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# Fusion filtration in LQG control for multisensor systems*

by

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**Abstract:** In the paper, state filtration in a LQG problem formulated for a multisensor system is considered. Control is determined by a central node as a linear form of a state estimate. It is assumed that control values are not available to local nodes. Because of the drawbacks of centralized filtration an optimal fusion of decentralized local Kalman filters is proposed. When control values are not available to local nodes, then control should be treated as a random variable in the synthesis of local state estimates. This leads to a non-classical estimation. It is shown that the proposed filter is equivalent to the centralized one.

**Keywords:** multisensor system, Kalman filter, LQG problem, fusion filtration

## 1. Introduction

Multisensor systems find applications in many areas, such as aerospace, robotics, image processing, military surveillance, fire protection or medical diagnosis. The advantage of using these systems over systems with a single sensor results from e.g. improved reliability, robustness, extended coverage, improved resolution etc. In these systems the state estimation problem is one of the critical concerns.

Theoretically, state estimate can be determined by using Kalman filter in a centralized structure. Conventional Kalman filtration requires that all process measurements be sent to a central station, which determines an estimate of the system state. The centralized architecture produces an optimal estimate in the minimum mean square error (MMSE) sense, but it may imply low survivability and requires high processing and communication loads.

In order to integrate data from distributed sensors estimation fusion algorithms and appropriate architectures are proposed.

Fusion approaches have been investigated for years and some results are known. In Hashemipour et al. (1988); Zhu et al. (2001); Song et al. (2007)

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and Duan and Li (2011) the centralized optimal state estimate is calculated from estimates determined by local nodes. The global estimate is equivalent to the optimal centralized one. In Chang et al. (1997), Chen et al. (2003) and Chang et al. (2004) fusion algorithms are presented, guaranteeing only local optimality.

In the majority of papers autonomous systems are considered (see Liggins et al., 1997; Schlosser and Kroschel, 2007; Sijs et al., 2008) or systems with control available to all the decision makers (see Hashemipour et al., 1988; Speyer, 1979). The latter case does not introduce additional difficulties to the estimation problem.

A decentralized Linear-Quadratic-Gaussian (LQG) problem is considered, e.g., in Speyer (1979), Mutambara (1998). In Speyer (1979) it is assumed that control values $u_n^j$, $j = 1, .., M$, are determined by $M$ local nodes and transmitted to other ones in order to calculate state estimates. Control and filtration are realized in a fully decentralized structure (without a central node).

In this paper a hierarchical LQG problem for a multisensor system is considered, in which control values determined by the central node are not known to local nodes. The control law is a linear form of the state estimate (see Meditch, 1969). The state estimate should be calculated on line basing on the available measurement information. Because of the drawbacks of centralized filtration, an optimal decentralized Kalman filter, which is equivalent to the centralized one, is proposed.

The state estimate is fused from local state estimates that are determined by local nodes, basing upon their own observations, and then transmitted to a central (fusion) node.

The main points of the paper are the nonconventional filtration problem formulation and its solution, because it is assumed that control values $u_n$ are unknown to local nodes. When control values are not available to local nodes then control should be treated as a random variable in the synthesis of local state estimates. This leads to a non-classical estimation.

## 2. Preliminaries

### 2.1. Filtration

It is well known that a minimum mean square error (MMSE) estimate $\hat{x}$ of a random signal $x$ given information $\vec{i}$ is a conditional expectation $\hat{x} = E(x|\vec{i})$. For dynamical systems a state estimate $\hat{x}_{n+1|j}$ at time $n+1$, given measurement information $\vec{i}_j = [i_0^T, i_1^T, ..., i_j^T]^T$ at time $j$ has the form

$$\hat{x}_{n+1|j} = E(x_{n+1}|\vec{i}_j). \tag{1}$$

Thus, for $j = n + 1$ we have

$$\hat{x}_{n+1|n+1} = E(x_{n+1}|\vec{i}_{n+1}) = E(x_{n+1}|\vec{i}_n, i_{n+1}) \tag{2}$$

where $\vec{i}_{n+1} = [\vec{i}_n^T, i_{n+1}^T]^T$.

