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A novel robust $\mathcal{H}_\infty$ fuzzy state-feedback control design on nonlinear Markovian jump systems with time-varying delay * †

by

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Abstract: This paper considers the problem of designing a robust $\mathcal{H}_\infty$ fuzzy state-feedback controller for a class of nonlinear Markovian jump systems with time-varying delay. A novel design methodology has been proposed for designing a controller that guarantees the $\mathcal{L}_2$-gain of the mapping from the exogenous input noise to the regulated output to be less than some prescribed value. Solutions to the problem are provided in terms of linear matrix inequalities. To illustrate the effectiveness of the design developed in this paper, a numerical example is also provided.

Keywords: $\mathcal{H}_\infty$ fuzzy control; Takagi-Sugeno (TS) fuzzy model; linear matrix inequalities (LMIs); Markovian jump parameters; time-varying delay

1. Introduction

Many physical systems may experience abrupt changes in their structure and parameter value shifts, caused by phenomena such as component and interconnection failures, parameters shifting, tracking, and the time required to measure some of the variables at different stages. Such systems can be modelled by a hybrid system with two components in the state vector. The first one, which varies continuously, is referred to as the continuous state of the system, and the second one, which varies discretely, is referred to as the mode of the system. There has been an increasing interest in these types of systems during the last decades, mostly due to the growing use of computers in the control of physical plants, but also as a result of the hybrid nature of physical processes. A special

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class of hybrid systems, known as Markovian jump systems, has been widely used to model manufacturing systems and communication systems.

In other words, Markovian jump systems are also referred to as hybrid systems, that is, the state space of the systems contains both continuous (differential equation) and discrete states (Markov process). Over the past two decades, the Markovian jump system has been extensively studied by many researchers; see Benjelloun, Boukas and Costa (1997), Boukas and Yang (1999), Boukas and Liu (2001), Dragan, Shi and Boukas (1999), Feng et al. (1992), Ji and Chizeck (1990), Rami and Ghaoui (1995), Shi and Boukas (1997), and Souza and Fragoso (1993). In Shi and Boukas (1997), the authors have considered the $\mathcal{H}_\infty$ control for Markovian jumping linear systems with parametric uncertainty. The delay-dependent robust stability and the $\mathcal{H}_\infty$ control of Markovian jump linear systems with time delay have been investigated in Boukas and Liu (2001). Although many researchers have studied the control design of Markovian jump linear system for many years, the design of control for Markovian jump nonlinear systems remains an open area. Indeed, recently, there has been some attempts in this domain. In Aliyu and Boukas (1998), Hamilton-Jacobi-equation-based sufficient conditions for nonlinear Markovian jump systems to have an $\mathcal{H}_\infty$ performance have been derived. However, until now, it is still very difficult to find a global solution to the HJE either analytically or numerically.

Over the past two decades, there has also been a rapidly growing interest in application of fuzzy logic to control problem. Researches have been focused on its application to industrial processes and a number of successful results have been reported in the literature. In spite of these successes, there are many basic issues that still remain to be addressed. One of them is how to achieve a systematic design that guarantees closed-loop stability. Recently, a great amount of effort has been devoted to describing a nonlinear system using the Takagi-Sugeno (TS) and Nguang fuzzy model; see Assawinchaichote and Nguang (2004) Assawinchaichote et al. (2008), Assawinchaichote (2012), Cao and Frank (2001), Chen, Tseng and He (2000), Han and Feng (1998), Ho et al. (2012), Lee et al. (2000), Lin, Chang and Hsu (2012), Nguang and Shi (2000, 2001), Tanaka et al. (1996, 2001), Wang, Tanaka and Griffin (1996), Wang, Tanaka and Ikeda (1996), Wang, Lin and Wang (2004), Wang, Tanaka and Ikeda (2000), Yoneyama (2000), and Zhang et al. (2012). In this TS fuzzy model, local dynamics in different state space regions are represented by local linear systems. The overall model of the system is obtained by "blending" of these linear models through nonlinear membership functions. In other words, a TS fuzzy model constitutes essentially a multi-model approach, in which simple sub-models are fuzzily combined to represent the global behavior of the system. Unlike conventional modelling techniques, which use a single model to describe the global behavior of a nonlinear system, fuzzy modelling combines simple (typically linear) sub-models to describe the global behavior of a nonlinear system. Based on this fuzzy model, a number of systematic model-based fuzzy control design methodologies have been developed.

During the past decades, the design of fuzzy $\mathcal{H}_\infty$ control for a class of non-

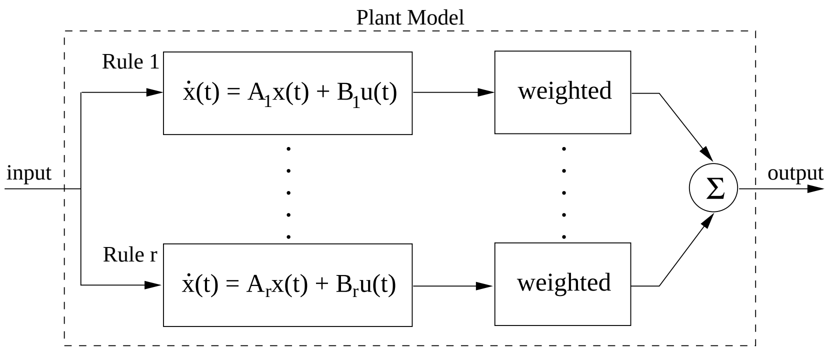

Figure 1. The TS type fuzzy system

linear systems without delays has been seriously investigated and many results have been reported; e.g., Chen, Tseng and He (2000), Han and Feng (1998), and Tanaka, Ikeda and Wang (1996). Furthermore, there have been also some attempts, reported in An and Wen (2011), Balasubramaniam, Krishnasamy and Rakkiyappan (2012), Cao and Frank (2001), Lee et al. (2000), Li, Liu and Chai (2009), Liu et al. (2010), Nguang and Shi (2000), Su et al. (2012, 2013), Tian and Peng (2006), Tian, Yue and Zhang (2009), Wang, Lin and Wang (2004), Wang and Lin (2003), and Yoneyama (2000, 2010), in which robust fuzzy control analysis and synthesis for nonlinear time-delay systems have been examined. Nevertheless, so far, to the best of our knowledge, the global robust $\mathcal{H}_\infty$ fuzzy state-feedback control problem for a class of uncertain nonlinear Markovian jump systems with time-varying delay via an LMI approach has not yet been considered in the literature.

Therefore, what we intend to do in this paper is to design an $\mathcal{H}_\infty$ fuzzy state-feedback controller for a class of nonlinear Markovian jump systems with time-varying delay, described by the Takagi-Sugeno (TS) fuzzy model. Based on the LMI approach, we develop a state-feedback controller that guarantees the $\mathcal{L}_2$-gain of the mapping from the exogenous input noise to the regulated output to be less than a prescribed value. The solutions are given in terms of a family of linear matrix inequalities. This paper is organized as follows. In Section 2, system description and definition are presented. In Section 3, based on an LMI approach we develop a technique for designing a robust $\mathcal{H}_\infty$ fuzzy state-feedback controller that guarantees the $\mathcal{L}_2$-gain of the mapping from the exogenous input noise to the regulated output to be less than a prescribed value. The validity of this approach is demonstrated by an example from the literature in Section 4. Finally in Section 5, the conclusion is given.

