta}) \\ \text{over} & \theta_i \geq 0. \end{array} \right.$$

The strategy is determined by the demand-bidding mechanism described below.

Definition 8 (Demand-bidding auction). *Demand-bidding auction* $\tilde{\mathbf{m}} = (\tilde{\boldsymbol{\xi}}, \tilde{\boldsymbol{\eta}})$ *is defined by the set of messages* $\Theta_i = \mathbb{R}_+$ *and the following coordination rules:*

$$\tilde{\xi}_i(\boldsymbol{\theta}) = \theta_i, \tag{26}$$

$$\tilde{\eta}_i(\boldsymbol{\theta}) = \theta_i p(\tilde{y}(\boldsymbol{\theta})) \tag{27}$$

for every $i = 1, .., n$, *and* $\tilde{y}(\boldsymbol{\theta}) = \sum_{i=1}^{n} \tilde{\xi}_i(\boldsymbol{\theta})$.

By the above definition a point-wise characterization of individual demand is directly communicated to the mechanism, rather than indirectly by means of the willingness to pay. Based on the revealed demand profile $\boldsymbol{\theta}$ the mechanism determines supply $\tilde{y}(\boldsymbol{\theta})$ and the corresponding price $\mu = p(\tilde{y}(\boldsymbol{\theta}))$. Notice that $\tilde{y}(\boldsymbol{\theta}) = S(p(\tilde{y}(\boldsymbol{\theta})))$, where $S(\mu) = p^{-1}(\mu)$.

It can be verified that there exists a Nash equilibrium point defined by the system $\texttt{AGENT}_i(\tilde{m}_i)$, $i = 1, ..., n$. The result follows from the Kakutani's fixed point theorem (Kakutani, 1941; Rosen, 1965). The necessary and sufficient conditions for Nash equilibrium are given below.

Theorem 3. *Suppose that Assumptions 1 and 3 hold. A vector of signals* $\tilde{\boldsymbol{\theta}}$ *constitutes a Nash equilibrium of the game defined by the demand-bidding auction*

$\tilde{\mathbf{m}}$, *if and only if the following conditions hold for every* $\tilde{x}_i > 0$*:*

$$\begin{cases} U_i'(\tilde{x}_i) \leq p(\tilde{y}) + \tilde{x}_i \dfrac{\partial^+ p(\tilde{y})}{\partial y}, \\ U_i'(\tilde{x}_i) \geq p(\tilde{y}) + \tilde{x}_i \dfrac{\partial^- p(\tilde{y})}{\partial y}, \end{cases} \tag{28}$$

where $i = 1, ..., n$, $\tilde{y} \equiv \tilde{y}(\tilde{\boldsymbol{\theta}})$ *and* $\tilde{x}_i \equiv \tilde{\xi}_i(\tilde{\boldsymbol{\theta}})$. *If function* p *is differentiable, then there exists a unique Nash equilibrium.*

Demand reduction strategy

Properties of allocations arising in equilibrium of demand-bidding auction are characterized in Theorem 4. Again, it is demonstrated that a solution to the coordination problem, $\texttt{SYSTEM}(\mathbf{U}, C)$, is not reached and that marginal gains from $\tilde{\xi}_i(\tilde{\boldsymbol{\theta}}) > 0$ exceed equilibrium price $p(\tilde{y}(\tilde{\boldsymbol{\theta}}))$.

THEOREM 4. *If Assumptions 1, 3, 5 hold,* p *is differentiable and* $\tilde{\mathbf{x}}$ *is a Nash equilibrium of the game defined by* $\tilde{\mathbf{m}}$*, then for every* $i = 1, ..., n$*:*

$$p(\tilde{y}) < U_i'(\tilde{x}_i) \leq p(\tilde{y})(1 + \varepsilon(\tilde{y})), \text{ when } \tilde{x}_i > 0. \tag{29}$$

Proof. By Theorem 3:

$$p(\tilde{y}) < p(\tilde{y}) + \tilde{x}_i p'(\tilde{y}) = U_i'(\tilde{x}_i) = p(\tilde{y}) \left(1 + \varepsilon(\tilde{y}) \frac{\tilde{x}_i}{\tilde{y}}\right) \leq p(\tilde{y})(1 + \varepsilon(\tilde{y}))$$

for every $i = 1, ..., n$ for which $\tilde{x}_i > 0$. ■

As in the case of the payment-bidding auction, the price-anticipating strategies for the demand-bidding auction can be conveniently expressed in terms of demand reduction. It can be defined by taking into account the condition formulated in Theorem 3. For the sake of our further analysis we also require that p satisfies Assumption 2.