If the random vector $[x_{n+1}^T, \vec{i}_n^T, i_{n+1}^T]^T$ is Gaussian, then

$$\begin{aligned}\hat{x}_{n+1|n+1} &= E(x_{n+1}|\vec{i}_{n+1}) = E(x_{n+1}|\vec{i}_n, i_{n+1}) = \\ &= E(x_{n+1}|\vec{i}_n) + E(x_{n+1}|\tilde{i}_{n+1|n}) - Ex_{n+1},\end{aligned} \tag{3}$$

where

$$\tilde{i}_{n+1|n} = i_{n+1} - E(i_{n+1}|\vec{i}_n). \tag{4}$$

If the random vector $[x_{n+1}^T, \tilde{i}_{n+1|n}^T]^T$ is Gaussian then

$$E(x_{n+1}|\tilde{i}_{n+1|n}) = Ex_{n+1} + P_{x_{n+1}\tilde{i}_{n+1|n}} P^{-1}_{\tilde{i}_{n+1|n}\tilde{i}_{n+1|n}} (\tilde{i}_{n+1|n} - E\tilde{i}_{n+1|n}), \tag{5}$$

where $P_{\alpha\beta}$ denotes the covariance matrix of the random vectors $\alpha$ and $\beta$.

Under the above assumptions, equation (3) can be written down in the form

$$\hat{x}_{n+1|n+1} = \hat{x}_{n+1|n} + K_{n+1}[i_{n+1} - E(i_{n+1}|\vec{i}_n)] \tag{6}$$

where

$$\hat{x}_{n+1|n} = E(x_{n+1}|\vec{i}_n) \tag{7}$$

and

$$K_{n+1} = P_{x_{n+1}\tilde{i}_{n+1|n}} P^{-1}_{\tilde{i}_{n+1|n}\tilde{i}_{n+1|n}}. \tag{8}$$

Equations (6)-(8) may be used for the determination of a recursive form of a state estimate $\hat{x}_{n|n}$ given available information $\vec{i}_n$.

## 2.2. The LQG problem

Consider a linear system described by the equations

$$\begin{aligned}x_{n+1} &= A_n x_n + B_n u_n + w_n \\ y_n &= C_n x_n + r_n\end{aligned} \tag{9}$$

where $x_n$, $u_n$, $y_n$ are the state, control and measurement, respectively; $A_n$, $B_n$, $C_n$ are known matrices, $w_n$, $r_n$ are the state and measurement noises, respectively. It is assumed that $x_0 \sim N(\bar{x}_0, X_0)$, $w_n \sim N(0, W_n)$, $r_n \sim N(0, R_n)$ and $x_n \in R^k$, $w_n \in R^k$, $y_n \in R^p$, $r_n \in R^p$; $A_n \in R^{k\times k}$, $C_n \in R^{p\times k}$. Additionally, $w_n$, $r_m$ are Gaussian white noise processes independent of each other and of the Gaussian initial state $x_0$.

The optimal control problem is to find

$$I^o = \min_{a_0(.), a_1(.), \ldots, a_N(.)} E[\frac{1}{2}\sum_{n=0}^{N}(x_n^T Q_n x_n + u_n^T H_n u_n)_{u_n = a_n(\vec{y}_n)}] \tag{10}$$

subject to the stochastic system (9), where $Q_n$ and $H_n$ are semipositive and positive, respectively, definite symmetric matrices and $\vec{y}_n = \{y_0, ..., y_n\}$ is the available measurement history.

It is an approach with the classical information pattern, because the control $u_n = a_n(\vec{y}_n)$ depends on all measurements.

The solution to this LQG problem is well known in the literature and has the form

$$u_n^o = S_n \hat{x}_{n|n}, \tag{11}$$

where the estimate, obtained using all the information, is

$$\hat{x}_{n|n} = E(x_n|\vec{y}_n). \tag{12}$$

The control gain $S_n$ is

$$S_n = -(H_n + B_n^T \Lambda_{n+1} B_n)^{-1} B_n^T \Lambda_{n+1} A_n, \tag{13}$$

where $\Lambda_n$ is propagated backwards in time as

$$\Lambda_n = A_n^T[\Lambda_{n+1} - \Lambda_{n+1} B_n (H_n + B_n^T \Lambda_{n+1} B_n)^{-1} B_n^T \Lambda_{n+1}] A_n + Q_n \tag{14}$$

with $\Lambda_N = Q_N$.