2. System description and definition

The class of uncertain nonlinear Markovian jump system with time-varying delay under consideration is described by the following TS fuzzy models:

Plant Rule i:

IF $\nu_1(t)$ is M_{i1} and $\cdots$ and $\nu_\vartheta(t)$ is $M_{i\vartheta}$ THEN

$$\begin{aligned}
\dot{x}(t) &= [A_i(\eta(t)) + \Delta A_i(\eta(t))]x(t) + A_{d_i}(\eta(t))x(t-\tau(t)) \\
&\quad +[B_{1_i}(\eta(t)) + \Delta B_{1_i}(\eta(t))]w(t) + [B_{2_i}(\eta(t)) + \Delta B_{2_i}(\eta(t))]u(t), \ x(0)=0, \\
z(t) &= [C_{1_i}(\eta(t)) + \Delta C_{1_i}(\eta(t))]x(t) + [D_{12_i}(\eta(t)) + \Delta D_{12_i}(\eta(t))]u(t) \\
x(t) &= \psi(t), \quad t \in [-\tau, 0], \ \ \tau(t) \le \tau
\end{aligned} \tag{1}$$

where $M_{iq}(j = 1, 2, \cdots, \vartheta)$ are fuzzy sets q for rule i, $\nu_i(t)$ are the premise variables, $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re^m$ is the input, $w(t) \in \Re^p$ is the disturbance, which belongs to $\mathcal{L}_2[0,\infty)$, $z(t) \in \Re^s$ is the controlled output, the matrices $A_i(\eta(t))$, $A_{d_i}(\eta(t))$, $B_{1_i}(\eta(t))$, $B_{2_i}(\eta(t))$, $C_{1_i}(\eta(t))$ and $D_{12_i}(\eta(t))$ are of appropriate dimensions, r is the number of IF-THEN rules, $\tau(t) \le \tau$ is the bounded time-varying delay in the state, and $\psi(t)$ is a vector-valued initial continuous function, defined on the interval $[-\tau, 0]$. $\{\eta(t)\}$, $t \ge 0$ is a continuous-time discrete-state homogenous Markov process taking values on a finite set $\mathcal{S} = \{1, 2, \cdots, s\}$ with transition probability matrix $Pr := \{P_{\imath k}(t)\}$ given by

$$\begin{aligned}
P_{\imath k}(t) &= Pr(\eta(t+\Delta) = k \mid \eta(t) = \imath) \\
&= \begin{cases} \lambda_{\imath k}\Delta + O(\Delta) & \text{if } \imath \ne k, \\ 1 + \lambda_{\imath\imath}\Delta + O(\Delta) & \text{if } \imath = k, \end{cases}
\end{aligned} \tag{2}$$

and $\sum_{k=1}^{s} P_{\imath k}(t) = 1$, where $\Delta > 0$; $\lim_{\Delta\to 0} \frac{O(\Delta)}{\Delta} = 0$; $\lambda_{\imath k} \ge 0$, $\imath \ne k$ is the transition rate from mode $\imath$ to mode k; $\lambda_{\imath\imath} = -\sum_{k=1,\ k\ne\imath}^{s} \lambda_{\imath k}$, $\imath, k \in \mathcal{S}$ gives the infinitesimal generator of the Markov process $\{\eta(t), t \ge 0\}$.

The matrices $\Delta A_i(\eta(t))$, $\Delta B_{1_i}(\eta(t))$, $\Delta B_{2_i}(\eta(t))$, $\Delta C_{1_i}(\eta(t))$ and $\Delta D_{12_i}(\eta(t))$ represent the uncertainties in the system and satisfy the following assumption.

Assumption 1

$$\Delta A_i(\eta(t)) = F(x(t),\eta(t),t)H_{1_i}(\eta(t)), \quad \Delta B_{1_i}(\eta(t)) = F(x(t),\eta(t),t)H_{2_i}(\eta(t)),$$

$$\Delta B_{2_i}(\eta(t)) = F(x(t),\eta(t),t)H_{3_i}(\eta(t)), \quad \Delta C_{1_i}(\eta(t)) = F(x(t),\eta(t),t)H_{4_i}(\eta(t)),$$

and $\Delta D_{12_i}(\eta(t)) = F(x(t),\eta(t),t)H_{5_i}(\eta(t))$,

where $H_{j_i}(\eta(t))$, $j = 1, 2, \cdots, 5$ *are known matrix functions which characterize the structure of the uncertainties. Furthermore, the following inequality holds:*

$$\|F(x(t),\eta(t),t)\| \le \rho(\eta(t)) \tag{3}$$

for any known positive constant $\rho(\eta(t))$.

Let $\varpi_i(\nu(t)) = \prod_{q=1}^{n} M_{iq}(\nu_q(t))$ and $\mu_i(\nu(t)) = \frac{\varpi_i(\nu(t))}{\sum_{i=1}^{r}\varpi_i(\nu(t))}$, where $M_{iq}(\nu_q(t))$ is the grade of membership of $\nu_q(t)$ in M_{iq}. It is assumed in this paper that

$$\varpi_i(\nu(t)) \geq 0, \quad i = 1, 2, \cdots, n; \quad \text{and} \quad \sum_{i=1}^{r} \varpi_i(\nu(t)) > 0$$

where r is the number of local plant rules, for all t. Therefore,

$$\mu_i(\nu(t)) \geq 0, \quad i = 1, 2, \cdots, n; \ \text{ and } \ \sum_{i=1}^{r} \mu_i(\nu(t)) = 1$$

for all t. For the convenience of notations, let $\varpi_i = \varpi_i(\nu(t))$, $\mu_i = \mu_i(\nu(t))$, $\eta = \eta(t)$ and any matrix $N(\mu, \eta(t) = \imath) = N(\mu, \imath)$.

The resulting fuzzy system model is inferred as the weighted average of the local models of the form:

$$\begin{aligned} \dot{x}(t) &= [A(\mu,\imath) + \Delta A(\mu,\imath)]x(t) + A_d(\mu,\imath)x(t-\tau(t)) \\ &\quad +[B_1(\mu,\imath) + \Delta B_1(\mu,\imath)]w(t) + [B_2(\mu,\imath) + \Delta B_2(\mu,\imath)]u(t), \quad x(0) = 0 \\ z(t) &= [C_1(\mu,\imath) + \Delta C_1(\mu,\imath)]x(t) + [D_{12}(\mu,\imath) + \Delta D_{12}(\mu,\imath)]u(t) \end{aligned} \tag{4}$$

where

$$A(\mu,\imath) = \sum_{i=1}^{r} \mu_i A_i(\imath), \quad A_d(\mu,\imath) = \sum_{i=1}^{r} \mu_i A_{d_i}(\imath), \quad B_1(\mu,\imath) = \sum_{i=1}^{r} \mu_i B_{1_i}(\imath),$$

$$B_2(\mu,\imath) = \sum_{i=1}^{r} \mu_i B_{2_i}(\imath), \quad C_1(\mu,\imath) = \sum_{i=1}^{r} \mu_i C_{1_i}(\imath), \quad D_{12}(\mu,\imath) = \sum_{i=1}^{r} \mu_i D_{12_i}(\imath),$$

$$\Delta A(\mu,\imath) = \sum_{i=1}^{r} \mu_i \Delta A_i(\imath) := F(x(t),\imath,t)H_1(\mu,\imath),$$

