Control and Cybernetics

vol. **42** (2013) No. 1

Constrained controllability of second order dynamical systems with delay*

by

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Abstract: The paper considers finite-dimensional dynamical control systems described by second order semilinear stationary ordinary differential state equations with delay in control. Using a generalized open mapping theorem, sufficient conditions for constrained local controllability in a given time interval are formulated and proved. These conditions require verification of constrained global controllability of the associated linear first-order dynamical control system. It is generally assumed that the values of admissible controls are in a convex and closed cone with vertex at zero. Moreover, several remarks and comments on the existing results for controllability of semilinear dynamical control systems are also presented. Finally, a simple numerical example which illustrates theoretical considerations is also given. It should be pointed out that the results given in the paper extend for the case of semilinear second-order dynamical systems constrained controllability conditions, which were previously known only for linear second-order systems.

Keywords: controllability, second-order dynamical systems, delayed control systems, semilinear control systems, constrained controls.

1. Introduction

Controllability is one of fundamental concepts in mathematical control theory. This is a qualitative property of dynamical control systems and is of particular importance in control theory. Systematic study of controllability started at the beginning of the 1960s, when the theory of controllability based on description in the form of state space for both time-invariant and time-varying linear control systems was worked out.

Roughly speaking, controllability generally means that it is possible to steer in some time interval a dynamical control system from an arbitrary initial state

*Submitted: January 2012; Accepted: October 2012.

to an arbitrary final state using controls taken from the set of admissible controls. The literature provides many different definitions of controllability, depending strongly on the class of dynamical control systems and on the set of admissible controls (Klamka, 1991, 1993, 1996, 2004).

In recent years, various controllability problems for different types of nonlinear dynamical systems have been considered in many publications and monographs. An extensive list of these publications can be found, for instance, in the monograph Klamka (1991) or in the survey paper Klamka (1993). However, it should be stressed, that the majority of literature in this direction has been mainly concerned with controllability problems for finite-dimensional nonlinear dynamical systems with unconstrained controls and without delays or for linear dynamical systems with constrained controls and with delays.

The monograph of Kaczorek (1993) presents controllability, observability and duality results for different types of continuous and discrete linear dynamical systems. Similarly, the monograph of Kaczorek (2002) presents controllability, observability and duality results for continuous and discrete positive linear dynamical systems. Moreover, in the papers by Kaczorek (2006, 2007a, b, and c) controllability and reachability of special kinds of linear stationary control dynamical systems are considered.

In the present paper, we shall consider constrained local controllability problems for second-order finite-dimensional semilinear stationary dynamical systems with point delay in control, described by the set of ordinary differential state equations. Let us recall that semilinear dynamical control systems contain both linear and pure nonlinear parts in the differential state equations.

We shall formulate and prove sufficient conditions for constrained local controllability in a prescribed time interval for semilinear second-order stationary dynamical systems, whose nonlinear term is continuously differentiable near the origin, and with single point delay in control. It is generally assumed that the values of admissible controls are in a given convex and closed cone with vertex at zero, or in a cone with nonempty interior. Proof of the main result is based on the so called generalized open mapping theorem presented in a simplified version by Bian and Webb (1999) and Klamka (2004).

Roughly speaking, it will be shown that under suitable assumptions constrained global relative controllability of a linear first-order associated approximated dynamical system implies constrained local relative controllability near the origin of the original semi-linear second-order dynamical system. This is a generalization to constrained controllability case of some previous results concerning controllability of linear dynamical systems with unconstrained controls (Klamka, 1991, 1993, 1996, 2004).

2. System description

Let us consider semi-linear finite-dimensional control system with single point delay in control, described by the following second order differential equation:

$$w''(t) = Gw(t) + f(w(t), u(t), u(t-h)) + Hu(t) + Ku(t-h) \text{ for } t \in [0, T] \quad (1)$$

where the state vector $w(t) \in R^n = W$ and the control vector $u(t) \in R^m = U, G$ is $n \times n$ dimensional constant matrix, H and K are $n \times m$ dimensional constant matrices, $h > 0$ is a single point delay.

Moreover, let us assume that nonlinear mapping $f : W \times U \times U \mapsto W$ is continuously differentiable near the origin and such that $f(0,0,0) = 0$.

For simplicity of considerations we assume zero initial conditions, i.e.

$w(0) = 0$ and $w\prime(0) = 0$.

