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Performance of robust portfolio optimization in crisis periods*

by

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Abstract: We examin empirical performances of two alternative robust optimization models, namely the worst-case conditional value-at-risk (worst-case CVaR) model and the nominal conditional value-at-risk (CVaR) model in crisis periods. Both models are based on historical value-at-risk methodology. These performances are compared by using a portfolio constructed on the basis of daily closing values of different stock indices in developed markets using data from 1990 to 2013. An empirical evidence is produced with RobustRisk software application. Both a Monte-Carlo simulation and an out-of-sample test show that robust optimization with worst-case CVaR model outperforms the nominal CVaR model in the crisis periods. However, the trade-off between model misspecification risk and return maximization depending on the market movements should be optimized in a robust model selection.

Keywords: robust control procedures, RobustRisk, portfolio optimization, Monte Carlo simulation, global crisis

1. Motivation

The background of robust procedures dates back to the 1960s and 1970s (Ellsberg, 1961; Jacobson, 1973; Kreps and Porteus 1978; Whittle, 1981). However, their applications in economics and finance literature have not succeeded until the last decade with improvements in financial software applications. A promising literature on robust portfolio optimization to minimize any model misspecification in risk estimation has recently emerged. This paper aims to provide an empirical evidence on performance of robust portfolio optimization

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models in crisis period on advanced markets. These models can be discussed under alternative portfolios but as impacts of the recent global crisis have been mostly observed in the advanced markets, the hypothetical portfolio elaborated in this paper is constructed by using daily returns on stock market indices from the US, the UK, Japan, Germany and France from 1990 and 2013.

Two alternative robust portfolio optimization methodologies are employed for empirical tests in this paper. Namely, the performance of worst-case CVaR with mixture uncertainty proposed by Zhu and Fukushima (2009) is compared with the performance of (nominal) CVaR introduced by Rockafellar and Uryasev (2000). Mixture uncertainty set is more proper for discrete data like time series of stock exchange indices and is preferred in worst-case CVaR methodology. The test results are produced by RobustRisk software application developed by Ozun and Balcilar (2013). Researchers can reproduce the results contained in this article, as well as perform empirical results for their own portfolios by means of RobustRisk software which is freely available for academic purposes on the internet.

The empirical findings show that worst-case robust portfolio optimization provides insurance for portfolio return in crisis periods. The out of sample performances of worst-case CVaR model compared to the nominal CVaR model support the conclusion that trade-off between risk and return should be carefully considered in market risk management where the worst–case can actually occur. The empirical analysis that treats the recent global crisis as worst-case and performs model applications on the advanced markets where the impacts of crisis have been more severe, demonstrates the fact that the nominal value-at-risk model might not be sufficient to capture the market risk.

A short but clear presentation of worst-case CVaR methodology is given in the next section. In the third section, the hypothetical portfolios constructed for empirical tests are introduced, and descriptive statistics for each stock market index are discussed. Empirical evidence with its implications for practical finance is discussed in the fourth section. The paper ends with conclusions and suggestions for future research. It offers two main contributions to the finance literature. Firstly, it provides empirical evidence on performance of the robust risk models in advanced markets in the recent crisis periods. This evidence has certain implications for portfolio and risk management practitioners, which are discussed in the empirical evidence section. Secondly, it uses a recently developed software application called RobustRisk. The paper provides an example of how to produce and interpret empirical results on robust portfolio optimization models with RobustRisk, which is freely available for academic purposes. The data used in this paper are also available on the same web-page and the researchers and practitioners can produce the same results themselves when they load RobustRisk on their computers.

2. Methods for robust portfolio optimization

This empirical paper compares the performance of two robust portfolio optimization methods, namely the nominal CVaR (Rockafellar and Uryasev, 2000) and the worst-case CVaR with mixture distribution uncertainty set (Zhu and Fukushima, 2009) in crisis periods. In the worst-case CVaR, we prefer to use the mixture distribution uncertainty set. Portfolio optimizations are based on time series data of financial assets, and mixture distribution uncertainty set fits continuous time series characteristics better than the discrete distributions. Since our motivation is empirical rather than methodological, we present portfolio optimization methodology with worst-case CVaR, briefly. Its derivation from a loss function in basic value-at-risk model can be found in Ozun and Balcilar (2013). Portfolio optimization with CVaR aims to achieve asset weights in the portfolio, x, that minimize the pre-defined limitations. In this case, as the limitation is conditional value-at-risk, the target is to find x values that minimize the conditional value-at-risk. The minimization function can be defined as in equation (1).

$$\min_{x\in X}\left(CVaR_{\beta}(x)\right) = \min_{(x,\alpha)\in XxR}\left(\alpha + \frac{1}{1-\beta}\int_{y\in R^m}^{\infty}[f(x,y)-\alpha]^{+}p(y)d(y)\right) \quad (1)$$

In the function, x and y vectors, which have m dimension, represent asset weights and their returns, respectively. The function $p(y)$ represents the probability of given return, y, while β is the confidence level for x asset weights. The loss function for asset weight (x) and their returns (y) is shown as f. The minimum boundary of loss value is shown as α. The special function under the integral in the equation (1), is defined as $[t]^{+} = max(t, 0)$.