$$\Delta B_1(\mu,\imath) = \sum_{i=1}^{r} \mu_i \Delta B_{1_i}(\imath) := F(x(t),\imath,t)H_2(\mu,\imath),$$

$$\Delta B_2(\mu,\imath) = \sum_{i=1}^{r} \mu_i \Delta B_{2_i}(\imath) := F(x(t),\imath,t)H_3(\mu,\imath),$$

$$\Delta C_1(\mu,\imath) = \sum_{i=1}^{r} \mu_i \Delta C_{1_i}(\imath) := F(x(t),\imath,t)H_4(\mu,\imath),$$

$$\Delta D_{12}(\mu,\imath) = \sum_{i=1}^{r} \mu_i \Delta D_{12_i}(\imath) := F(x(t),\imath,t)H_5(\mu,\imath)$$

with $H_1(\mu,\imath) = \sum_{i=1}^{r} \mu_i H_{1_i}(\imath)$, $H_2(\mu,\imath) = \sum_{i=1}^{r} \mu_i H_{2_i}(\imath)$, $H_3(\mu,\imath) = \sum_{i=1}^{r} \mu_i H_{3_i}(\imath)$, $H_4(\mu,\imath) = \sum_{i=1}^{r} \mu_i H_{4_i}(\imath)$, and $H_5(\mu,\imath) = \sum_{i=1}^{r} \mu_i H_{5_i}(\imath)$.

DEFINITION 1 *Suppose γ is a given positive real number. A system of the form (4) is said to have $\mathcal{L}_2[0, T_f]$ gain less than or equal to γ if*

$$\mathbf{E}\left[\int_0^{T_f} \left\{z^T(t)z(t) - \gamma^2 w^T(t)w(t)\right\} \, dt\right] < 0 \tag{5}$$

where $\mathbf{E}[\cdot]$ denotes the expectation operator.

In this paper, we consider the following $\mathcal{H}_\infty$ fuzzy state-feedback, which is inferred as the weighted average of the local models of the form:

$$u(t) = K(\mu, \imath)x(t) \tag{6}$$

where $K(\mu, \imath) = \sum_{j=1}^{r} \mu_j K_j(\imath)$. Before ending this section, we describe the problem under our study as follows.

Problem Formulation: Given a prescribed $\mathcal{H}_\infty$ performance $\gamma > 0$, design an $\mathcal{H}_\infty$ fuzzy state-feedback controller of the form (6) such that the inequality (5) is guaranteed.

3. Main results

This section provides LMI-based solutions to the problem of designing a robust $\mathcal{H}_\infty$ controller that guarantees the $\mathcal{L}_2$-gain of the mapping from the exogenous input noise to the regulated output to be less than some prescribed value.

THEOREM 1 *Given the system (4) and a prescribed $\mathcal{H}_\infty$ performance $\gamma > 0$, the inequality (5) holds if for $\imath = 1, 2, \cdots, s$ there exist positive definite symmetric matrices $P(\imath)$, $W(\imath)$, and positive constants $\delta(\imath)$ such that the following conditions hold:*

$$\Omega_{ii}(\imath) \quad < \quad 0, \quad i = 1, 2, \cdots, r, \tag{7}$$

$$\Omega_{ij}(\imath) + \Omega_{ji}(\imath) \quad < \quad 0, \quad i < j \leq r \tag{8}$$

where

$$\Omega_{ij}(\imath)=\begin{pmatrix} \Psi_{ij}(\imath) & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T \\ \mathcal{R}(\imath)\tilde{B}_{1_i}^T(\imath) & -\gamma\mathcal{R}(\imath) & (*)^T & (*)^T & (*)^T & (*)^T \\ W(\imath)A_{d_i}(\imath) & 0 & -W(\imath) & (*)^T & (*)^T & (*)^T \\ P(\imath) & 0 & 0 & -W(\imath) & (*)^T & (*)^T \\ \Upsilon_{ij}(\imath) & 0 & 0 & 0 & -\gamma\mathcal{R}(\imath) & (*)^T \\ \mathcal{Z}^T(\imath) & 0 & 0 & 0 & 0 & -\mathcal{P}(\imath) \end{pmatrix}, \tag{9}$$

$$\begin{aligned} \Psi_{ij}(\imath) &= A_i(\imath)P(\imath)+P(\imath)A_i^T(\imath) + B_{2_i}(\imath)Y_j(\imath)+Y_j^T(\imath)B_{2_i}^T(\imath)+\lambda_{\imath\imath}P(\imath), && (10)\\ \Upsilon_{ij}(\imath) &= \tilde{C}_{1_i}(\imath)P(\imath) + \tilde{D}_{12_i}(\imath)Y_j(\imath), && (11)\\ \mathcal{R}(\imath) &= diag\{\delta(\imath)I, I, \delta(\imath)I, I\}, && (12)\\ \mathcal{Z}(\imath) &= \left(\sqrt{\lambda_{\imath 1}}P(\imath)\cdots\sqrt{\lambda_{\imath(\imath-1)}}P(\imath)\sqrt{\lambda_{\imath(\imath+1)}}P(\imath)\ \cdots\sqrt{\lambda_{\imath s}}P(\imath)\right), && (13)\\ \mathcal{P}(\imath) &= diag\{P(1), \cdots, P(\imath-1), P(\imath+1), \cdots, P(s)\}, && (14) \end{aligned}$$

with

$$\begin{aligned}
\tilde{B}_{1_i}(\imath) &= \left[\begin{array}{cccc} I & I & I & B_{1_i}(\imath) \end{array}\right] && (15)\\
\tilde{C}_{1_i}(\imath) &= \left[\begin{array}{cccc} \gamma\rho(\imath)H_{1_i}^T(\imath) & \sqrt{2}\aleph(\imath)\rho(\imath)H_{4_i}^T(\imath) & 0 & \sqrt{2}\aleph(\imath)C_{1_i}^T(\imath) \end{array}\right]^T && (16)\\
\tilde{D}_{12_i}(\imath) &= \left[\begin{array}{cccc} 0 & \sqrt{2}\aleph(\imath)\rho(\imath)H_{5_i}^T(\imath) & \gamma\rho(\imath)H_{3_i}^T(\imath) & \sqrt{2}\aleph(\imath)D_{12_i}^T(\imath) \end{array}\right]^T && (17)\\
\aleph(\imath) &= \left(1+\rho^2(\imath)\sum_{i=1}^{r}\sum_{j=1}^{r}\left[\|H_{2_i}(\imath)^T H_{2_j}(\imath)\|\right]\right)^{\frac{1}{2}}. && (18)
\end{aligned}$$