DEFINITION 9. *Let* p *be a function that satisfies Assumption 2. For each agent* $i = 1, ..., n$*, the demand reducing factor* $\tilde{\chi}_i$ *corresponding to supply level* y *is defined for mechanism* $\tilde{\mathbf{m}}$ *as follows:*

$$\tilde{\chi}_i(x_i, z_i) = \frac{p(x_i + z_i)}{p(x_i + z_i) + x_i p'(x_i + z_i)}, \tag{30}$$

where $z_i = y - x_i$, $i = 1, ..., n$.

PROPOSITION 2. *Demand reducing factor* $\tilde{\chi}_i$ *corresponding to supply* $y = x_i + z_i$ *is equal to:*

$$\tilde{\chi}_i(x_i, z_i) = 1 - \beta(y) \frac{x_i}{y - \beta(y) z_i}. \tag{31}$$

Furthermore, it satisfies the following conditions:

$$x \geq 0 \wedge z > 0 \Rightarrow \tilde{\chi}_i(x,z) \in (0,1], \tag{32}$$

$$x > 0 \wedge z \geq 0 \Rightarrow \tilde{\chi}_i(x,z) \in (0,1). \tag{33}$$

Proof. The result is obtained from (30) and from the following observations:

$$z_i > 0 \Rightarrow \lim_{x_i \downarrow 0} \tilde{\chi}_i(x_i, z_i) = 1,$$

$$x_i > 0 \Rightarrow \lim_{z_i \downarrow 0} \tilde{\chi}_i(x_i, z_i) = \frac{p(x_i)}{p(x_i) + x_i p'(x_i)} < 1.$$

■

By the above characterization, in Nash equilibrium of the price-anticipation game, defined by $\tilde{\mathbf{m}}$, agent i reveals a modified demand profile. Truthful bidding is again optimal only if no amount of the resource is allocated to the agent. In other words, the messages communicated by the price-anticipating agent i reveal to the coordinator a preference indicator optimally modified by the demand reducing factor $\tilde{\chi}_i$. Finally, verify that for $(x_i, z_i) \neq (0,0)$ we have

$$\hat{\chi}_i(x_i, z_i) - \tilde{\chi}_i(x_i, z_i) \geq 0. \tag{34}$$

In the following section the above relation between the demand reducing factors $\hat{\chi}_i$ and $\tilde{\chi}_i$ is used in a comparative study of outcomes attainable in Nash equilibrium of the payment-bidding and demand-bidding auctions.

6. Nash equilibrium characterization

In this section a sequence of results is derived that characterize outcomes attainable *individually* by each agent in Nash equilibrium of the studied price-anticipation games. It is demonstrated that the payoff received by a price-anticipating agent may weakly dominate the payoff reached in optimal solution to the coordination problem, attainable under the price-taking strategy. Furthermore, it is demonstrated that some of the price-anticipating agents may also improve their allocations.

For simplicity we apply the following notation: $\bar{x}_i = \bar{\xi}_i(\bar{\boldsymbol{\theta}})$, $\hat{x}_i = \hat{\xi}_i(\hat{\boldsymbol{\theta}})$, $\tilde{x}_i = \tilde{\xi}_i(\tilde{\boldsymbol{\theta}})$, $\bar{y} = \sum_{i=1}^n \bar{x}_i$, $\hat{y} = \sum_{i=1}^n \hat{x}_i$, $\tilde{y} = \sum_{i=1}^n \tilde{x}_i$, $\bar{\mu} = p(\bar{y})$, $\hat{\mu} = p(\hat{y})$, $\tilde{\mu} = p(\tilde{y})$, $\bar{w}_i = \bar{x}_i \bar{\mu}$, $\hat{w}_i = \hat{x}_i \hat{\mu}$, $\tilde{w}_i = \tilde{x}_i \tilde{\mu}$.

Lemma 1. *Suppose that Assumptions 1, 2 and 3 hold. Suppose also that function $g_i \colon \mathbb{R}_+^2 \to \mathbb{R}_+$ is continuous and characterized by the following properties for every $i = 1, ..., n$:*

$$\begin{cases} x \geq 0 \wedge z > 0 \Rightarrow g_i(x,z) \in (0,1] \\ x > 0 \wedge z \geq 0 \Rightarrow g_i(x,z) \in (0,1). \end{cases} \tag{35}$$