The estimate $\hat{x}_{n|n}$ can be found using equations (6)-(8). The estimate is propagated as

$$\hat{x}_{n+1|n+1} = \hat{x}_{n+1|n} + K_{n+1}(y_{n+1} - C_{n+1}\hat{x}_{n+1|n}), \tag{15}$$

where

$$\hat{x}_{n+1|n} = E(x_{n+1}|\vec{y}_n) = A_n \hat{x}_{n|n} + B_n u_n. \tag{16}$$

The Kalman gain $K_{n+1}$ is given as

$$\begin{aligned} K_{n+1} &= P_{n+1|n} C_{n+1}^T (C_{n+1} P_{n+1|n} C_{n+1}^T + R_{n+1})^{-1} = \\ &= P_{n+1|n+1} C_{n+1}^T R_{n+1}^{-1}, \end{aligned} \tag{17}$$

where
$P_{n+1|n+1} = E(x_{n+1} - \hat{x}_{n+1|n+1})(x_{n+1} - \hat{x}_{n+1|n+1})^T$
and $P_{n+1|n} = E(x_{n+1} - \hat{x}_{n+1|n})(x_{n+1} - \hat{x}_{n+1|n})^T$
are the error covariance matrices are described as

$$P_{n+1|n+1} = (\mathbf{1} - K_{n+1} C_{n+1}) P_{n+1|n} \tag{18}$$

$$P_{n+1|n} = A_n P_{n|n} A_n^T + W_n \tag{19}$$

with $P_{0|-1} = X_0$.

Equations (15)-(19) form the solution to filtration problem related to LQG control problem.

## 3. Problem formulation

Consider a linear system described by the equations

$$\begin{aligned} x_{n+1} &= A_n x_n + B_n u_n + w_n \\ y_n^j &= C_n^j x_n + r_n^j, \quad j = 1, ..., M \end{aligned} \tag{20}$$

where $x_n$, $y_n^j$ are the state and the measurement from the $j$th sensor, respectively; $A_n$, $C_n^j$ are the system and observation models, $w_n$, $r_n^j$ are the state and measurement noises, respectively. It is assumed that $x_0 \sim N(\bar{x}_0, X_0)$, $w_n \sim N(0, W_n)$, $r_n^j \sim N(0, R_n^j)$ and $x_n \in R^k$, $w_n \in R^k$, $y_n^j \in R^{p^j}$, $r_n^j \in R^{p^j}$; $A_n \in R^{k\times k}$, $C_n^j \in R^{p_j \times k}$. Additionally, $w_n$, $r_m^j$ are Gaussian white noise processes independent of each other and of the Gaussian initial state $x_0$.

The stacked measurement equation is written as

$$y_n = C_n x_n + r_n \tag{21}$$

where $y_n = [y_n^{1T}, ..., y_n^{MT}]^T$, $C_n = [C_n^{1T}, ..., C_n^{MT}]^T$, $r_n = [r_n^{1T}, ..., r_n^{MT}]^T$, $R_n = E r_n r_n^T = blockdiag\{R_n^1, ..., R_n^M\}$.

Let us consider the LQG problem presented in Section 2.2.

The control $u_n = a_n(\vec{y}_n)$ is described by the equations (11). It is a linear form of the state estimate $\hat{x}_{n|n}$. The conventional Kalman filter is described by the equations (15)-(19).

The system (20) can be considered as a multisensor system in which the measurements $y_n^j, \quad j = 1, ..., M$, are transmitted to and processed by a central controller, and the resulting state estimate is optimal in LMS sense. For reasons presented in the introduction, every local node with local processing capability, carries out Kalman filtering upon its own available information $\vec{y}_n^j = \{y_0^j, ..., y_n^j\}$ and then transmits the local state estimates to the fusion center. Two different situations can be identified. In the first one the information on values of control $u_n$, determined by the central node, is sent to local nodes. In the second situation the values of control are not available to local nodes. Then the control should be treated as a random variable when determining local estimates. This case is considered in the paper.