Furthermore, a suitable choice of the fuzzy controller is

$$u(t)=\sum_{j=1}^{r}\mu_j K_j(\imath)x(t) \qquad (19)$$

where

$$K_j(\imath)=Y_j(\imath)(P(\imath))^{-1}. \qquad (20)$$

Proof. The closed-loop state space form of the fuzzy system model (4) with the controller (6) is given by

$$\begin{aligned}
\dot{x}(t) &= [A(\mu,\imath)+B_2(\mu,\imath)K(\mu,\imath)]x(t)+A_d(\mu,\imath)x(t-\tau(t)) \qquad (21)\\
&+[\Delta A(\mu,\imath)+\Delta B_2(\mu,\imath)K(\mu,\imath)]x(t)+\Big[B_1(\mu,\imath)+\Delta B_1(\mu,\imath)\Big]w(t),\ x(0)=0,
\end{aligned}$$

or, in a more compact form,

$$\begin{aligned}
&\dot{x}(t)=\\
&[A(\mu,\imath)+B_2(\mu,\imath)K(\mu,\imath)]x(t)+A_d(\mu,\imath)x(t-\tau(t))+\tilde{B}_1(\mu,\imath)\mathcal{R}(\imath)\tilde{w}(t),\\
&x(0)=0,
\end{aligned} \qquad (22)$$

where

$$\begin{aligned}
\tilde{B}_1(\mu,\imath) &= \left[\begin{array}{cccc} I & I & I & B_1(\mu,\imath) \end{array}\right] && (23)\\
\tilde{w}(t) &= \mathcal{R}^{-1}(\imath)\left[\begin{array}{c} F(x(t),\imath,t)H_1(\mu,\imath)x(t)\\ F(x(t),\imath,t)H_2(\mu,\imath)w(t)\\ F(x(t),\imath,t)H_3(\mu,\imath)K(\mu,\imath)x(t)\\ w(t) \end{array}\right]. && (24)
\end{aligned}$$

Consider a Lyapunov-Krasovskii functional candidate as follows:

$$V(x(t),\imath)=\gamma x^T(t)Q(\imath)x(t)+\gamma\int_{t-\tau(t)}^{t}x^T(v)G(\imath)x(v)dv,\ \ \forall\,\imath\in\mathcal{S} \qquad (25)$$

where $Q(\imath)=P^{-1}(\imath)>0$ and $G(\imath)=W^{-1}(\imath)>0$. For this choice, we have $V(0,\imath_0)=0$ and $V(x(t),\imath)\to\infty$ only when $\|x(t)\|\to\infty$.

Now, consider the weak infinitesimal operator $\tilde{\Delta}$ of the joint process $\{(x(t),\imath), t \geq 0\}$, which is the stochastic analog of the deterministic derivative; see Krusher (1997). $\{(x(t),\imath), t \geq 0\}$ is a Markov process with infinitesimal operator given by, see Souza (1993),

$$\begin{array}{l}\tilde{\Delta}V(x(t),\imath)\\ = \gamma x^T(t)\Big[Q(\imath)\Big(A(\mu,\imath)+B_2(\mu,\imath)K(\mu,\imath)\Big)\\ +\Big(A(\mu,\imath)+B_2(\mu,\imath)K(\mu,\imath)\Big)^T Q(\imath)+G(\imath)\Big]x(t)\\ +\gamma x^T(t)Q(\imath)\tilde{B}_1(\mu,\imath)\mathcal{R}(\imath)\tilde{w}(t)\\ +\gamma \tilde{w}^T(t)\mathcal{R}(\imath)\tilde{B}_1^T(\mu,\imath)Q(\imath)x(t)+\gamma x^T(t)\sum_{k=1}^{s}\lambda_{\imath k}Q(k)x(t)\\ -\gamma x^T(t-\tau(t))G(\imath)x(t-\tau(t))+\gamma x^T(t)Q(\imath)A_d(\mu,\imath)x(t-\tau(t))\\ +\gamma x^T(t-\tau(t))A_d^T(\mu,\imath)Q(\imath)x(t).\end{array} \tag{26}$$

Using the fact that for any vector $x(t)$ and $x(t-\tau(t))$

$$\begin{array}{l}x^T(t)Q(\imath)A_d(\mu,\imath)x(t-\tau(t))+x^T(t-\tau(t))A_d^T(\mu,\imath)Q(\imath)x(t)\\ \leq x^T(t)Q(\imath)A_d(\mu,\imath)G^{-1}(\imath)A_d^T(\mu,\imath)Q(\imath)x(t)+x^T(t-\tau(t))G(\imath)x(t-\tau(t)),\end{array}$$

(26) becomes

$$\begin{array}{l}\tilde{\Delta}V(x(t),\imath)=\\ \gamma x^T(t)\Big[Q(\imath)\Big(A(\mu,\imath)+B_2(\mu,\imath)K(\mu,\imath)\Big)+\Big(A(\mu,\imath)+B_2(\mu,\imath)K(\mu,\imath)\Big)^T Q(\imath)\\ +Q(\imath)A_d(\mu,\imath)G^{-1}(\imath)A_d^T(\mu,\imath)Q(\imath)+G(\imath)+\sum_{k=1}^{s}\lambda_{\imath k}Q(k)\Big]x(t)\\ +\gamma x^T(t)Q(\imath)\tilde{B}_1(\mu,\imath)\mathcal{R}(\imath)\tilde{w}(t)+\gamma\tilde{w}^T(t)\mathcal{R}(\imath)\tilde{B}_1^T(\mu,\imath)Q(\imath)x(t).\end{array} \tag{27}$$

By adding and subtracting $-\aleph^2(\imath)z^T(t)z(t)+\gamma^2\tilde{w}^T(t)\mathcal{R}(\imath)\tilde{w}(t)$ to and from (27), we get

$$\begin{array}{l}\tilde{\Delta}V(x(t),\imath)=\\ -\aleph^2(\imath)z^T(t)z(t)+\gamma^2\tilde{w}^T(t)\mathcal{R}(\imath)\tilde{w}(t)+\aleph^2(\imath)z^T(t)z(t)+\gamma\begin{bmatrix}x(t)\\ \tilde{w}(t)\end{bmatrix}^T\times\\ \left(\begin{array}{cc}\left(\begin{array}{c}\Big[A(\mu,\imath)+B_2(\mu,\imath)K(\mu,\imath)\Big]^T Q(\imath)\\ +Q(\imath)\Big[A(\imath)+B_2(\mu,\imath)K(\mu,\imath)\Big]\\ +\sum_{k=1}^{s}\lambda_{\imath k}Q(k)+G(\imath)\\ +Q(\imath)A_d(\mu,\imath)G^{-1}(\imath)A_d^T(\mu,\imath)Q(\imath)\end{array}\right) & Q(\imath)\tilde{B}_1(\mu,\imath)\mathcal{R}(\imath)\\ \mathcal{R}(\imath)\tilde{B}_1^T(\mu,\imath)Q(\imath) & -\gamma\mathcal{R}(\imath)\end{array}\right)\begin{bmatrix}x(t)\\ \tilde{w}(t)\end{bmatrix}.\end{array} \tag{28}$$

Now, let us consider the following terms:

$$\gamma^2\tilde{w}^T(t)\mathcal{R}(\imath)\tilde{w}(t) = \gamma^2\begin{bmatrix} F(x(t),\imath,t)H_1(\mu,\imath)x(t) \\ F(x(t),\imath,t)H_2(\mu,\imath)w(t) \\ F(x(t),\imath,t)H_3(\mu,\imath)K(\mu,\imath)x(t) \\ w(t)\end{bmatrix}^T \mathcal{R}(\imath)\times \begin{bmatrix} F(x(t),\imath,t)H_1(\mu,\imath)x(t) \\ F(x(t),\imath,t)H_2(\mu,\imath)w(t) \\ F(x(t),\imath,t)H_3(\mu,\imath)K(\mu,\imath)x(t) \\ w(t)\end{bmatrix}$$

$$\leq \frac{\rho^2(\imath)\gamma^2}{\delta(\imath)}x^T(t)\Big\{H_1^T(\mu,\imath)H_1(\mu,\imath)+K^T(\mu,\imath)H_3^T(\mu,\imath)H_3(\mu,\imath)K(\mu,\imath)\Big\}x(t) + \aleph^2(\imath)\gamma^2 w^T(t)w(t) \quad (29)$$