If the following system of conditions is satisfied by $\bar{\mathbf{x}} \in \mathbb{R}_+^n$ *and* $\mathbf{x}^* \in \mathbb{R}_+^n$*:*

$$\begin{cases} \bar{x}_i \Big[U_i'(\bar{x}_i) - p(\bar{y}) \Big] = 0, \\ U_i'(\bar{x}_i) - p(\sum_{i=1}^n \bar{x}_i) \leq 0, \\ \bar{x}_i \geq 0, \;\; \bar{y} = \sum_{i=1}^n \bar{x}_i, \;\; i = 1, ..., n, \end{cases} \tag{36}$$

$$\begin{cases} x_i^* \Big[U_i'(x_i^*) g_i(x_i^*, z_i^*) - p(y^*) \Big] = 0, \\ U_i'(x_i^*) g_i(x_i^*, z_i^*) - p(y^*) \leq 0, \\ x_i^* \geq 0, \;\; y^* = x_i^* + z_i^*, \;\; i = 1, ..., n, \end{cases} \tag{37}$$

then $p(y^*) < p(\bar{y})$ *and for every* $i = 1, ..., n$, *if* $x_i^* = 0$ *then* $\bar{x}_i = 0$. *Furthermore, it is not true that* $x_i^* > 0$, $\bar{x}_i > 0$ *and* $x_i^* > \bar{x}_i$ *for every* $i = 1, ..., n$.

Proof. Notice first that from (36) and (37) it follows that $\bar{\mathbf{x}} \neq 0$ and $\mathbf{x}^* \neq 0$. Thus, we have $y^* > 0$ and $\bar{y} > 0$. Let us now assume that $p(y^*) \geq p(\bar{y})$. Then, by Theorem 2, for all $i = 1, ..., n$, if $x_i^* > 0$ then $U_i'(x_i^*) > p(y^*) \geq p(\bar{y}) \geq U_i'(\bar{x}_i)$. This, however, implies that $y^* < \bar{y}$, which is a contradiction. We conclude that $p(y^*) < p(\bar{y})$.

Suppose next that for some i we have $x_i^* = 0$ and $\bar{x}_i > 0$. This implies that $U_i'(\bar{x}_i) = p(\bar{y}) > p(y^*) \geq U_i'(0)$, which is not possible under the assumption that function U_i' is decreasing. Thus, if $x_i^* = 0$, then $\bar{x}_i = 0$ for every $i = 1, ..., n$.

Finally, whenever $x_i^* > 0$, $\bar{x}_i > 0$ and $x_i^* > \bar{x}_i$ for every $i = 1, ..., n$, then $p(y^*) > p(\bar{y})$, which cannot hold by the arguments presented above. It is, therefore, not true that $x_i^* > 0$, $\bar{x}_i > 0$ and $x_i^* > \bar{x}_i$ for every $i = 1, ..., n$. ∎

LEMMA 2. *Suppose that Assumptions 1 and 3 hold, and* p *is continuously differentiable. For every* $i = 1, ..., n$ *let functions* $g_{ik} \colon \mathbb{R}_+^2 \to \mathbb{R}_+$, $k = 1, 2$, *be continuous and characterized by the following properties for* $(x, z) \in \mathbb{R}_+^2$, *with* $x + z = y \in \mathbb{R}_+$*:*

$$\begin{cases} xz \geq 0 \Rightarrow g_{i1}(x, z) - g_{i2}(x, z) \geq 0, \\ xz > 0 \Rightarrow g_{i1}(x, z) - g_{i2}(x, z) > 0, \\ x \geq 0 \wedge z > 0 \Rightarrow g_{ik}(x, z) \in (0, 1], \;\; k = 1, 2, \\ x > 0 \wedge z \geq 0 \Rightarrow g_{ik}(x, z) \in (0, 1), \;\; k = 1, 2, \\ x_{i2} > x_{i1} \wedge z_{i2} > z_{i1} \Rightarrow g_{ik}(x_{i1}, z_{i1}) > g_{ik}(x_{i2}, z_{i2}), \;\; k = 1, 2. \end{cases} \tag{38}$$

If the following system of conditions is satisfied for $k = 1, 2$*:*

$$\begin{cases} x_{ik} \Big[U_i'(x_{ik}) g_{ik}(x_{ik}, z_{ik}) - p(y_k) \Big] = 0, \\ U_i'(x_{ik}) g_{ik}(x_{ik}, z_{ik}) - p(y_k) \leq 0, \\ x_{ik} \geq 0, \;\; y_k = x_{ik} + z_{ik}, \;\; i = 1, ..., n. \end{cases} \tag{39}$$

then it is not true that $x_{i1} > 0$, $x_{i2} > 0$ *and* $x_{i2} > x_{i1}$ *for every* $i = 1, ..., n$.