When the integral in the equation is transformed into a definite sum, and u_k term for transformation is used as shown in equation (2), equation (1) can be re-defined as equation (3).

$$u_k = [f(x, y_k) - \alpha]^{+} \quad (2)$$

$$\min_{x\in X}\left(CVaR_{\beta}(x)\right) = \min_{(x,\alpha)\in XxR}\left(\alpha + \frac{1}{1-\beta}\sum_{k=1}^{S}u_k\pi_k\right). \quad (3)$$

In equation (3), S refers to the total number of observations, π_k refers to the probability of k^{th} observation. If the distribution of returns is known for k, by using the probability density function of the distribution, values can be produced in a parametric model. Alternatively, equal probability assumption can be pre-defined, $\pi_k = 1/S$, as used in Rockafellar and Uryasev (2000).

The set of X consists of all possible asset weights, x. To limit this set, the sum of weights can be defined as 1, each individual weight can be appointed as higher than zero, or certain special requirements on asset weights can be described. The inputs, target, limitations and the output can be formalized as equations (4)-(10).

Inputs:

$$y, \beta, \mu, \pi \tag{4}$$

Target:

$$\min\left(\alpha + \frac{1}{1-\beta} u^T \pi\right) \tag{5}$$

Limitations:

$$x \in X \tag{6}$$

$$u_k \geq f(x, y_k) - \alpha \qquad k = 1..S \tag{7}$$

$$u_k \geq 0 \tag{8}$$

$$x^T \bar{y} \geq \mu \tag{9}$$

Outputs:

$$\alpha, u, x. \tag{10}$$

The first limitation defined through equation (6) sets the conditions for asset weights; the sum of the weights should be 1, and each weight should be positive. Limitations in equations (7) and (8) stem from the fact that vector of u equals to the defined special function because according to the definition of the special function, when the function input is negative, output should be zero, otherwise, it should be equal to the input itself. In this framework, each u value should be equal or higher than zero, and u value should be equal or higher than the function input. The last limitation defined in equation (9) guarantees a total return equal or higher than a desired amount (μ) in the optimization. In this equation, the mean value of return vector is shown as $\bar{y}$.

In robust optimization under worst-case conditional value-at-risk model, the target is to find the asset weights in the portfolio that minimize the worst-case conditional value-at-risk. The risk weights are calculated with following equation:

$$\min_{x \in X} (WCVaR_\beta(x)) = \min_{(x,\alpha,\theta) \in XxRxR} \left(\theta : \alpha + \frac{1}{1-\beta} \int_{y \in R^m}^{\infty} [f(x, y^i) - \alpha]^+ p^i(y) d(y) \leq \theta, \quad i = 1..L\right). \tag{11}$$

In equation (11), we have L time periods that show different characteristics for return distributions, and define the probability distribution of returns for each period as $p^i(.)$ $i = 1..L$. The return vectors, which have m dimensions, are represented as y^i. The maximum loss value of different L periods is shown as θ. In the equation, when we apply the u term transformation on the special function as shown in equation (12), and create an integral with finite sum, the equation (11) can be written as equation (13).

$$u_k^i = [f(x, y_k^i) - \alpha]^+ \tag{12}$$

$$\min_{x\in X}(WCVaR_\beta(x)) = \min_{(x,\alpha,\theta)\in XxRxR}\left(\theta : \alpha + \frac{1}{1-\beta}\sum_{k=1}^{S^i} u_k^i \pi_k^i \leq \theta,\ i = 1..L\right). \tag{13}$$

In the equation, S^i equals to the number of observations in period i, π_k^i refers to probability of k^{th} return value in period i. In the model, it is assumed that $\pi_k^i = 1/S^i$. The optimization parameters can be listed as follows.

Inputs:

$$y, \beta, \mu, \pi \tag{14}$$

Target:

$$\min(\theta) \tag{15}$$

Limitations:

$$x \in X \tag{16}$$

$$\alpha + \frac{1}{1-\beta}\left(\pi^i\right)^T u^i \leq \theta \quad i = 1..L \tag{17}$$

$$u_k^i \geq f(x, y_k^i) - \alpha \quad i = 1..L,\ \ k = 1..S \tag{18}$$

$$u_k^i \geq 0 \tag{19}$$

$$x^T(\bar{y^i}) \geq \mu \tag{20}$$

Outputs:

$$\alpha, u, \theta, x. \tag{21}$$

The first limitation provides the condition that asset weights are feasible, i.e. their sum is 1 and each weight is equal or higher than zero. The second limitation derives from the worst–case conditional value-at-risk definition. The third and fourth limitations result from the fact that the u value equals to the defined specific function. The latest limitation guarantees a target return equal or higher than a minimum limit for each different period.