The main problem solved in the paper is to compute local state estimates using measurements available only at the respective nodes i.e. $\hat{x}_{n|n}^j = E(x_n|\vec{y}_n^j)$. These local estimates are sent to the central node and fused to the form equivalent to (15).

## 4. Global information filter

The Kalman filter described by the equations (15)-(19) can be called a covariance filter.

In multisensor systems a number of useful properties characterize the information filters, that are Kalman covariance filters recast in terms of an information state vector and an information matrix.

Let us notice that equation (15) can be written down in the form

$$\hat{x}_{n+1|n+1} = (\mathbf{1} - K_{n+1}C_{n+1})\hat{x}_{n+1|n} + K_{n+1}y_{n+1}. \tag{22}$$

Next

$$\mathbf{1} - K_{n+1}C_{n+1} = \overbrace{(\mathbf{1} - K_{n+1}C_{n+1})P_{n+1|n}}^{P_{n+1|n+1}(18)} P^{-1}_{n+1|n} = P_{n+1|n+1}P^{-1}_{n+1|n}. \tag{23}$$

Inserting (23) and (17) into (22) gives

$$\hat{x}_{n+1|n+1} = P_{n+1|n+1}P^{-1}_{n+1|n}\hat{x}_{n+1|n} + P_{n+1|n+1}C^T_{n+1}R^{-1}_{n+1}y_{n+1}. \tag{24}$$

By multiplying both sides of the equation (24) by $P^{-1}_{n+1|n+1}$ and using the definition of $C_{n+1}$, $R_{n+1}$ and $y_{n+1}$, we obtain

$$\hat{x}^*_{n+1|n+1} = \hat{x}^*_{n+1|n} + \sum_{j=1}^{M} C^j_{n+1}(R^j_{n+1})^{-1}y^j_{n+1} = \hat{x}^*_{n+1|n} + \sum_{j=1}^{M} i^j_{n+1}, \tag{25}$$

where

$$\begin{aligned}
\hat{x}^*_{n+1|n+1} &= P^{-1}_{n+1|n+1}\hat{x}_{n+1|n+1} \\
\hat{x}^*_{n+1|n} &= P^{-1}_{n+1|n}\hat{x}_{n+1|n} \\
i^j_{n+1} &= C^j_{n+1}(R^j_{n+1})^{-1}y^j_{n+1}.
\end{aligned} \tag{26}$$

Equation (26) is a measurement–update information filter equation and $i^j_{n+1}$ is defined as an information state vector.

In order to determine $\hat{x}^*_{n+1|n}$, multiply both sides of the equation (16) by $P^{-1}_{n+1|n}$. Using (26) we obtain

$$\begin{aligned}
P^{-1}_{n+1|n}\hat{x}_{n+1|n} &= P^{-1}_{n+1|n}(A_nP_{n|n}P^{-1}_{n|n}\hat{x}_{n|n} + B_nu_n) \\
\hat{x}^*_{n+1|n} &= P^{-1}_{n+1|n}(A_nP_{n|n}\hat{x}^*_{n|n} + B_nu_n).
\end{aligned} \tag{27}$$

Equation (27) is a time-update information filter equation.

It can be shown that the recursive form of the matrix $P^{-1}_{n+1|n+1}$ is

$$\begin{aligned}
P^{-1}_{n+1|n+1} &= P^{-1}_{n+1|n} + C^T_{n+1}R^{-1}_{n+1}C_{n+1} = \\
&= P^{-1}_{n+1|n} + \sum_{j=1}^{M} C^{iT}_{n+1}(R^j_{n+1})^{-1}C^j_{n+1} = \\
&= P^{-1}_{n+1|n} + \sum_{j=1}^{M} I^j_{n+1},
\end{aligned} \tag{28}$$

where

$$I^j_{n+1} = C^{jT}_{n+1}(R^j_{n+1})^{-1}C^j_{n+1} \tag{29}$$

is defined as the information matrix.