and

$$\aleph^2(\imath)z^T(t)z(t) = \aleph^2(\imath)x^T(t)\Big[C_1(\mu,\imath)+F(x(t),\imath,t)H_4(\mu,\imath)+D_{12}(\mu,\imath)K(\mu,\imath)+ F(x(t),\imath,t)H_5(\mu,\imath)K(\mu,\imath)\Big]^T\Big[C_1(\mu,\imath)+F(x(t),\imath,t)H_4(\mu,\imath)+ D_{12}(\mu,\imath)K(\mu,\imath)+F(x(t),\imath,t)H_5(\mu,\imath)K(\mu,\imath)\Big]x(t)$$

$$\leq 2\aleph^2(\imath)x^T(t)\Big\{\left[C_1(\mu,\imath)+D_{12}(\mu,\imath)K(\mu,\imath)\right]^T\left[C_1(\mu,\imath)+D_{12}(\mu,\imath)K(\mu,\imath)\right] + \left[F(x(t),\imath,t)H_4(\mu,\imath)+F(x(t),\imath,t)H_5(\mu,\imath)K(\mu,\imath)\right]^T \left[F(x(t),\imath,t)H_4(\mu,\imath)+F(x(t),\imath,t)H_5(\mu,\imath)K(\mu,\imath)\right]\Big\}x(t) \quad (30)$$

$$\leq 2\aleph^2(\imath)x^T(t)\Big\{\left[C_1(\mu,\imath)+D_{12}(\mu,\imath)K(\mu,\imath)\right]^T\left[C_1(\mu,\imath)+D_{12}(\mu,\imath)K(\mu,\imath)\right]+ \rho^2(\imath)\left[H_4(\mu,\imath)+H_5(\mu,\imath)K(\mu,\imath)\right]^T\left[H_4(\mu,\imath)+H_5(\mu,\imath)K(\mu,\imath)\right]\Big\}x(t)$$

where $\aleph(\imath) = \|I+\rho^2(\imath)[H_2^T(\mu,\imath)H_2(\mu,\imath)\|^{\frac{1}{2}}$. Hence,

$$\gamma^2\tilde{w}^T(t)\mathcal{R}(\imath)\tilde{w}(t)+\aleph^2(\imath)z^T(t)z(t) \leq x^T(t)\Big[\tilde{C}_1(\mu,\imath)+\tilde{D}_{12}(\mu,\imath)K(\mu,\imath)\Big]^T\mathcal{R}^{-1}(\imath)\times \Big[\tilde{C}_1(\mu,\imath)+\tilde{D}_{12}(\mu,\imath)K(\mu,\imath)\Big]x(t)+\aleph^2(\imath)\gamma^2w^T(t)w(t) \quad (31)$$

where

$$\begin{aligned}\tilde{C}_1(\mu,\imath) &= \left[\begin{array}{cccc}\gamma\rho(\imath)H_1^T(\mu,\imath) & \sqrt{2}\aleph(\imath)\rho(\imath)H_4^T(\mu,\imath) & 0 & \sqrt{2}\aleph(\imath)C_1^T(\mu,\imath)\end{array}\right]^T \\ \tilde{D}_{12}(\mu,\imath) &= \left[\begin{array}{cccc}0 & \sqrt{2}\aleph(\imath)\rho(\imath)H_5^T(\mu,\imath) & \gamma\rho(\imath)H_3^T(\mu,\imath) & \sqrt{2}\aleph(\imath)D_{12}^T(\mu,\imath)\end{array}\right]^T.\end{aligned}$$

Substituting (31) into (28) yields

$$\tilde{\Delta}V(x(t),\imath) \leq -\aleph^2(\imath)z^T(t)z(t)+\gamma^2\aleph^2(\imath)w^T(t)w(t)+\gamma\begin{bmatrix}x(t)\\ \tilde{w}(t)\end{bmatrix}^T\Phi(\mu,\imath)\begin{bmatrix}x(t)\\ \tilde{w}(t)\end{bmatrix} \quad (32)$$

where

$$\Phi(\mu,\imath)=\begin{pmatrix}\begin{pmatrix}\Big[A(\mu,\imath)+B_2(\mu,\imath)K(\mu,\imath)\Big]^T Q(\imath)\\+Q(\imath)\Big[A(\mu,\imath)+B_2(\mu,\imath)K(\mu,\imath)\Big]\\+\frac{1}{\gamma}\Big[\tilde{C}_1(\mu,\imath)+\tilde{D}_{12}(\mu,\imath)K(\mu,\imath)\Big]^T\times\\ \mathcal{R}^{-1}(\imath)\Big[\tilde{C}_1(\mu,\imath)+\tilde{D}_{12}(\mu,\imath)K(\mu,\imath)\Big]\\+\sum_{k=1}^{s}\lambda_{\imath k}Q(k)+G(\imath)\\+Q(\imath)A_d(\mu,\imath)G^{-1}(\imath)A_d^T(\mu,\imath)Q(\imath)\end{pmatrix} & Q(\imath)\tilde{B}_1(\mu,\imath)\mathcal{R}(\imath)\\ \mathcal{R}(\imath)\tilde{B}_1^T(\mu,\imath)Q(\imath) & -\gamma\mathcal{R}(\imath)\end{pmatrix}. \tag{33}$$

Using the fact that

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n M_{ij}^T(\imath)N_{mn}(\imath)\leq$$
$$\frac{1}{2}\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j[M_{ij}^T(\imath)M_{ij}(\imath)+N_{ij}(\imath)N_{ij}^T(\imath)],$$

we can rewrite (3) as follows:

$$\begin{aligned}&\tilde{\Delta}V(x(t),\imath)\leq\\&-\aleph^2(\imath)z^T(t)z(t)+\gamma^2\aleph^2(\imath)w^T(t)w(t)+\gamma\textstyle\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\begin{bmatrix}x(t)\\\tilde{w}(t)\end{bmatrix}^T\Phi_{ij}(\imath)\begin{bmatrix}x(t)\\\tilde{w}(t)\end{bmatrix}\\&=-\aleph^2(\imath)z^T(t)z(t)+\gamma^2\aleph^2(\imath)w^T(t)w(t)+\gamma\textstyle\sum_{i=1}^{r}\mu_i^2\begin{bmatrix}x(t)\\\tilde{w}(t)\end{bmatrix}^T\Phi_{ii}(\imath)\begin{bmatrix}x(t)\\\tilde{w}(t)\end{bmatrix}\\&+\gamma\textstyle\sum_{i=1}^{r}\sum_{i<j}^{r}\mu_i\mu_j\begin{bmatrix}x(t)\\\tilde{w}(t)\end{bmatrix}^T\Big(\Phi_{ij}(\imath)+\Phi_{ji}(\imath)\Big)\begin{bmatrix}x(t)\\\tilde{w}(t)\end{bmatrix}\end{aligned}$$

where

$$\Phi_{ij}(\imath)=$$

$$\begin{pmatrix}\begin{pmatrix}\Big[A_i(\imath)+B_{2_i}(\imath)K_j(\imath)\Big]^T Q(\imath)\\+Q(\imath)\Big[A_i(\imath)+B_{2_i}(\imath)K_j(\imath)\Big]\\+\frac{1}{\gamma}\Big[\tilde{C}_{1_i}(\imath)+\tilde{D}_{12_i}(\imath)K_j(\imath)\Big]^T\mathcal{R}^{-1}(\imath)\Big[\tilde{C}_{1_i}(\imath)+\tilde{D}_{12_i}(\imath)K_j(\imath)\Big]\\+\sum_{k=1}^{s}\lambda_{\imath k}Q(k)+G(\imath)\\+Q(\imath)A_{d_i}(\imath)G^{-1}(\imath)A_{d_i}^T(\imath)Q(\imath)\end{pmatrix} Q(\imath)\tilde{B}_{1_i}(\imath)\mathcal{R}(\imath)\\ \mathcal{R}(\imath)\tilde{B}_{1_i}^T(\imath)Q(\imath)-\gamma\mathcal{R}(\imath)\end{pmatrix}. \tag{34}$$