In practice, inputs, limitation and outputs defined in equations from (4) to (10) for nominal CVaR model, and inputs, targets, limitation and outputs defined in equations from (14) to (21) for the robust CVaR model are optimized by using linear programming. In the test stage, assets weights produced with both optimization models are used to compare the expected returns. Three different but complementary test approaches are employed. The first approach is called in-sample test and it shows returns generated with both models by using the data for optimization. The second performance test is based on out-of- sample test. We measure and compare returns generated with both models in out-of-sample data. Finally, we implement a Monte-Carlo simulation process which produces random returns in case of periods in the future. These periods are produced in line with the same characteristics observed in each sub-period in the historical data. For each simulated period, optimized weights are used to the compare return performance of the models. As it is shown in empirical results, performance of robust model is better in the simulated new period even though there is a period which is similar to the crisis period within all the sub-periods.

3. Data and preliminary analysis

We construct a hypothetical portfolio based on daily historical values of stock market indices from the US, the UK, Japan, Germany and France. As the aim of the paper is to test the performance of the robust optimization models in the recent global crisis, we select the stock market indices of the advanced economies, where the observed impacts of the crisis have been more severe. The performance of alternative portfolios can be examined by RobustRisk software application. The total data set includes 10 907 daily observations of closing values of the selected indices from 02/01/1990 to 31/05/2013. Data from 02/01/1990 to 27/05/2008 with 4 801 observations is selected as in sample data for training purposes. The rest of the data from 02/06/2008 to 31/05/2013 with 1 305 observations representing the crisis period are reserved for purposes of the out of sample test.

Table 1. Stock indices used for portfolio construction

Country	Index Name	Symbol
The US	Dow Jones Industrial Index-500	DJI
The US	Standard and Poors 100 Index	S&P
The UK	Financial Times Stock-Exchange-100	FTSE
Japan	Nikkei Stock Exchange	NKY
Germany	Deutsche Borse AG Index-50	DAX
France	Paris Stock Exchange-30	CAC

For the worst-case CVaR estimation, the in–sample data are divided into 4 different sub-periods. In other words, the number of S is 4. However, the numbers of observations in each period are different. The rationale behind the determination of sub-periods is based on the main observed trends in the markets, i.e., bull and bear market trends are clustered under different time periods. In addition, the mean and variance of each sub-period also encourage us to divide the full time periods into 4 sub-periods. As the data and RobustRisk software application are publicly available, the readers can produce the empirical evidence with different sub-periods determined by their own judgments. As it is suggested in the conclusion section, the robust risk models can be improved by allowing the model to assign proper sub-periods with certain data clustering methods rather than arbitrary sub-period determination.

Table 2. The number of sub-periods (S) and the numbers of observations in each period (S^i)

Period ID (S)	Time period	Sample size (S^i)
Period 1	02/01/1990 - 29/12/1995	1564
Period 2	01/01/1996 - 31/12/1999	1045
Period 3	03/01/2000 - 31/12/2002	782
Period 4	01/01/2003 - 27/05/2008	1410
Out of sample period	02/06/2008 - 31/05/2013	1305

The returns of the indices in each period are presented in the Fig. 1. The graphs indicatively show that the volatilities of the markets are especially higher in the third segment, representing the non-stable period between 2000 and 2002.

In Table 3, the mean and variance values of returns in the stock market index in each sub-period are presented.

Table 3. Means and standard deviations of the indices

	Period ID	DJI	S&P	DAX	CAC	NKY	FTSE
Mean	Period 1	0.00042	0.00038	0.00021	0.00002	-0.00032	0.00030
	Period 2	0.00063	0.00065	0.00090	0.00097	-0.00018	0.00046
	Period 3	-0.00039	-0.00067	-0.00130	-0.00110	-0.00077	-0.00076
	Period 4	0.00033	0.00037	0.00071	0.00041	0.00044	0.00035
Variance	Period 1	0.00006	0.00005	0.00012	0.00013	0.00023	0.00007
	Period 2	0.00012	0.00013	0.00020	0.00017	0.00020	0.00011
	Period 3	0.00021	0.00022	0.00047	0.00037	0.00029	0.00024
	Period 4	0.00007	0.00008	0.00015	0.00013	0.00015	0.00009

The preliminary analysis of the time series data indicates that there is a risk-return imbalance with return distribution in period 3. Negative return with probability distribution observed in period 3 and higher variance (with positive return) are material in the time series data. It is highly probable that the robust CVaR model will assign period 3 as the worst-case and optimize the portfolio with the distributional characteristics of that period.