Using standard substitutions

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} w(t) \\ w'(t) \end{bmatrix} \in R^{2n},$$

we may transform the second-order semilinear dynamical system (1) into the equivalent first-order semilinear stationary $2n$-dimensional control system described by the following ordinary differential state equation

$$x'(t) = Ax(t)+F(x(t),u(t),u(t-h))+Bu(t)+Du(t-h) \text{ for } t \in [0,T], T>0 \quad (2)$$

with zero initial conditions:

$x(0) = 0 \quad u(t) = 0 \quad$ for $t \in$[-h,0]

where the state vector $x(t) \in R^{2n} = X$ and the control $u(t) \in R^{2m} = U$, A is $2n \times 2n$ dimensional constant matrix, B and D are $2n \times m$ dimensional constant matrices,

$$A = [\begin{matrix} 0 & I \\ G & 0 \end{matrix}] \quad B = [\begin{matrix} 0 \\ H \end{matrix}] \quad D = \begin{bmatrix} 0 \\ K \end{bmatrix},$$

and the nonlinear term has the form

$$F(x(t),u(t),u(t-h)) = [\begin{matrix} 0 \\ f(x(t),u(t),u(t-h)) \end{matrix}] \in R^{2n}.$$

Moreover, from previous assumptions concerning the nonlinear term $f(x(t),$ $u(t), u(t-h))$ it follows that the nonlinear mapping $F : X \times U \times U \rightarrow X$ is also continuously differentiable near the origin and such that F(0,0,0)=0.

It is well known that in practical applications admissible controls are always required to satisfy certain additional constraints. Generally, for arbitrary control constraints it is rather very difficult to give easily computable criteria for constrained controllability even in the linear and in finite dimensional cases (Klamka, 1991, 1993). However, for some special cases of the constraints it is possible to formulate and prove simple algebraic constrained controllability conditions.

Therefore, in the sequel we shall assume that the set of values of admissible controls $U_c \subset U$ is a given closed and convex cone with nonempty interior and vertex at zero. Then, the set of admissible controls for the dynamical control systems (1) and (2) has the following form: $U_{ad} = L_\infty([0,T],U_c)$.

Then for a given admissible control $u(t)$ there exists a unique solution $w(t;u) \in R^n$ of the second-order differential equation (1) and similarly, unique solution $x(t;u) \in R^{2n}$ of the first-order ordinary differential state equation (2), with zero initial condition. By transforming the semilinear differential equation (2) into the nonlinear integral equation we get (Klamka, 1996)

$$x(t;u) = \int_0^t S(t-s)(F(x(s;u(s)),u(s),u(s-h)) + Bu(s) + Du(s-h))ds \quad (3)$$

where the matrix semigroup $S(t) = \exp(At)$ for $t \geqslant 0$ is $2n \times 2n$ dimensional exponential transition matrix for the linear part of the semilinear first-order control system (2).

For the semilinear stationary finite-dimensional second-order dynamical system (1) or, equivalently, for the first-order dynamical system (2), it is possible to define many different concepts of controllability. However, in the sequel we shall focus our attention on the so called constrained controllability in a given time interval [0,T].

In order to do that, first of all let us introduce the notion of the so called attainable or reachable set for dynamical system (2) at given final time $T>0$ from zero initial conditions, denoted shortly by $K_T(U_c)$ and defined as follows (Klamka, 1991, 1993, 1996, 2004):

$$K_T(U_c) = \{x \in X : x = x(T,u), u(t) \in U_c \text{ for a.e. } t \in [0,T]\} \quad (4)$$

where $x(t,u)$, $t > 0$ is the unique solution of the differential first-order state equation (2) with zero initial conditions and a given admissible control $u \in U_{ad} = L_\infty([0,T],U_c)$.

Now, using the concept of the attainable set $K_T(U_c)$, let us recall the well known (see, e.g., Klamka, 1991, 1993, 1996, 2004) definitions of local and global constrained controllability in [0,T] for semilinear second-order dynamical system (1).

Definition 2.1 The dynamical system (1) is said to be U_c-locally controllable in [0,T] if the attainable set $K_T(U_c)$ contains a neighborhood of zero in the space X.

Definition 2.2 The dynamical system (1) is said to be U_c-globally controllable in [0,T] if $K_T(U_c) = X$.