Summarizing, the measurement-update equations (25) and (28) are computationally simpler than equations (15) and (18), at the cost of increased complexity in the time-update equation (27) compared with (16).

# 5. Local Kalman filtration

## 5.1. Covariance filter

Let us consider the multisensor system (20) for $u_n^o$ described by equation (11), i.e.

$$x_{n+1} = A_n x_n + B_n S_n \hat{x}_{n|n} + w_n$$
$$y_n^j = C_n^j x_n + r_n^j, \quad j = 1, ..., M. \tag{30}$$

It is assumed that parameters defining the state model in (20) are known to the local node.

The local filtration problem is to find the local estimates $\hat{x}^j_{n+1|n+1}$, $j = 1, ..., M$, defined as

$$\hat{x}^j_{n+1|n+1} = E(x_{n+1}|\vec{y}^j_{n+1}). \tag{31}$$

Note that $\hat{x}_{n|n}$ in (30) should be treated as a random variable for local nodes.

The estimate $\hat{x}^j_{n+1|n+1}$ can be found using equations (6)-(8). It is propagated as

$$\hat{x}^j_{n+1|n+1} = \hat{x}^j_{n+1|n} + K^j_{n+1}(y^j_{n+1} - C^j_{n+1}\hat{x}^j_{n+1|n}), \tag{32}$$

where

$$K^j_{n+1} = P_{x_{n+1}\tilde{y}^j_{n+1|n}} P^{-1}_{\tilde{y}^j_{n+1|n}\tilde{y}^j_{n+1|n}}. \tag{33}$$

We write the state equation (30) in the form

$$x_{n+1} = (A_n + B_n S_n)x_n - B_n S_n \tilde{x}_{n|n} + w_n \tag{34}$$

where $\tilde{x}_{n|n} = x_n - \hat{x}_{n|n}$.

The term $\hat{x}^j_{n+1|n}$ in (32) becomes

$$\hat{x}^j_{n+1|n} = E(x_{n+1}|\vec{y}^j_n) = (A_n + B_n S_n)\hat{x}^j_{n|n} - B_n S_n E(\tilde{x}_{n|n}|\vec{y}^j_n). \tag{35}$$

We find that

$$\begin{aligned} E(\tilde{x}_{n|n}|\vec{y}^j_n) &= E[(x_n - \hat{x}_{n|n})|\vec{y}^j_n] = E(x_n|\vec{y}^j_n) - E(\hat{x}_{n|n}|\vec{y}^j_n) = \\ &= E(x_n|\vec{y}^j_n) - E\{[E(x_n|\vec{y}_n)]|\vec{y}^j_n\} = \\ &E(x_n|\vec{y}^j_n) - E(x_n|\vec{y}^j_n) = 0. \end{aligned} \tag{36}$$

Thus, the last term in (35) is equal to zero and $\hat{x}^j_{n+1|n}$ in (32) has the form

$$\hat{x}^j_{n+1|n} = (A_n + B_n S_n)\hat{x}^j_{n|n}. \tag{37}$$

The matrix gain $K^j_{n+1}$, defined by (33), has the classical form

$$K^j_{n+1} = P^j_{n+1|n} C^{jT}_{n+1} (C^j_{n+1} P^j_{n+1|n} C^{jT}_{n+1} + R^j_{n+1})^{-1}. \tag{38}$$

The covariance matrix $P^j_{n+1|n}$ is defined as

$$P^j_{n+1|n} = E(\tilde{x}^j_{n+1|n} \tilde{x}^{jT}_{n+1|n}), \tag{39}$$

where $\tilde{x}^j_{n+1|n} = x_{n+1} - \hat{x}^j_{n+1|n}$.

From (34) and (37) we have

$$\begin{aligned} \tilde{x}^j_{n+1|n} &= (A_n + B_n S_n)x_n - B_n S_n \tilde{x}_{n|n} + w_n - (A_n + B_n S_n)\hat{x}^j_{n|n} = \\ &= (A_n + B_n S_n)\tilde{x}^j_{n|n} - B_n S_n \tilde{x}_{n|n} + w_n, \end{aligned} \tag{40}$$

where $\tilde{x}^j_{n|n} = x_n - \hat{x}^j_{n|n}$.