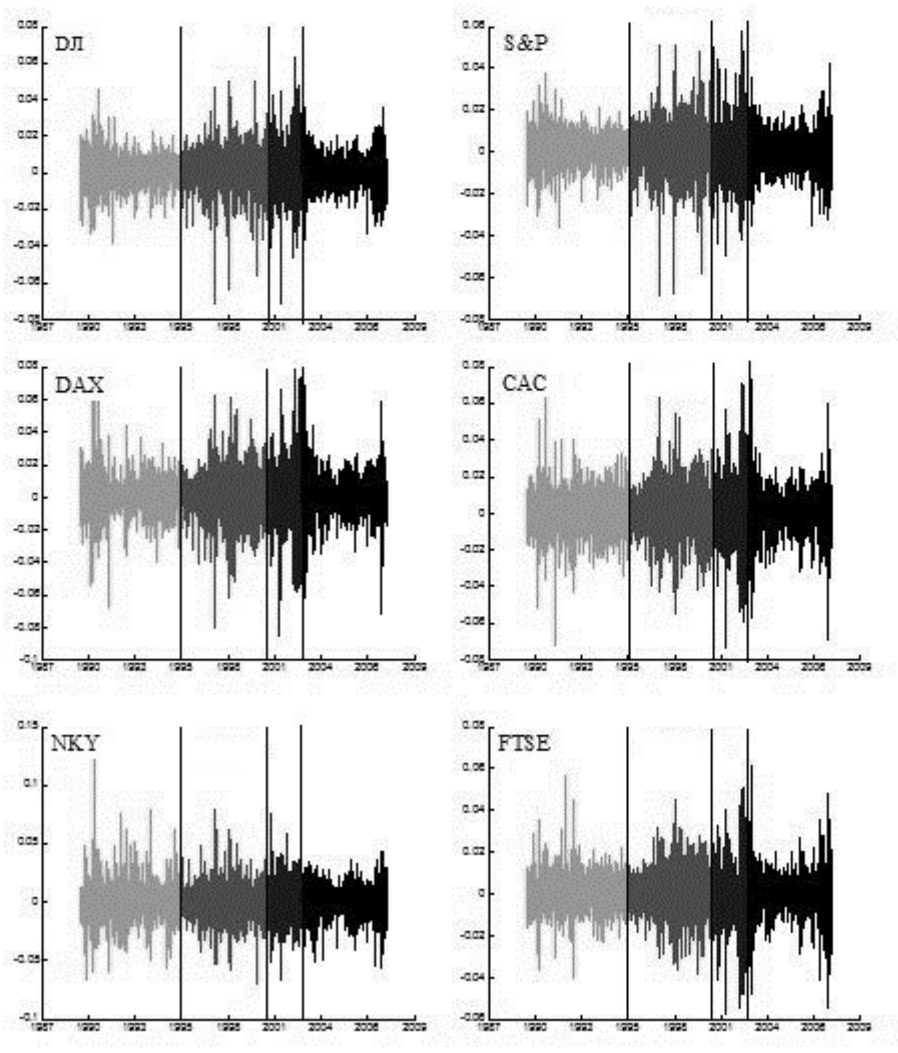

Figure 1. Daily returns on the stock indices for the in-sample data period. Each vertical line indicates period boundaries

4. Empirical findings and their implications in practice

We run the models with in–sample period data under alternative target expected returns (μ). However, the worst-case CVaR optimization does not guarantee any positive return with in–sample period data due to severe bear market conditions observed in period 3. As it can be remembered from the methodology section, when the distribution of returns is known, values can be assigned in a parametric model. In the nominal CVaR model, alternatively, equal probability assumption

Table 4. Asset weight of optimization result with $\beta = 0.99$ and $\mu = 0$

Method	DJI	S&P	DAX	CAC	NKY	FTSE
WCVaR	0	0	0	0	0	0
CVaR	0.287	0.1365	0	0	0.3096	0.2669

can be pre-defined, $\pi_k = 1/S$. However, in the worst-case CVaR model, risk estimation is based on the worst distribution of returns within the universe of all the possible distributions in sub-periods. As the distributions of assets do not remain same in time, the returns might fit into different distributions in the worst-cases in order to provide a return guarantee for the whole period.

Table 5. Return and Risk value of optimization result with $\beta = 0.99$ and $\mu = 0$

		Period 1	Period 2	Period 3	Period 4
Return	WCVaR	0	0	0	0
	CVaR	0.00016	0.00034	-0.00064	0.00037
Risk	WCVaR	0.0406	0.0483	0.1853	0.00019
	CVaR	0.0228	0.0279	0.0347	0.0228

In our case, the worst-case CVaR model could not produce any results. In other words, the model does not guarantee any positive minimum estimated return for the portfolio as the model does not work with any positive (π) values. Using lower significance level, such as 0.95, does not change the result.