Proof. Suppose that $x_{i1} > 0$, $x_{i2} > 0$ and $x_{i2} > x_{i1}$ for every $i = 1, ..., n$. Then $y_1 < y_2$ and by the assumed properties of functions $g_{ik},\ k = 1, 2$ we must have:

$$U_i'(x_{i1})g_{i1}(x_{i1}, z_{i1}) = p(y_1) \text{ and } U_i'(x_{i2})g_{i1}(x_{i2}, z_{i2}) < p(y_2),$$
$$U_i'(x_{i1})g_{i2}(x_{i1}, z_{i1}) > p(y_1) \text{ and } U_i'(x_{i2})g_{i2}(x_{i2}, z_{i2}) = p(y_2).$$

However, under Assumptions 1 and 3, the second relation cannot hold for $x_{i1} < x_{i2}$. Furthermore, both relations imply that:

$$U_i'(x_{i1})(g_{i2}(x_{i1}, z_{i1}) - g_{i1}(x_{i1}, z_{i1})) > 0,$$
$$U_i'(x_{i2})(g_{i1}(x_{i2}, z_{i2}) - g_{i2}(x_{i2}, z_{i2})) < 0,$$

which is a contradiction to the assumed properties of functions $g_{ik},\ k = 1, 2$. Therefore, it is not true that $x_{i1} > 0$, $x_{i2} > 0$ and $x_{i2} > x_{i1}$ for every $i = 1, ..., n$ ■

The following results provide the desired characterization of the outcomes reachable in the studied games.

Theorem 5. *Suppose that Assumptions 1, 2, 3, 4 and 5 hold. Suppose also that p defines the demand reducing factors $\hat{\chi}_i$ and $\tilde{\chi}_i$ that satisfy the assumptions of Lemma 2. Then the following relations hold:*

- *$p(\hat{y}) < p(\bar{y})$ and $p(\tilde{y}) < p(\bar{y})$,*
- *for every $i = 1, ..., n$, if $\bar{x}_i > 0$ then $\hat{x}_i > 0$ and $\tilde{x}_i > 0$,*
- *it is not true that for every $i = 1, ..., n$ we have $\hat{x}_i > 0$, $\bar{x}_i > 0$ and $\hat{x}_i > \bar{x}_i$,*
- *it is not true that for every $i = 1, ..., n$ we have $\tilde{x}_i > 0$, $\hat{x}_i > 0$ and $\tilde{x}_i > \hat{x}_i$.*

Proof. Follows from Lemma 1 and 2. ■

Theorem 6. *Suppose that Assumptions 1, 2, 3, 4 and 5 hold. The following relations hold for every $i = 1, ..., n$:*

- *if $\hat{x}_i \geq \bar{x}_i$, then $Q_i(\hat{m}_i(\hat{\boldsymbol{\theta}})) \geq Q_i(\bar{m}_i(\bar{\boldsymbol{\theta}}))$,*
- *if $\tilde{x}_i \geq \bar{x}_i$, then $Q_i(\tilde{m}_i(\tilde{\boldsymbol{\theta}})) \geq Q_i(\bar{m}_i(\bar{\boldsymbol{\theta}}))$.*

Proof. Consider $\hat{x}_i \geq \bar{x}_i$. By concavity of U_i we have:

$$U_i(\bar{x}_i) \leq U_i(\hat{x}_i) + U_i'(\hat{x}_i)(\bar{x}_i - \hat{x}_i).$$

From Theorem 2 we conclude that:

$$U_i(\bar{x}_i) - U_i'(\hat{x}_i)\bar{x}_i \leq U_i(\hat{x}_i) - U_i'(\hat{x}_i)\hat{x}_i \leq U_i(\hat{x}_i) - p(\hat{y})\hat{x}_i.$$

Since $0 < \bar{x}_i \leq \hat{x}_i \Rightarrow U_i'(\bar{x}_i) = p(\bar{y}) \geq U_i'(\hat{x}_i)$, it follows that: $U_i(\bar{x}_i) - p(\bar{y})\bar{x}_i \leq U_i(\bar{x}_i) - U_i'(\hat{x}_i)\bar{x}_i$. Consequently:

$$Q_i(\bar{m}_i(\bar{\boldsymbol{\theta}})) = U_i(\bar{x}_i) - p(\bar{y})\bar{x}_i \leq U_i(\hat{x}_i) - p(\hat{y})\hat{x}_i = Q_i(\hat{m}_i(\hat{\boldsymbol{\theta}})).$$

By the same arguments we can show that $Q_i(\tilde{m}_i(\tilde{\boldsymbol{\theta}})) \geq Q_i(\bar{m}_i(\bar{\boldsymbol{\theta}}))$ if $\tilde{x}_i \geq \bar{x}_i$. ■

THEOREM 7. *Suppose that Assumptions 1, 2, 3, 4 and 5 hold. Let* $\Pi(\mu, y) = \mu y - C(y)$ *denote the coordinator's payoff. Then* $\Pi(\hat{\mu}, \hat{y}) \leq \Pi(\bar{\mu}, \bar{y})$ *and* $\Pi(\tilde{\mu}, \tilde{y}) \leq \Pi(\bar{\mu}, \bar{y})$.