In the last part of this section we shall discuss the relationships between controllability of the first-order system (2) for $F(x(t),u(t)) = 0$, and linear second-order dynamical system (1) for the case when $f(w(t),u(t)) = 0$. Therefore, for comparison, we shall consider the following two linear dynamical systems:

$$w''(t) = Gw(t) + Hu(t) + Ku(t-h) t \in [0,T] \quad (5)$$

$$x'(t) = Ax(t) + Bu(t) + Du(t-h) t \in [0,T]. \quad (6)$$

Corollary 2.1 *Second-order linear dynamical system (5) is controllable without any control constraints in a given time interval if and only if the associated first-order 2n-dimensional dynamical system (6) is controllable without any control constraints in the same given time interval.*

Remark 2.1 However, it should be pointed out, that for the controllability problem with constrained controls the above Corollary 2.1 does not hold and there are no general direct relationships between constrained controllability of first-order and second-order linear dynamical systems.

3. Preliminaries

In this section, for completeness of considerations, we shall introduce certain notations and present some rather known important facts from the general theory of nonlinear operators.

Let U and X be given spaces and $g(u)$:$U \to X$ be a mapping continuously differentiable near the origin 0 of U. Let us suppose for convenience that $g(0)$=0. It is well known from the implicit-function theorem (Klamka, 2004) that, if the derivative $Dg(0)$: $U \to X$ maps the space U onto the whole space X, then the nonlinear map g transforms the neighborhood of zero in the space U onto some neighborhood of zero in the space X.

Now, let us consider the more general case when the domain of the nonlinear operator g is Ω, an open subset of U containing 0. Let U_c denote a closed and convex cone in U with vertex at 0.

In the sequel, we shall use for controllability investigations some property of the nonlinear mapping g which is a consequence of the generalized open-mapping theorem. This result seems to be widely known, but for the sake of completeness we shall present it here, though without proof and in a slightly less general form, sufficient for our purpose.

Lemma 3.1 *Let X, U, U_c, and Ω be as described above. Let g:$\Omega \to$X be a nonlinear mapping and suppose that on Ω nonlinear mapping g has derivative Dg, which is continuous at 0. Moreover, suppose that g(0) = 0 and assume that linear map Dg(0) maps U_c onto the whole space X. Then there exist neighborhoods $N_0 \subset X$ about 0$\in$X and $M_0 \subset \Omega$ about 0$\in$U such that the nonlinear equation x=g(u) has, for each x$\in N_0$, at least one solution u$\in M_0 \cap U_c$, where $M_0 \cap U_c$ is a so called conical neighborhood of zero in the space U.*

4. Controllability conditions

In this section we shall study constrained local relative controllability in [0,T] for semilinear dynamical system (1) using the associated linear $2n$-dimensional control dynamical system

$$z'(t) = Cz(t) + Eu(t) + Gu(t-h) \quad \text{for} \quad t \in [0,T] \tag{7}$$

with zero initial condition, $z(0) = 0$, where

$$C = A + D_x F(0,0,0)$$

$$E = B + D_u F(0,0,0)$$

$$G = D + D_{u(t-h)} F(0,0,0) \tag{8}$$

The main result of the paper is the following sufficient condition for constrained local controllability in a given time interval of the semilinear dynamical system with single point delay in control (1).

Theorem 4.1 *Suppose that*

(i) $F(0,0,0) = 0$,

(ii) $U_c \subset U$ is a closed and convex cone with vertex at zero,

(iii) the associated linear control system (7) is U_c-globally controllable in [0,T].

Then the semilinear stationary dynamical control system (1) is U_c-locally controllable in [0,T].

Proof. Let us define for the nonlinear dynamical system (5) a nonlinear map $g{:}L_\infty([0,T],U_c) \to X$ by $g(u) = x(T,u)$.

Similarly, for the associated linear dynamical system (7), we define a linear map $H{:}L_\infty([0,T],U_c) \to X$ by $Hv = z(T,v)$.

By the assumption (iii) the linear dynamical system (7) is U_c-globally relative controllable in $[0,T]$. Therefore, by Definition 2.2, the linear operator H is surjective i.e., it maps the cone U_{ad} onto the whole space X. Furthermore, by Lemma 3.1 we have that $Dg(0)= H$.

Since U_c is a closed and convex cone, then the set of admissible controls $U_{ad} = L_\infty([0,T],U_c)$ is also a closed and convex cone in the function space $L_\infty([0,T],U)$. Therefore, the nonlinear map g satisfies all the assumptions of the generalized open mapping theorem stated in Lemma 3.1.

So, the nonlinear map g transforms a conical neighborhood of zero in the set of admissible controls U_{ad} onto some neighborhood of zero in the state space X. This is by Definition 2.1 equivalent to the U_c-local relative controllability in $[0,T]$ of the semilinear dynamical control system (1). Hence, our theorem follows.