Using (20) and pre and post multiplying (34) by

$$\Xi(\imath) = \begin{pmatrix} P(\imath) & 0 \\ 0 & I \end{pmatrix},$$

we obtain

$$\Xi(\imath)\Phi_{ij}(\imath)\Xi(\imath) =$$

$$\left(\begin{pmatrix} P(\imath)A_i^T(\imath) + Y_j^T(\imath)B_{2_i}^T(\imath) + A_i(\imath)P(\imath) + B_{2_i}(\imath)Y_j(\imath) \\ +\frac{1}{\gamma}\left[\tilde{C}_{1_i}(\imath)P(\imath) + \tilde{D}_{12_i}(\imath)Y_j(\imath)\right]^T \mathcal{R}^{-1}(\imath)\times \\ \left[\tilde{C}_{1_i}(\imath)P(\imath) + \tilde{D}_{12_i}(\imath)Y_j(\imath)\right] + \sum_{k=1}^{s} \lambda_{\imath k}P(\imath)P^{-1}(k)P(\imath) \\ +P(\imath)G(\imath)P(\imath) + A_{d_i}(\imath)G^{-1}(\imath)A_{d_i}^T(\imath) \end{pmatrix} |1 \; \tilde{B}_{1_i}(\imath)\mathcal{R}(\imath) \\ \mathcal{R}(\imath)\tilde{B}_{1_i}^T(\imath) - \gamma\mathcal{R}(\imath) \right). \quad (35)$$

Note that (35) is the Schur complement of $\Omega_{ij}(\imath)$, defined in (9). Using (7), (8) and (35), we learn that

$$\Phi_{ii}(\imath) < 0 \quad (36)$$

$$\Phi_{ij}(\imath) + \Phi_{ji}(\imath) < 0. \quad (37)$$

Following from (3), (36) and (37), we know that

$$\tilde{\Delta}V(x(t),\imath) < -\aleph^2(\imath)z^T(t)z(t) + \gamma^2\aleph^2(\imath)w^T(t)w(t). \quad (38)$$

Applying the operator $\mathbf{E}[\int_0^{T_f}(\cdot)dt]$ to both sides of (38), we obtain

$$\mathbf{E}\left[\int_0^{T_f} \tilde{\Delta}V(x(t),\imath)dt\right] < \mathbf{E}\left[\int_0^{T_f} (-\aleph^2(\imath)z^T(t)z(t) + \gamma^2\aleph^2(\imath)w^T(t)w(t))dt\right]. \quad (39)$$

From the Dynkin's (1965) formula, it follows that

$$\mathbf{E}\left[\int_0^{T_f} \tilde{\Delta}V(x(t),\imath)dt\right] = \mathbf{E}[V(x(T_f),\imath(T_f))] - \mathbf{E}[V(x(0),\imath(0))]. \quad (40)$$

Substitution of (40) into (39) yields

$$0 < \mathbf{E}\left[\int_0^{T_f} (-\aleph^2(\imath)z^T(t)z(t) + \gamma^2\aleph^2(\imath)w^T(t)w(t))dt\right] - \mathbf{E}[V(x(T_f),\imath(T_f))] + \mathbf{E}[V(x(0),\imath(0))].$$

Using (38) and the fact that $V(x(0)=0,\imath(0))=0$ and $V(x(T_f),\imath(T_f))>0$, we have

$$\mathbf{E}\left[\int_0^{T_f} \left\{z^T(t)z(t) - \gamma^2 w^T(t)w(t)\right\}dt\right] < 0. \quad (41)$$

Hence, the inequality (5) holds. This completes the proof of Theorem 1. □

It is necessary to note that in Theorem 1, the inequalities in (7) and (8) are not only linear with respect to matrix variables, but are also linear with respect to the performance index gamma, which implies that the $\mathcal{H}_\infty$ performance γ_{min} can be optimized by solving a convex optimization algorithm with LMI solver toolbox.

In order to demonstrate the effectiveness and advantages of the proposed design methodology, an illustrative example is given in the next section.

4. An illustrative example

Consider a modified nonlinear mass-spring-damper system as shown in Fig. 2. The mass-spring-damper system, i.e., mass attached to spring and damper, is a common control experimental device frequently used in laboratory. The dynamics of the modified nonlinear mass-spring-damper system is governed by the following state equation; see Tanaka, Ikeda and Wang (1996) and Lee et al. (2000):

$$\begin{aligned}
\dot{x}_1(t) &= -[0.1125+\Delta R]x_1(t) - \beta x_1(t-\tau(t)) - 0.02x_2(t) - 0.67x_2^3(t) \\
&\quad -0.1x_2^3(t-\tau(t)) - 0.005x_2(t-\tau(t)) + u(t) + 0.1w_1(t) \\
\dot{x}_2(t) &= x_1(t) + 0.1w_2(t) \\
z(t) &= \begin{bmatrix} x_1(t) \\ \\ x_2(t) \end{bmatrix}
\end{aligned} \tag{42}$$

where $x_1(t)$ and $x_2(t)$ are the state vectors, which represent the velocity and distance, respectively, $u(t)$ is the control input, $w_1(t)$ and $w_2(t)$ are the disturbance inputs, $z(t)$ is the regulated output, β is the delay parameter, ΔR is an uncertain term, which is bounded in $[0 \;\; 0.1125]$, and the time-varying delay $\tau(t) = 4 + 0.5\cos(0.9t)$. It is assumed that

$$x_1(t) \in [-1.5 \;\; 1.5] \text{ and } x_2(t) \in [-1.5 \;\; 1.5].$$

Based on Tanaka, Ikeda and Wang (1996), the nonlinear term can be represented as

$$\begin{aligned}
-0.67x_2^3(t) &= M_1 \cdot 0 \cdot x_2(t) - (1-M_1)\cdot 1.5075x_2(t), \\
-0.1x_2^3(t-\tau(t)) &= M_1 \cdot 0 \cdot x_2(t-\tau(t)) - (1-M_1)\cdot 0.225x_2(t-\tau(t)).
\end{aligned}$$

Upon solving the above equations, M_1 is obtained as follows:

$$\begin{aligned}
M_1(x_2(t)) &= 1 - \frac{x_2^2(t)}{2.25} \\
M_2(x_2(t)) &= 1 - M_1(x_2(t)) = \frac{x_2^2(t)}{2.25}.
\end{aligned}$$

Note that $M_1(x_2(t))$ and $M_1(x_2(t))$ can be interpreted as the membership functions of fuzzy sets.