Proof. By convexity of C we have: $C(\hat{y}) \geq C(\bar{y}) + \bar{\mu}(\hat{y} - \bar{y}) \geq C(\bar{y}) + \hat{\mu}\hat{y} - \bar{\mu}\bar{y}$, which implies that $\hat{\mu}\hat{y} - C(\hat{y}) \leq \bar{\mu}\bar{y} - C(\bar{y})$. The same holds for $\tilde{\mu} = p(\tilde{y})$. ■

It is noteworthy that the identified properties of Nash equilibrium outcomes, as far as the addressed class of price-anticipation games is considered, are in fact determined by the properties of function p. Namely, agent's demand corresponding to a given value of price, satisfying the interaction balancing equation, is reduced in equilibrium by the factor that depends on the elasticity of price function. It is therefore clear that the obtained conclusions should be sensitive to any violations of Assumptions 1 and 2. This, indeed, is the case, as illustrated below.

EXAMPLE 1 (Monotonicity of demand reducing factors). *Consider* $p(y) = y^b$. *Elasticity of* p *is nondecreasing,* $\varepsilon(y) = b$. *However, as it can be deduced from the expressions presented below, functions* $g_{i1} = \hat{\chi}_i$ *and* $g_{i2} = \tilde{\chi}_i$ *may violate the monotonicity conditions of Lemma 2:*

$$\hat{\chi}_i(x + \Delta x, z + \Delta z) - \hat{\chi}_i(x, z) = \frac{b\,(\Delta z x - \Delta x z)}{(x+z)\,(b+1)\,(\Delta x + \Delta z + x + z)},$$
$$\tilde{\chi}_i(x + \Delta x, z + \Delta z) - \tilde{\chi}_i(x, z) = \frac{b\,(\Delta z x - \Delta x z)}{(x+z+bx)\,(\Delta x + \Delta z + x + z + b\Delta x + bx)}.$$

On the other hand, function $p(y) = 1/(C - y)$, *characterized by nondecreasing elasticity* $\varepsilon(y) = y/(C-y)$, *satisfies the required conditions:*

$$\hat{\chi}_i(x + \Delta x, z + \Delta z) - \hat{\chi}_i(x, z) = -\frac{\Delta x}{Z},$$
$$\tilde{\chi}_i(x + \Delta x, z + \Delta z) - \tilde{\chi}_i(x, z) = -\frac{(C - z)\Delta x + x\Delta z}{(Z - z)(Z - (z + \Delta \mathrm{z}))}.$$

EXAMPLE 2 (Demand reduction with decreasing price elasticity). *Consider the following example due to Johari (2004). Suppose* $p(y) = a + \max\{y + 1, 0\}$ *for some* $a < 1$. *Function* p *does not satisfy Assumption 2. Indeed, for* $y \geq 1$ *we have* $\varepsilon'(y) = (a-1)/(a+y-1)^2 < 0$, *which shows that* p *is characterized by decreasing elasticity. This example can be used to demonstrate that when Assumption 2 is violated, relation (20) of Theorem 2 is no longer valid.*

Let us assume that $U_i(x_i) = \gamma_i x_i$ *for* $\boldsymbol{\gamma} = (1, a)$ *and* $i = 1, 2$. *It can be verified that equilibrium conditions (18), corresponding to the payment-bidding game, are satisfied by* $\hat{\boldsymbol{\theta}} = (a - a^3, a^3)$. *Precisely, there holds* $\hat{\mathbf{x}} = (1 - a^2, a^2)$, $\hat{y} = 1$, $p(\hat{y}) = a$, $\beta^-(\hat{y}) = 0$ *and* $\beta^+(\hat{y}) = 1/(1 + a)$. *However, in such a case* $U_2'(\hat{x}_2) = p(\hat{y}) = a$, *which means that agent* $i = 2$ *bids truthfully, contrary to what Theorem 2 claims.*