Hence

$$\begin{aligned} P^j_{n+1|n} &= E[(A_n + B_n S_n)\tilde{x}^j_{n|n} - B_n S_n \tilde{x}_{n|n} + w_n][(A_n + B_n S_n)\tilde{x}^j_{n|n} - \\ &\quad - B_n S_n \tilde{x}_{n|n} + w_n]^T = (A_n + B_n S_n) P^j_{n|n} (A_n + B_n S_n)^T - \\ &\quad - (A_n + B_n S_n) P^{j*}_{n|n} S^T_n B^T_n - B_n S_n P^{*j}_{n|n} (A_n + B_n S_n)^T + \\ &\quad + B_n S_n P_{n|n} S^T_n B^T_n + W_n, \end{aligned} \tag{41}$$

where

$$\begin{aligned} &P^j_{n|n} = E(\tilde{x}^j_{n|n} \tilde{x}^{jT}_{n|n}), \quad P^{j*}_{n|n} = E(\tilde{x}^j_{n|n} \tilde{x}^T_{n|n}), \\ &P^{*j}_{n|n} = E(\tilde{x}_{n|n} \tilde{x}^{jT}_{n|n}) = (P^{j*}_{n|n})^T. \end{aligned} \tag{42}$$

By subtracting both sides of (32) from the identity $x_{n+1} = x_{n+1}$ we obtain

$$\tilde{x}^j_{n+1|n+1} = (\mathbf{1} - K^j_{n+1} C^j_{n+1})\tilde{x}^j_{n+1|n} - K^j_{n+1} r^j_{n+1} \tag{43}$$

and similarly for (15)

$$\tilde{x}_{n+1|n+1} = (\mathbf{1} - K_{n+1} C_{n+1})\tilde{x}_{n+1|n} - K_{n+1} r_{n+1}. \tag{44}$$

The covariance matrix $P^j_{n+1|n+1} = E(\tilde{x}^j_{n+1|n+1} \tilde{x}^{jT}_{n+1|n+1})$ can be found in the classical way and has the form

$$P^j_{n+1|n+1} = (\mathbf{1} - K^j_{n+1} C^j_{n+1}) P^j_{n+1|n}. \tag{45}$$

It follows that $K^j_{n+1}$ described by (38) can be expressed as

$$K^j_{n+1} = P^j_{n+1|n+1} C^{jT}_{n+1} (R^j_{n+1})^{-1}. \tag{46}$$

The matrix $P^{j*}_{n+1|n+1} = E(\tilde{x}^j_{n+1|n+1}\tilde{x}^T_{n+1|n+1})$ may be expressed in the form

$$\begin{aligned} P^{j*}_{n+1|n+1} &= E[(\mathbf{1} - K^j_{n+1}C^j_{n+1})\tilde{x}^j_{n+1|n} - K^j_{n+1}r^j_{n+1}] \\ &\quad [(\mathbf{1} - K_{n+1}C_{n+1})\tilde{x}_{n+1|n} - K_{n+1}r_{n+1}]^T = \\ &= (\mathbf{1} - K^j_{n+1}C^j_{n+1})P^{j*}_{n+1|n}(\mathbf{1} - K_{n+1}C_{n+1})^T + K^j_{n+1}R^{j*}_{n+1}K^T_{n+1}, \end{aligned} \tag{47}$$

where
$P^{j*}_{n+1|n} = E(\tilde{x}^j_{n+1|n}\tilde{x}^T_{n+1|n})$
and $R^{j*}_{n+1} = [R^{j1}_{n+1}, ...., R^j_{n+1}, ..., R^{jM}_{n+1}]_{R^{jl}_{n+1}=0}$.