Table 6. Asset weight of optimization result with $\beta = 0.99$ and $\mu = -0.0005$

Method	DJI	S&P	DAX	CAC	NKY	FTSE
WCVaR	0.6988	0	0	0	0.0303	0.271
CVaR	0.287	0.1365	0	0	0.3096	0.2669

Given those findings, our motivation was to determine a minimum loss level with robust CVaR model for the worst case. When the model is run with $\mu = -0.0005$, the worst-case CVaR produces estimated returns as shown in Tables 6-7 and Fig. 2. As presented in Table 7, with the distributional characteristics

Table 7. Return and Risk value of optimization result with $\beta = 0.99$ and $\mu = -0.0005$

		Period 1	Period 2	Period 3	Period 4
Return	WCVaR	0.00037	0.00056	-0.0005	0.00034
	CVaR	0.00016	0.00034	-0.00064	0.00037
Risk	WCVaR	0.0231	0.034	0.0403	0.0243
	CVaR	0.0228	0.0279	0.0347	0.0228

of period 3, the robust model guarantees that the loss is restricted with 0.0005; and also outperforms the nominal CVaR.

In order to analyze the model performances, we have run Monte-Carlo simulations with 5 million iterations. The Sharp ratios indicate that the performance of robust model outperforms the nominal model when the return distribution structures of period 1, 2 and 3 are used as the representative for the whole in-sample period. Only with the distribution in period 4 which represents a bull market, the nominal model outperforms the robust model, but the magnitude of success is not material. As Table 7 shows, robust model provides a comfort of 0.0005 losses with period 3 return distribution characteristics and produces lower loss as represented through the Sharp ratio.

Table 8. Monte-Carlo simulation results (5 million iterations)

Period	Method	Return	Variance	Sharp Ratio
1	Nominal	0.00016	0.00005	0.02187
	Robust	0.00037	0.00004	0.0577
2	Nominal	0.00033	0.00007	0.04031
	Robust	0.00056	0.00009	0.06058
3	Nominal	-0.00064	0.00012	-0.05788
	Robust	-0.00050	0.00016	-0.03970
4	Nominal	0.00038	0.00005	0.05337
	Robust	0.00034	0.00005	0.04588

After determining the optimized weights of the assets in the portfolio under the minimized loss level, the performances of the optimized portfolios resulting from the worst-case CVaR and nominal CVaR models can be compared for the out of sample data from 02/06/2008-31/05/2013 representing the recent global financial crisis. In this way, we compare the performances of the models in the recent crisis period which is chosen as out of sample period with optimized assets weights trained between 1990 and 2008. The worst-case model returns are immunized against a loss of 0.005.