γ	$\bar{\mathbf{x}}$	$\hat{\mathbf{x}}$	$\tilde{\mathbf{x}}$	$\bar{\mathbf{w}}$	$\hat{\mathbf{w}}$	$\tilde{\mathbf{w}}$	$\bar{\mathbf{Q}}$	$\hat{\mathbf{Q}}$	$\tilde{\mathbf{Q}}$
1.00	0.193	0.687	0.912	0.162	0.379	0.313	0.015	0.145	0.335
4.00	3.774	3.591	3.151	3.162	1.978	1.083	3.090	4.118	4.611
5.00	4.967	4.186	3.682	4.162	2.306	1.265	4.769	5.924	6.454

$\bar{\mu}$	$\hat{\mu}$	$\tilde{\mu}$	$\bar{y}$	$\hat{y}$	$\tilde{y}$
0.838	0.551	0.344	8.934	8.464	7.745

$\bar{\Pi}$	$\hat{\Pi}$	$\tilde{\Pi}$	$\bar{W}$	$\hat{W}$	$\tilde{W}$
6.141	3.636	1.946	14.015	13.822	13.345

γ	$\nabla_i Q_i(\bar{m}_i(\bar{\boldsymbol{\theta}}))$	$\nabla_i Q_i(\hat{m}_i(\bar{\boldsymbol{\theta}}))$	$\nabla_i Q_i(\bar{m}_i(\tilde{\boldsymbol{\theta}}))$
1.00	-0.000	-0.000	-0.000
4.00	0.000	-0.000	0.000
5.00	0.000	0.000	0.000

Table 1. Numerical results for $p(y) = -\frac{1}{y-10} - \frac{1}{10}$ and $U_i(x_i) = \gamma_i \ln(x_i + 1)$.

The results of numerical experiments* illustrate the relations identified in this section. Tables 1-4 present allocations $\mathbf{x}$, payments $\mathbf{w}$, agents' payoffs $\mathbf{Q}$, coordinator's payoff Π, equilibrium price μ and supply y, and efficiency index of equilibrium point $W = \sum_{i=1}^{n} U_i(x_i) - C(y)$ that can be reached when the agents' actions, modeled by system $\texttt{AGENT}_i, i = 1, ..., n$, are coordinated by the rules of mechanisms $\bar{\mathbf{m}}$, $\hat{\mathbf{m}}$ and $\tilde{\mathbf{m}}$. The presented outcomes were obtained in computations in which index $\|(\nabla_1 Q_1(m_1(\boldsymbol{\theta})), ..., \nabla_n Q_n(m_1(\boldsymbol{\theta}))\|$ is minimized over $\boldsymbol{\theta} \in \mathbb{R}^n_+$ for $m_i \in \{\bar{m}_i, \hat{m}_i, \tilde{m}_i\}$, $i = 1, ..., n$, subject to feasibility constraints. Notice that the presented numerical data correspond to the outcomes located in a *neighborhood* of the theoretical equilibrium points.

Table 1 shows the results obtained when Assumption 2 and 3 are satisfied. In particular, notice that function p is characterized by nondecreasing elasticity, $\varepsilon(y) = 10/(10 - y)$, over domain $y \in [0, 10)$. It can be seen that the following relations hold: $\bar{\mu} > \hat{\mu} > \tilde{\mu}$ and $\bar{y} > \hat{y} > \tilde{y}$, $\bar{\Pi} > \hat{\Pi}$ and $\bar{\Pi} > \tilde{\Pi}$. Furthermore, there holds $\bar{Q}_i < \hat{Q}_i$ for $\bar{x}_i < \hat{x}_i$ and $\bar{x}_i \geq \hat{x}_i$, $\bar{Q}_i < \tilde{Q}_i$ both for $\bar{x}_i < \tilde{x}_i$ and $\bar{x}_i \geq \tilde{x}_i$.

Table 2 illustrates the results obtained for linear utility functions. What should be noticed is the high value of tolerance for the first-order optimality conditions at $\hat{\boldsymbol{\theta}}$, indicating that the applied solver may have reached a local solution. Nonetheless, as can be seen, the examined relations between the equilibrium points are still satisfied. In particular, observe that $\bar{x}_i = 0$ for $\hat{x}_i = 0$

*Matlab 7.12.0, `fmincon` SQP solver.