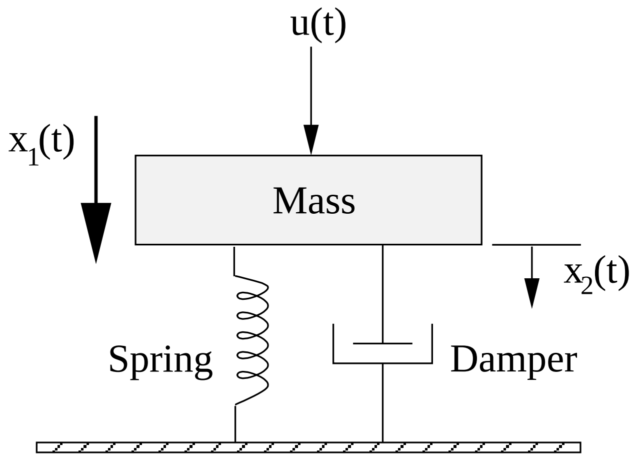

Figure 2. The mass-spring-damper system

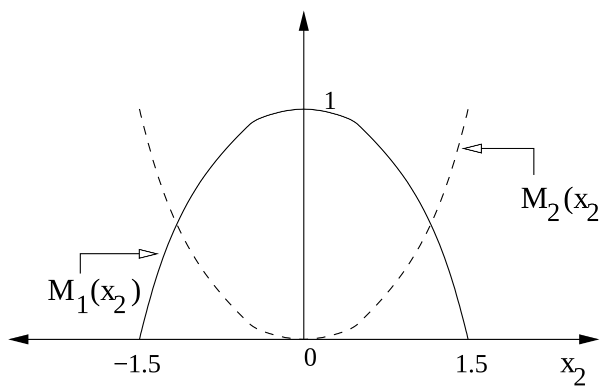

Figure 3. Membership functions for two fuzzy sets

Suppose that the system could be aggregated into three modes as shown in Table 1 and the transition probability matrix that relates the three operation

Table 1. System terminology.

Mode $\imath$	$\beta(\imath)$
1	0.0120
2	0.0125
3	0.0130

modes is given as follows:

$$P_{\imath k} = \begin{bmatrix} 0.67 & 0.17 & 0.16 \\ 0.30 & 0.47 & 0.23 \\ 0.26 & 0.10 & 0.64 \end{bmatrix}.$$

Using these two fuzzy sets, the uncertain nonlinear Markovian jump system with time-varying delay can be represented by the following TS fuzzy model:

Plant Rule 1: IF $x_2(t)$ is $M_1(x_2(t))$ THEN

$$\begin{aligned} \dot{x}(t) &= [A_1(\imath) + \Delta A_1(\imath)]x(t) + A_{d_1}(\imath)x(t-\tau(t)) + B_1(\imath)w(t) + B_2(\imath)u(t), \\ x(0) &= 0 \\ z(t) &= C_1(\imath)x(t), \end{aligned}$$

Plant Rule 2: IF $x_2(t)$ is $M_2(x_2(t))$ THEN

$$\begin{aligned} \dot{x}(t) &= [A_2(\imath) + \Delta A_2(\imath)]x(t) + A_{d_2}(\imath)x(t-\tau(t)) + B_1(\imath)w(t) + B_2(\imath)u(t), \\ x(0) &= 0, \\ z(t) &= C_1(\imath)x(t) \end{aligned}$$

where

$$A_1(\imath) = \begin{bmatrix} -0.1125 & -0.02 \\ 1 & 0 \end{bmatrix}, \quad A_2(\imath) = \begin{bmatrix} -0.1125 & -1.5275 \\ 1 & 0 \end{bmatrix},$$

$$A_{d_1}(\imath) = \begin{bmatrix} -\beta(\imath) & -0.005 \\ 0 & 0 \end{bmatrix}, \quad A_{d_2}(\imath) = \begin{bmatrix} -\beta(\imath) & -0.23 \\ 0 & 0 \end{bmatrix},$$

$$B_1(\imath) = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad B_2(\imath) \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C_1(\imath) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$\Delta A_1(\imath) = F(x(t),t)H_{1_1}(\imath)$, $\Delta A_2(\imath) = F(x(t),t)H_{1_2}(\imath)$, $x(t) = [x_1^T(t)\ x_2^T(t)]^T$ and $w(t) = [w_1^T(t)\ w_2^T(t)]^T$.

Now, by assuming that $\|F(x(t),t)\| \le \rho = 1$, we have

$$H_{1_1}(\imath) = H_{1_2}(\imath) = \begin{bmatrix} -0.1125 & 0 \\ 0 & 0 \end{bmatrix}.$$

Using the LMI optimization algorithm and Theorem 1 with $\gamma = 1$, we obtain

$$P(1) = \begin{bmatrix} 3.0235 & -0.3160 \\ -0.3160 & 0.0612 \end{bmatrix},\ W(1) = \begin{bmatrix} 1.9167 & -0.2101 \\ -0.2101 & 11.2435 \end{bmatrix},$$
$$\delta(1) = 0.0807,$$

$$Y_1(1) = \begin{bmatrix} -19.2632 & -0.1261 \end{bmatrix},\ Y_2(1) = \begin{bmatrix} -19.6783 & -0.0566 \end{bmatrix},$$

$$K_1(1) = \begin{bmatrix} -14.3014 & -75.8701 \end{bmatrix},\ K_2(1) = \begin{bmatrix} -14.3418 & -74.9435 \end{bmatrix},$$

$$P(2) = \begin{bmatrix} 2.8945 & -0.3304 \\ -0.3304 & 0.0762 \end{bmatrix},\ W(2) = \begin{bmatrix} 1.9319 & -0.2654 \\ -0.2654 & 11.2661 \end{bmatrix},$$
$$\delta(2) = 0.0784,$$

$$Y_1(2) = \begin{bmatrix} -18.2470 & 0.0231 \end{bmatrix},\ Y_2(2) = \begin{bmatrix} -18.6784 & 0.1096 \end{bmatrix},$$

$$K_1(2) = \begin{bmatrix} -12.4132 & -53.5244 \end{bmatrix},\ K_2(2) = \begin{bmatrix} -12.4517 & -52.5558 \end{bmatrix}$$

$$P(3) = \begin{bmatrix} 2.9778 & -0.3218 \\ -0.3218 & 0.0666 \end{bmatrix},\ W(3) = \begin{bmatrix} 1.9270 & -0.2294 \\ -0.2294 & 11.2507 \end{bmatrix},$$
$$\delta(3) = 0.0801,$$

$$Y_1(3) = \begin{bmatrix} -18.8926 & -0.0678 \end{bmatrix},\ Y_2(3) = \begin{bmatrix} -19.3150 & 0.0081 \end{bmatrix},$$

$$K_1(3) = \begin{bmatrix} -13.5067 & -66.2681 \end{bmatrix},\ K_2(3) = \begin{bmatrix} -13.5460 & -65.3188 \end{bmatrix}.$$

The resulting fuzzy controller is

$$u(t) = \sum_{j=1}^{2} \mu_j K_j(\imath) x(t) \tag{43}$$

where

$$\mu_1 = M_1(x_2(t)) \ \text{ and } \ \mu_2 = M_2(x_2(t)).$$

Remark 1 *The fuzzy controller (43) guarantees that the inequality (5) holds. The system terminology is shown in Table 1, while Fig. 4 shows the result of the switching between modes during the simulation with the initial mode 2. The histories of the state variables, $x_1(t)$, which is the velocity, and $x_2(t)$, which is the distance, are given in Fig. 5. The simulation results indicate that the* *trajectories of the state variables finally converge to zero appropriately. The disturbance input signal,* $w(t)$*, which was used during simulation is the rectangular signal (magnitude 0.1 and frequency 1 Hz) shown in Fig. 6. The ratio of the regulated output energy to the disturbance input noise energy obtained by using the* $\mathcal{H}_\infty$ *fuzzy controller (43) is depicted in Fig. 7. After three seconds, the ratio of the regulated output energy to the disturbance input noise energy tends to a constant value, which is about* 0.505*. So,* $\gamma = \sqrt{0.505} = 0.711$*, which is less than the prescribed value* 1*.*