Using (18), (45) and (17), (46) in (47) yields

$$\begin{aligned} P^{j*}_{n+1|n+1} &= P^j_{n+1|n+1}(P^j_{n+1|n})^{-1}P^{j*}_{n+1|n}P^{-1}_{n+1|n}P_{n+1|n+1} + \\ &\quad + P^j_{n+1|n+1}C^{jT}_{n+1}(R^j_{n+1})^{-1}\overbrace{R^{j*}_{n+1}R^{-1}_{n+1}C_{n+1}}^{C^j_{n+1}}P_{n+1|n+1}. \end{aligned} \tag{48}$$

In order to determine $P^{j*}_{n+1|n} = E(\tilde{x}^j_{n+1|n}\tilde{x}^T_{n+1|n})$ we have from (9), (16)

$$\tilde{x}_{n+1|n} = A_n\tilde{x}_{n|n} + w_n \tag{49}$$

and from (40)

$$\begin{aligned} P^{j*}_{n+1|n} &= E[(A_n + B_nS_n)\tilde{x}^j_{n|n} - B_nS_n\tilde{x}_n + w_n][A_n\tilde{x}_{n|n} + w_n]^T = \\ &= (A_n + B_nS_n)P^{j*}_{n|n}A^T_n - B_nS_nP_{n|n}A^T_n + W_n. \end{aligned} \tag{50}$$

Equations (32), (37), (38), (41), (45), (48) and (50) form the solution to local filtration problem.

We should initialize the local filter with

$$\hat{x}^j_{0|-1} = E(x_0 - \hat{x}^j_{0|-1}) = \bar{x}_0 \tag{51}$$

and

$$\begin{aligned} P^j_{0|-1} &= E[(x_0 - \hat{x}^j_{0|-1})(x_0 - \hat{x}^j_{0|-1})^T] = X_0 \\ P^{j*}_{0|-1} &= E[(x_0 - \hat{x}^j_{0|-1})(x_0 - \hat{x}_{0|-1})^T] = X_0. \end{aligned} \tag{52}$$

### 5.2. Information filter

Let us notice that equation (32) can be written down in the form

$$\hat{x}^j_{n+1|n+1} = (\mathbf{1} - K^j_{n+1}C^j_{n+1})\hat{x}^j_{n+1|n} + K^j_{n+1}y^j_{n+1}. \tag{53}$$

Next

$$\begin{aligned} &\mathbf{1} - K^j_{n+1}C^j_{n+1} = \overbrace{(\mathbf{1} - K^j_{n+1}C^j_{n+1})P^j_{n+1|n}}^{P^j_{n+1|n+1}(45)}(P^j_{n+1|n})^{-1} = \\ &= P^j_{n+1|n+1}(P^j_{n+1|n})^{-1}. \end{aligned} \tag{54}$$

Inserting (54) and (46) into (53) leads to

$$\hat{x}^j_{n+1|n+1} = = P^j_{n+1|n+1}(P^j_{n+1|n})^{-1}\hat{x}^j_{n+1|n} + P^j_{n+1|n+1}C^{jT}_{n+1}(R^j_{n+1})^{-1}y^j_{n+1}. \quad (55)$$

By multiplying both sides of equation (55) by $(P^j_{n+1|n+1})^{-1}$ we obtain

$$\hat{x}^{j*}_{n+1|n+1} = \hat{x}^{j*}_{n+1|n} + C^j_{n+1}(R^j_{n+1})^{-1}y^j_{n+1} = \hat{x}^{j*}_{n+1|n} + i^j_{n+1} \quad (56)$$

where

$$\hat{x}^{j*}_{n+1|n+1} = (P^j_{n+1|n+1})^{-1}\hat{x}^j_{n+1|n+1},\ \hat{x}^{j*}_{n+1|n} = (P^j_{n+1|n})^{-1}\hat{x}^j_{n+1|n}. \quad (57)$$

In order to determine $\hat{x}^*_{n+1|n}$ we multiply both sides of the equation (37) by $(P^j_{n+1|n})^{-1}$. We obtain

$$(P^j_{n+1|n})^{-1}\hat{x}^j_{n+1|n} = (P^j_{n+1|n})^{-1}(A_n + B_nS_n)P^j_{n|n}(P^j_{n|n})^{-1}\hat{x}^j_{n|n}$$
$$\hat{x}^{j*}_{n+1|n} = (P^j_{n+1|n})^{-1}(A_n + B_nS_n)P^j_{n|n}\hat{x}^{j*}_{n|n}. \quad (58)$$