γ	$\bar{\mathbf{x}}$	$\hat{\mathbf{x}}$	$\tilde{\mathbf{x}}$	$\bar{\mathbf{w}}$	$\hat{\mathbf{w}}$	$\tilde{\mathbf{w}}$	$\bar{\mathbf{Q}}$	$\hat{\mathbf{Q}}$	$\tilde{\mathbf{Q}}$
1.00	0.000	0.000	0.213	0.000	0.000	0.175	0.000	0.000	0.038
4.00	0.000	4.136	3.759	0.000	8.645	3.082	0.000	7.899	11.96
5.00	9.804	5.407	4.941	49.02	11.30	4.051	0.000	15.73	20.66

$\bar{\mu}$	$\hat{\mu}$	$\tilde{\mu}$	$\bar{y}$	$\hat{y}$	$\tilde{y}$
5.000	2.090	0.820	9.804	9.543	8.913

$\bar{\Pi}$	$\hat{\Pi}$	$\tilde{\Pi}$	$\bar{W}$	$\hat{W}$	$\tilde{W}$
46.068	17.816	5.979	46.068	41.449	38.625

γ	$\nabla_i Q_i(\bar{m}_i(\bar{\boldsymbol{\theta}}))$	$\nabla_i Q_i(\hat{m}_i(\bar{\boldsymbol{\theta}}))$	$\nabla_i Q_i(\bar{m}_i(\tilde{\boldsymbol{\theta}}))$
1.00	-4.000	-1.090	-0.000
4.00	-1.000	0.252	-0.000
5.00	0.000	0.200	0.000

Table 2. Numerical results for $p(y) = -\frac{1}{y-10} - \frac{1}{10}$ and $U_i(x_i) = \gamma_i x_i$.

and $\tilde{x}_i > 0$.

Finally, Tables 3 and 4 show that with respect to the resource allocation levels, x_i, $i = 1, ..., n$, the price-taking strategy cannot be strictly dominated by the price-anticipating strategy. Namely, for every $i = 1, ..., n$, there holds: $\bar{x}_i > \hat{x}_i$ and $\bar{x}_i > \tilde{x}_i$. However, what can also be noticed is that the price-anticipating agents receive higher payoffs anyway, i.e. $\bar{Q}_i < \hat{Q}_i$ and $\bar{Q}_i < \tilde{Q}_i$. On the other hand, the allocations reached in payment-bidding auction dominate the allocations in demand-bidding auction, $\hat{x}_i > \tilde{x}_i$ for every $i = 1, ..., n$. In other words, it is not true that for every $i = 1, ..., n$ we have $\hat{x}_i > 0$, $\bar{x}_i > 0$ and $\hat{x}_i > \bar{x}_i$, and that $\tilde{x}_i > 0$, $\hat{x}_i > 0$ and $\tilde{x}_i > \hat{x}_i$. Notice that Assumption 3 is satisfied only locally.

7. Summary

In this paper the perspective of an individual agent has been taken in a theoretical study of the implications that imperfect information may have for coordinability of the system in which the uniform-price-based coordination rules are applied. It has been demonstrated how the actively competing agents may reach an equilibrium point that cannot be viewed as a solution to the coordination problem and, therefore, how the coordinability condition can be violated in the analyzed setting. More precisely, the following conclusions can be drawn from the analysisp resented. If each agent applies the price-anticipating strat-

γ	$\bar{\mathbf{x}}$	$\hat{\mathbf{x}}$	$\tilde{\mathbf{x}}$	$\bar{\mathbf{w}}$	$\hat{\mathbf{w}}$	$\tilde{\mathbf{w}}$	$\bar{\mathbf{Q}}$	$\hat{\mathbf{Q}}$	$\tilde{\mathbf{Q}}$
1.00	1.486	1.465	1.440	0.126	0.123	0.119	0.870	0.872	0.873
4.00	1.550	1.544	1.537	0.131	0.129	0.127	3.868	3.869	3.871
5.00	1.554	1.549	1.544	0.132	0.130	0.127	4.867	4.869	4.871

$\bar{\mu}$	$\hat{\mu}$	$\tilde{\mu}$	$\bar{y}$	$\hat{y}$	$\tilde{y}$
0.085	0.084	0.083	4.589	4.558	4.521

$\bar{\Pi}$	$\hat{\Pi}$	$\tilde{\Pi}$	$\bar{W}$	$\hat{W}$	$\tilde{W}$
0.234	0.229	0.224	9.840	9.839	9.838

γ	$\nabla_i Q_i(\bar{m}_i(\bar{\boldsymbol{\theta}}))$	$\nabla_i Q_i(\hat{m}_i(\bar{\boldsymbol{\theta}}))$	$\nabla_i Q_i(\bar{m}_i(\tilde{\boldsymbol{\theta}}))$
1.00	0.000	-0.000	-0.000
4.00	-0.000	-0.000	-0.000
5.00	0.000	0.000	0.000

Table 3. Numerical results for $p(y) = -\frac{1}{y-10} - \frac{1}{10}$ and $U_i(x_i) = \gamma_i \sin(x_i)$.