In practical applications of Theorem 4.1, the most difficult problem is to verify the assumption (iii) of constrained global controllability of the linear stationary dynamical system (7) (Klamka, 1991, 1993, 1996, 2004). In order to avoid this disadvantage, we may use the following theorem.

Theorem 4.2 (Klamka, 1991, 1993, 1996, 2004). *Suppose that the set U_c is a given convex cone with vertex at zero and a nonempty interior in the space of control values R^m. Then the associated linear dynamical control system with single point delay in control (7) is U_c-globally controllable in given time interval [0,T] for $T \leqslant h$ if and only if*

1. *it is controllable without any constraints, i.e.* $rank[E, CE, C^2E, ..., C^{2n-1}E] = 2n$,
2. *there is no real eigenvector $v \in R^{2n}$ of the matrix C^{tr} satisfying inequalities $v^{tr}Eu \leqslant 0$, for all $u \in U_c$.*

   Moreover, the associated linear dynamical control system (7) is U_c-globally controllable in [0,T] for $T>h$ if and only if
3. *it is controllable without any constraints, i.e.* $rank[E,G,CE,CG,C^2E,C^2G,...,C^{2n-1}E,C^{2n-1}G] = 2n$,
4. *there is no real eigenvector $v \in R^{2n}$ of the matrix C^{tr} satisfying inequalities*

$v^{tr}Du \leqslant 0$, *for all* $u \in U_c$.

It should be pointed out that for the single input associated linear dynamical control system (7), i.e. for the case of scalar controls and m =1, Theorem 4.2 reduces to the following Corollary.

Corollary 4.1 *Suppose that the dynamical system (1) has single input, i.e., m=1 and* $U_c = R^+$.

Then the associated linear dynamical control system (7) is U_c*-globally controllable in [0,T], for* $T \leqslant h$ *if and only if it is controllable without any constraints, i.e.*

$rank[E, CE, C^2E, ..., C^{2n-1}E] = 2n$,

and matrix C has only complex eigenvalues.

Moreover, the associated linear dynamical control system with single point delay in control (7) is U_c*-globally controllable in given time interval [0,T], for T>h if and only if it is controllable without any constraints, i.e.*

$rank[E, G, CE, CG, C^2E, CG^2, ..., C^{2n-1}E, C^{2n-1}G] = 2n$,

and matrix C has only complex eigenvalues.

Remark 4.1. It should be stressed that the important advantage of the Corollary 4.1 is that instead of a rather difficult condition 2 given in Theorem 4.2 it is enough to verify only eigenvalues of the matrix C.

Remark 4.2. Since all the dynamical systems considered in the previous sections are system with constant coefficient and control values are restricted only in direction and are not restricted in their values, then in fact all the results presented in Section 4 are valid for any time interval [0,T].

Remark 4.3. For dynamical systems with delays, controllability strongly depends on the length of the time interval [0,T]. It is well known (Klamka, 1991) that dynamical system with delay h>0 may be uncontrollable for $T \leqslant h$ however, this system may be controllable for the final time T>h.

Remark 4.4. The general assumption that all initial conditions given in Section 2 are zero is not essential for controllability considerations for linear dynamical systems with cone constrained values of controls. It should be pointed out that the same controllability conditions hold for any nonzero initial conditions.

5. Example

Finally, let us consider constrained controllability of a simple illustrative example of dynamical systems presented in the previous sections.

Let the semilinear second-order finite-dimensional dynamical control system with point delay in control defined on a given time interval [0,T], have the following form

$$\begin{array}{l} w_1''(t) = -w_1(t) + u(t-h) + e^{u(t)} - 1 \\ w_2''(t) = -2w_2(t) + \sin w_2(t) + u(t) \end{array} . \tag{9}$$

Therefore, taking into account previous notations and equations we have $n = 2,\ m = 1,\ w(t) = (w_1(t), w_2(t))^{tr} \in R^2 = W,\ u(t) \in U_c = R^+$.

Now, let us introduce the following standard substitution for the state variables:

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} = \begin{bmatrix} w_1(t) \\ w_1'(t) \\ w_2(t) \\ w_2'(t) \end{bmatrix}.$$

Then, using the notations given in the previous sections, matrices A, B, C and D and the nonlinear mapping f have the following forms:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -2 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}; \quad D = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix};$$

$$f(w(t), u(t), u(t-h)) = \begin{bmatrix} e^{u(t-h)} - 1 \\ \sin w_2(t) \end{bmatrix}. \tag{10}$$

Therefore, taking into account the form of equations (9) we have

$$F(w(t), u(t), u(t-h)) = \begin{bmatrix} 0 \\ e^{u(t-h)} - 1 \\ 0 \\ \sin w_2(t) \end{bmatrix}.$$

Moreover, let the cone of values of controls be a cone of positive numbers i.e., $U_c = R^+$, and therefore, the set of admissible controls is a cone of the following form $U_{ad} = L_\infty([0,T],R^+)$.