Both Figs. 3 and 4 clearly point out the fact that the performance of robust

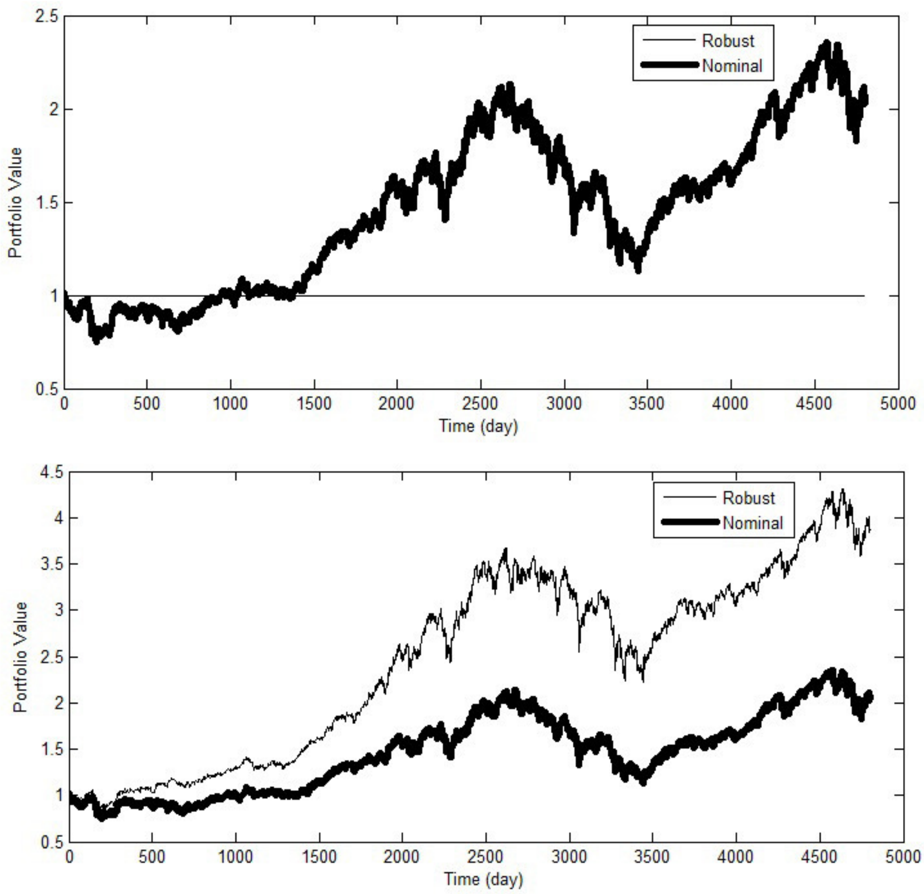

Figure 2. Visual presentation of the optimized portfolio values during the in-sample period

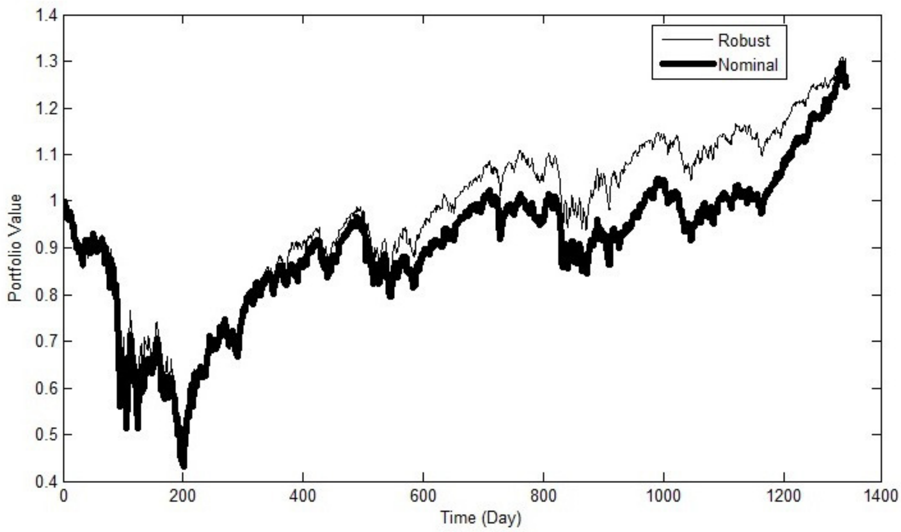

Figure 3. Out of sample performances of the models

model is better than that of the nominal model; however, as the markets start to recover after 2012, the nominal CVaR performance catches the one of the robust model.

In this paper, we contribute to the finance literature by providing a performance comparison of robust optimization procedures in the crisis period. The evidence points out the fact that robust optimization immunes the portfolio against loss in the crisis period with a pre-determined guaranteed level. In our case, the robust portfolio optimization outperforms the nominal CVaR optimization for the data corresponding to the recent global crisis period. In that sense, the empirical findings in this paper do not support the argument that the contribution of robust optimization in portfolio performance is limited as compared to stochastic models (Bertsimas et al., 2011: 465). It is empirically shown that using robust optimization provides more favorable returns in the crisis periods.

The empirical results in this paper clearly suggest considering robust optimization procedures especially in crisis periods in order to immune the portfolios against a higher level of loss than pre-determined or tolerated levels. On the other hand, as the out of sample test results after 2012 indicate, the robust model provides a capacity to construct a budget of uncertainty. It provides the portfolio managers alternatives in the trade-off between robustness and performance, and in choosing the corresponding level of probabilistic guarantee. However, trade-off between risk and return should be considered under alternative risk tolerance and market conditions determined by portfolio managers or institutions.