egy, $\mathtt{AGENT}_i(\hat{m}_i)$ or $\mathtt{AGENT}_i(\tilde{m}_i)$, then, referring to the outcomes attainable under the price-taking strategy, $\mathtt{AGENT}_i(\bar{m}_i)$, in Nash equilibrium of the studied price-anticipation games:

- each agent reveals a reduced level of demand, which results in a reduced value of equilibrium price, supply and the coordinator's payoff,
- some of the price-anticipating agents (but not all of them) may receive improved allocations, in case of which they also receive improved payoffs,
- some of the price-anticipating agents (but not all of them) may receive improved payoffs with reduced allocations,
- the payment-bidding auction cannot be strictly dominated by the demand-bidding auction with respect to the resource allocation levels individually received by each agent,
- the commonly applied price-taking strategy cannot be strictly dominated by the commonly applied price-anticipating strategy with respect to the resource allocation levels individually received by each agent.

Price-based coordinability implies the existence of uniform prices that support optimal solution to the system coordination problem. Equivalently, the goal of price-based coordination is achieved when a set of prices can be found in response to which the performance index of the coordinator is optimized and no agent can derive controls that improve his/her individual performance index. The results presented in this paper show why such a coordination signal cannot be reached in the environments with asymmetric information, in which

γ	$\bar{\mathbf{x}}$	$\hat{\mathbf{x}}$	$\tilde{\mathbf{x}}$	$\bar{\mathbf{w}}$	$\hat{\mathbf{w}}$	$\tilde{\mathbf{w}}$	$\bar{\mathbf{Q}}$	$\hat{\mathbf{Q}}$	$\tilde{\mathbf{Q}}$
0.00	1.362	1.386	1.370	0.282	0.214	0.176	0.696	0.769	0.804
0.50	1.549	1.579	1.545	0.321	0.243	0.198	0.647	0.730	0.768
1.00	3.831	3.100	2.707	0.793	0.478	0.348	0.782	0.933	0.963

$\bar{\mu}$	$\hat{\mu}$	$\tilde{\mu}$	$\bar{y}$	$\hat{y}$	$\tilde{y}$
0.207	0.154	0.128	6.743	6.065	5.622

$\bar{\Pi}$	$\hat{\Pi}$	$\tilde{\Pi}$	$\bar{W}$	$\hat{W}$	$\tilde{W}$
0.948	0.609	0.458	3.074	3.041	2.993

γ	$\nabla_i Q_i(\bar{m}_i(\bar{\boldsymbol{\theta}}))$	$\nabla_i Q_i(\hat{m}_i(\bar{\boldsymbol{\theta}}))$	$\nabla_i Q_i(\bar{m}_i(\tilde{\boldsymbol{\theta}}))$
0.00	-0.000	-0.000	0.000
0.50	0.000	0.000	0.000
1.00	0.000	0.000	-0.000

Table 4. Numerical results for $p(y) = -\frac{1}{y-10} - \frac{1}{10}$ and $U_i(x_i) = \gamma_i \ln(x_i + 1) + (1 - \gamma_i)\sin(x_i)$.

the agents actively try to reach a Nash equilibrium point of a game induced by the uniform-price-based coordination mechanisms. Indeed, as it is well-known, whenever there are actively interacting agents in the system that know something that others do not, then their individual goals should not be expected harmonize with a specific goal of the coordinator. The *invisible hand* of selfish (market) competition may therefore be often expected to lead to the outcomes that are inefficient, at least as long as the utilitarian efficiency indicator is concerned; see Stiglitz (2006).

In light of what has been stated above it is quite natural to pose the question whether it is possible to control the expected outcomes of the anticipative control strategies. A general and rather pessimistic answer to this question has been given by the theory of mechanism design; see Karpowicz (2012a). Namely, in the considered setting, due to asymmetry of information the coordinator is forced to face a trade-off between either reaching the system-wide goals or meeting expectations of the agents interacting in the system. Certainly, it is possible for the coordinator to design a game with Nash equilibrium point at which coordinability condition is reached. However, in principle, such a design comes at a cost that is high enough to determine the coordinator's decision regarding the actual implementation of the game in the system. In practice, gains from reaching a solution to the coordination problem may be outweighed by the equilibrium-implementation costs which must be incurred by the coordinator, through violation of the interaction-balancing conditions, or by the agents, through violation of the game participation conditions.

Indeed, there are many factors that affect performance of a distributed system, including the very structure of the system's interactions. There are, therefore, many ways to resolve the design trade-offs faced by the coordinator. The results presented in this paper are intended to explain the role played in this context by a choice of price-based coordination rules.

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