5. Conclusion

In this paper, we have developed a technique for designing a robust $\mathcal{H}_\infty$ fuzzy state-feedback controller for a class of nonlinear Markovian jump systems with time-varying delay that guarantees the $\mathcal{L}_2$-gain of the mapping from the exogenous input noise to the regulated output to be less than some prescribed value. In addition, solutions to the problem are given in terms of linear matrix inequalities which make them more useful. Finally, an illustrative example is provided to demonstrate the effectiveness and advantages of the proposed design methodology. However, failure of components can take place in many real physical control problems, so further results on robust fuzzy dynamic system over the nonlinear Markovian jump system with time-varying delays together with D-stability constraints can be considered in future research work.

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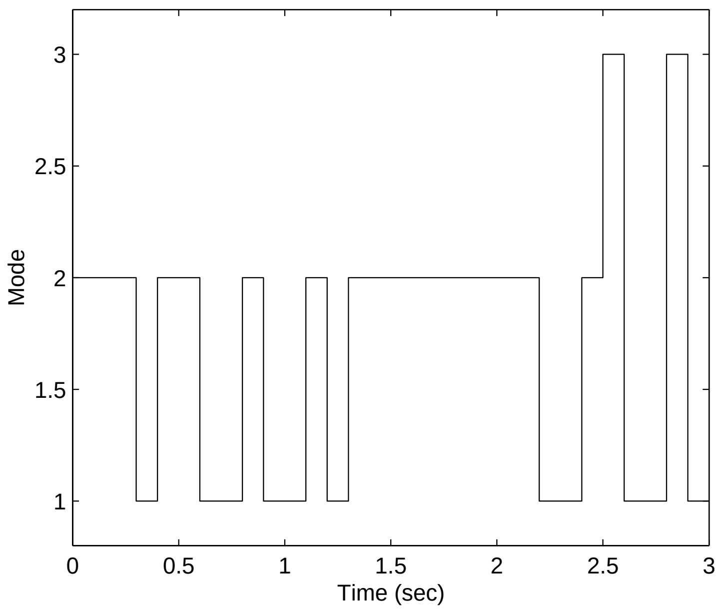

Figure 4. The result of the switching between modes during the simulation with the initial mode being mode 2

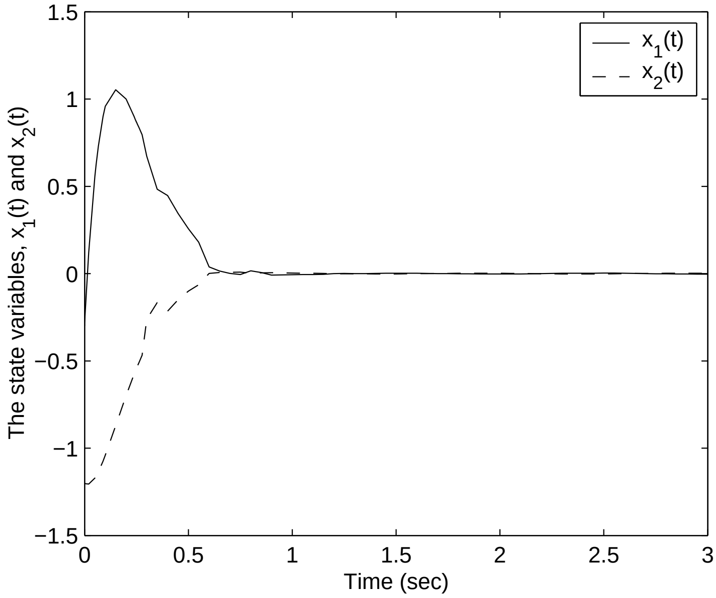

Figure 5. The histories of the state variables, $x_1(t)$ and $x_2(t)$

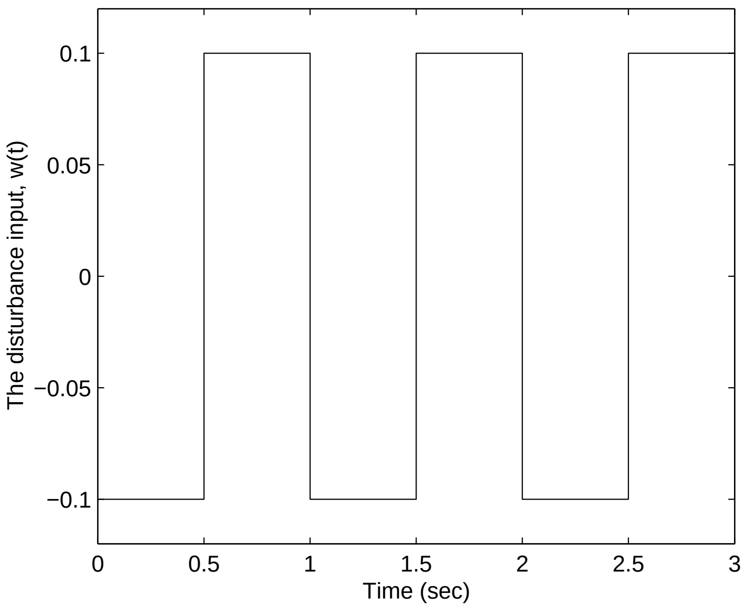

Figure 6. The disturbance noise input, $w(t)$

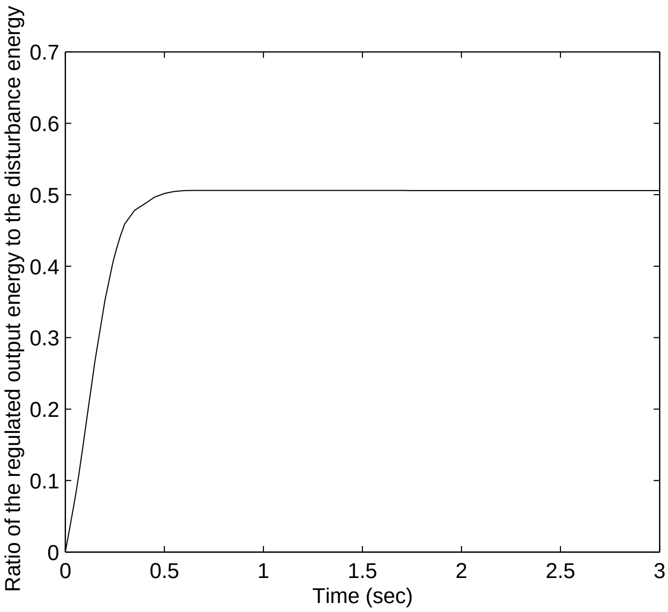

Figure 7. The ratio of the regulated output energy to the disturbance noise energy, $\left(\frac{\int_0^{T_f} z^T(t)z(t)dt}{\int_0^{T_f} w^T(t)w(t)dt}\right)$

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