Hence, we have

$$F(0,0,0) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Moreover,

$$\mathrm{D_X} F(x(t), u(t), u(t-h)) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \cos w_2(t) & 0 \end{bmatrix}.$$

Therefore,

$$\mathrm{D_X} F(0,0,0) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

and, consequently, we have

$$\mathrm{C} = \mathrm{A} + \mathrm{D_X}F(0,0,0) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}.$$

Similarly,

$$\mathrm{D_u}F(x(t),u(t),u(t-h)) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}; \quad E = B + D_uF(0,0,0) = B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix},$$

and, finally,

$$\mathrm{D_{u(t\text{-}h)}}F(x(t),u(t),u(t-h)) = \begin{bmatrix} 0 \\ e^{u(t-h)} \\ 0 \\ 0 \end{bmatrix}; \quad \mathrm{D_{u(t\text{-}h)}}F(0,0,0) = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}.$$

Hence,

$$\mathrm{G} = \mathrm{D} + \mathrm{D_{u(t\text{-}h)}}F(0,0,0) = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}.$$

Thus, we have

$$\mathrm{C}^2 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}; \quad \mathrm{C}^3 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

Moreover,

$$\mathrm{sI\text{-}C} = \begin{bmatrix} s & -1 & 0 & 0 \\ 1 & s & 0 & 0 \\ 0 & 0 & s & -1 \\ 0 & 0 & 1 & s \end{bmatrix}.$$

Therefore, the characteristic equation for the matrix C is as follows
$\det(sI\text{-}C) = (s^2{+}1)(s^2{+}1){=}0$,
and so matrix C has only two different complex eigenvalues i and $-i$, each of multiplicity 2.

Moreover, using the well known controllability matrix and rank controllability condition for linear dynamical system (7) for time interval $[0,T]$, $T \leqslant h$

(Klamka, 1991, 1993, 1996, 2004) we have

$$rank\left[E, CE, C^2E, C^3E, C^3\right] = rank\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix} = 3 < 4 = 2n.$$

Thus, the associated first order linear dynamical system with point delay in control is not controllable in the time interval [0,T] for $T \leqslant h$.

However, using the well known controllability matrix and rank controllability condition for linear dynamical system (7) for time interval [0,T], T>h (Klamka, 1991, 1993, 1996, 2004), we have

$$rank\left[E, G, CE, CG, C^2E, C^2G, C^3E, C^3G\right] =$$
$$= \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & -1 & 0 \\ 1 & 1 & 0 & 0 & -1 & -1 & 0 & 0 \end{bmatrix}$$
$$= 4 = 2n.$$

Hence, both assumptions of Theorem 4.2 are satisfied and therefore, the associated linear $2n$-dimensional dynamical control system (7) with the above matrices C, E and G is R^+ -globally controllable in a given time interval [0,T], for T >h. Moreover, all the assumptions stated in Theorem 4.1 are also satisfied and thus the second-order semilinear dynamical control system with point delay in control (9) is R^+-locally controllable in [0,T].

This example shows that controllability strongly depends on time interval and delayed control.

6. Concluding remarks

In this paper, sufficient conditions for constrained local controllability near the origin for semilinear second-order stationary finite-dimensional dynamical control systems with point delay in control have been formulated and proved. It was generally assumed that control values are in a given convex cone with vertex at zero and nonempty interior. In the proof of the main result the generalized open mapping theorem has been used.

These conditions extend to the case of constrained controllability of second-order finite-dimensional semilinear dynamical control systems with point delay in control the results published previously in Klamka (1991, 1993, 1996, 2004).

The method presented in the paper is quite general and covers a wide class of semilinear dynamical control systems. Therefore, similar constrained controllability results may be derived for more general class of semilinear dynamical control systems.

For example, it seems that it is possible to extend sufficient constrained controllability conditions given in the previous sections for more general class of semilinear dynamical control systems with multiple point delays in the control or with multiple point delays in the controls and in the state variables and for the discrete-time semilinear control systems. Moreover, quite similar method can be used to derive sufficient conditions for local controllability of semilinear dynamical systems with nonlinear term containing both state variables and control function.

Acknowledgement

This paper was supported by the Polish National Centre of Science grant no. N N514 51 33 40.

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