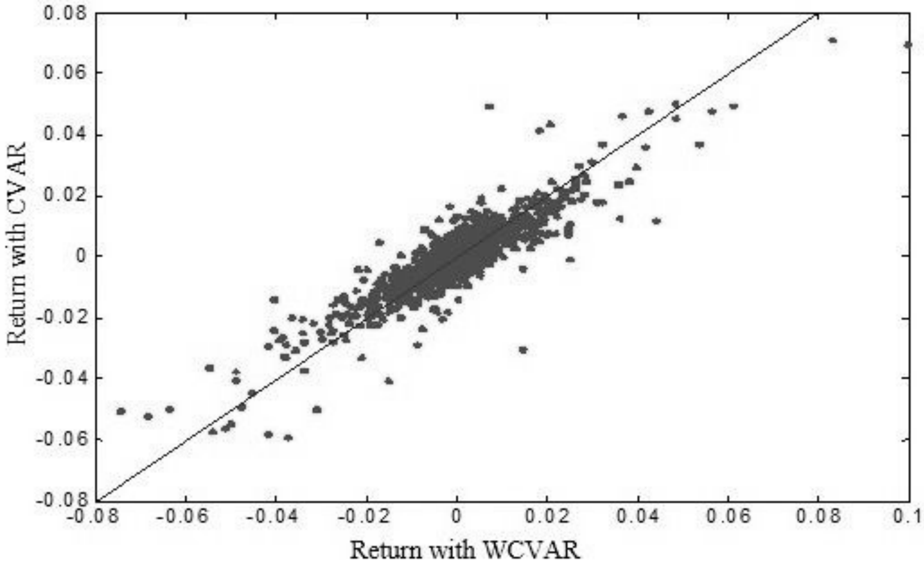

Figure 4. Distribution of optimized portfolio returns in comparison during out of sample period

5. Conclusion

In this paper, we compared the worst-case CVaR and the nominal CVaR optimization procedures for the data of the recent crisis period. Robust optimization procedures have gained popularity due to recent improvements of software coding in the last decade. However, the preliminary research was based on theoretical and methodological strengths of robust optimization with limited empirical evidence.

Recent empirical research provides results showing that the robust portfolio optimization outperforms the conventional value-at-risk models (Zymler et al. 2011; Chen and Kwon, 2012). On the other hand, there exists also certain empirical evidence supporting the argument that robust optimization procedures do not provide any advantage as there is a cost of robustness in risk and return trade-off in practice (Bertsimas et al., 2011: 465).

In this paper, we use RobustRisk software to compare the performances of optimization models with worst-case CVaR and nominal CVaR in the recent global crisis period. The out of sample and Monte-Carlo simulation results point out the fact that robust optimization outperforms the CVaR optimization in crisis periods on the advanced markets. The model performances resemble each other when the crisis is over.

The practitioners or academicians can use the worst-case CVaR optimization model and compare its performance with the nominal CVaR optimization results with alternative portfolios and time-periods by using RobustRisk. As the performances of the models with return distribution characteristics in period 4 indicate, by adopting a conservative approach in risk-return trade-off, CVaR can sometimes produce inefficient portfolio optimization results. The mixed quantitative results encourage us to reach a conclusion that both the quantitative and qualitative procedures should be considered in risk management. Expert judgments in estimation of market character, i.e., operating in a bear or bull market are considered as crucial as estimation of robust optimized risk parameters. Failure in estimating the market trend may produce inefficient optimization decisions.

Future research may concentrate on methodological improvements of robust optimization procedures in data modeling for determination of sub-periods. Instead of arbitrary sub-periods determined by the users' own judgments, data clustering methodologies that divide the whole sample periods into homogeneous sub-periods by examining the distributional characteristics of the observations, can be embedded in the robust optimization procedures.

A. How to use RobustRisk?

This additional note has been prepared to explain how to use RobustRisk. Users should follow the steps described below to run robust portfolio optimization and produce robust parameters with RobustRisk.

Stage 1: Loading RobustRisk on the desktop.
RobustRisk software application is publicly available and can be freely loaded on users desktops. The computers should have Matlab Compiler Runtime Installer v7.9.

Stage 2: Preparing MS Excel® data file and data loading.
The MS Excel® data file should include the time series of returns for each financial instruments in the portfolio under examination. Each column in the spreadsheets should have time series for a financial instrument. There are no quantitative limitations on the financial instrument in the portfolio apart from MS Excel® limitations. RobustRisk allows the users to appoint different time periods to reach the worst-case period in the data set. If the users prefer to divide the data into sub-periods with their own judgment, data for each period should be prepared on different spreadsheets in the same MS Excel® file. In order to load the in sample and out of sample MS Excel® files from the external sources, File function is selected and the MS Excel® data file can be loaded into "RobustRisk" by "Import_InSampleDataExcel" or "Import_outSampleDataExcel" options.

When data is loaded, RobustRisk automatically asks the users to assign the number of periods in the data, and calculates the sample size, numbers of assets and periods, mean and variance for the financial instruments in each period separately and prints them on the interface screen.

Stage 3: Selecting the confidence level (β) and minimum guaranteed return (μ).