It can be shown that a recursive form of the covariance matrix $(P^j_{n+1|n+1})^{-1}$ has the form

$$(P^j_{n+1|n+1})^{-1} = (P^j_{n+1|n})^{-1} + C^{jT}_{n+1}(R^j_{n+1})^{-1}C^j_{n+1} =$$
$$= (P^j_{n+1|n})^{-1} + I^j_{n+1}. \quad (59)$$

Equations (56), (58), (59) establish the information form of the local Kalman filtration. This form will be used in a hierarchical filtration presented in the next section.

## 6. Hierarchical filtration

One of possible implementations of equations (25) and (28) is a hierarchical structure in which the $j$th local node calculates at time $n$ the information vector $i^j_n$ and the matrix information $I^j_n$ and transmits them to the central node (fusion center) where a global estimate is determined. The information state prediction is calculated by the central node.

Another implementation of equations (25) and (28) is a hierarchical structure in which the $j$th local node calculates at time $n$ the information state vector $x^{j*}_{n|n}$ and the matrix $(P^j_{n|n})^{-1}$ and transmits them in the information form to the central node (fusion center) where a global estimate is determined.

From (56) and (59) we have

$$i^j_{n+1} = \hat{x}^{j*}_{n+1|n+1} - \hat{x}^{j*}_{n+1|n} \quad (60)$$

$$I^j_{n+1} = (P^j_{n+1|n+1})^{-1} - (P^{-j}_{n+1|n})^{-1}. \quad (61)$$

Inserting (60) and (61) into (25) and (28), respectively, yields

$$\hat{x}^*_{n+1|n+1} = \hat{x}^*_{n+1|n} + \sum_{j=1}^{M} (\hat{x}^{j*}_{n+1|n+1} - \hat{x}^{j*}_{n+1|n}) \tag{62}$$

$$P^{-1}_{n+1|n+1} = \sum_{j=1}^{M} [(P^j_{n+1|n+1})^{-1} - (P^j_{n+1|n})^{-1}]. \tag{63}$$

The above filtration is equivalent to the corresponding centralized Kalman filtration.

In Zhu et al. (2001) a hierarchical structure for autonomous multisensor systems is analysed, being an implementation of the equations

$$\hat{x}^*_{n+1|n+1} = \hat{x}^*_{n+1|n} + \sum_{j=1}^{M} (\hat{x}^{*j}_{n+1|n+1} - \hat{x}^*_{n+1|n}) \tag{64}$$

and

$$P^{-1}_{n+1|n+1} = P^{-1}_{n+1|n} + \sum_{j=1}^{M} [(P^j_{n+1|n+1})^{-1} - P^{-1}_{n+1|n}]. \tag{65}$$

In the above structure, local information state estimates $\hat{x}^{*j}_{n|n}$ and information matrices $(P^j_{n|n})^{-1}$, $j = 1, ..., M$, are calculated at local nodes and transmitted to the central node where a global state estimate $\hat{x}_{n|n}$ and information matrix $(P_{n|n})^{-1}$ are calculated.

The covariance matrix $(P^j_{n+1|n+1})^{-1}$ depends on $P^j_{n+1|n}$ according to (45). The matrix $P^j_{n+1|n}$ depends on the covariance matrix $P_{n|n}$ according to (41). Thus, the matrix $P_{n|n}$ should be sent back from central node to local nodes.

## 7. Conclusions

The paper presents the nonconventional Kalman filtration related to LQG control problem formulated for multisensor systems. Filtration is realized in the hierarchical structure. It is assumed that values of controls are not available to local nodes and should be treated as random variables. This leads to a nonconventional local filtration based on local measurement information. The estimation equations may be implemented in a hierarchical structure in which local state estimates and appropriate covariance matrices in the information form are sent to the central node and the information on global covariance matrix is sent back. For time-invariant systems with stationary noises and steady state conditions, covariance matrix $P_{n|n}$ reaches a constant, steady-state value. In this case only local state estimates $\hat{x}^{*j}_{n|n}$ should be transmitted to the central node.

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