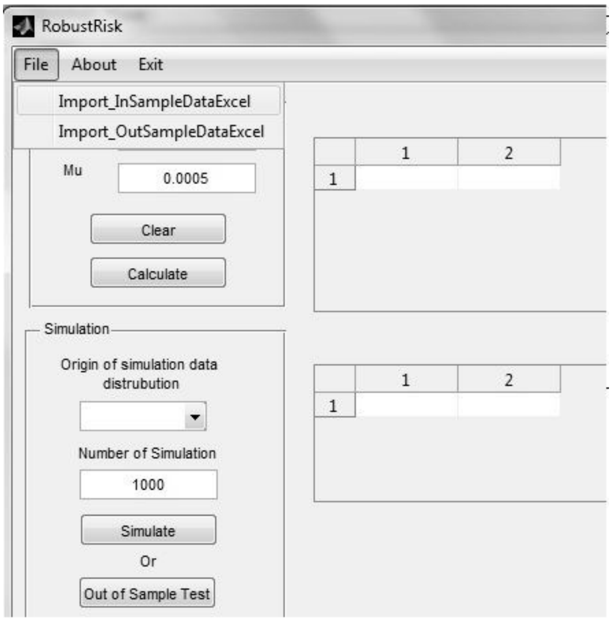

Figure 5. The MS Excel® load function

The interface of RobustRisk has two separate boxes where the users can enter their choices on the confidence level for the optimization process (β) and minimum required return on the optimization (μ). The (μ) value is used by the worst-case CVaR and is not a parameter for CVaR estimation. The default values for the parameters are 0.99 and 10^{-3}, but can be changed by the users according to their choices for the risk tolerance and confidence level. The (μ) value can be determined as negative to set up a stop loss as the maximum tolerated return in the worst-case CVaR.

Stage 4: Estimating robust portfolio outputs.
The "calculate" button on the interface produces basic descriptive statistics for each weights, expected returns and risk for the optimized portfolio asset in different sub-periods. The historical optimized portfolio values are also printed on the screen. In the worst-case CVAR model, RobustRisk performs optimization with data in each period, and produce optimized parameters with the worst-case period characteristics for the whole period of analysis.

The users can reach the optimum (minimum) guaranteed expected return by using different μ values. However, after a break-even threshold level for μ, the optimization does not produce any robust solution, which means that the required minimum return is not guaranteed. In this case, the worst-case CVaR does not produce any value for return as the system cannot guarantee any minimum return.

Stage 5: Simulation and out of sample performances of the robust opti-

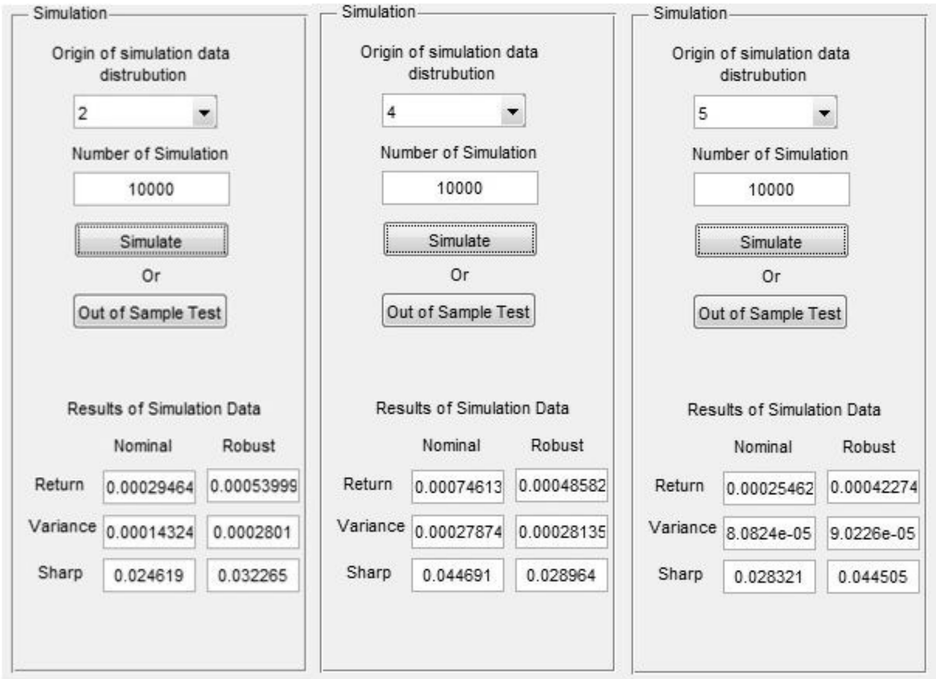

Figure 6. Performances of the models with Monte-Carlo simulation

mization models.

In order to test the accuracy of the performances of the models, RobustRisk has two alternative out-of-sample procedures. As a first procedure, users can perform Monte-Carlo simulation by selecting any period as a representative for the worst-case and compare the model performances with mean, variance and Sharp ratio. The number of simulations can be determined by the users, and RobustRisk calculates the performance indices and show them on the interface.

As a second procedure, the users can perform out-of-sample tests with reserved test data to measure the performances of the models. The out of sample data can be loaded into the system by using the same File function. RobustRisk calculates mean, variance and Sharp ratios for each model, and also print a visual comparison for the models on a window.

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