Control and Cybernetics

vol. **43** (2014) No. 2

Application of the simple additive modeling of the first principle model inaccuracies for the offset–free process control*

by

Jacek Czeczot, Piotr Łaszczyk

Silesian University of Technology
Faculty of Automatic Control, Electronics and Computer Science
Institute of Automatic Control
ul. Akademicka 16, 44-100 Gliwice, Poland
jacek.czeczot@polsl.pl, piotr.laszczyk@polsl.pl

Abstract: In this paper, the method for simple additive modeling of the first principle model inaccuracies for the offset-free process control is presented. Starting from transformation of the general nonlinear state model into the input-affine dynamical equation describing directly the controlled variable, it is shown how to compensate for the potential modeling inaccuracies by lumping them into a single additive parameter. Its on-line estimation procedure based only on the measurement data collected from the process is very simple and effective and the estimate converges without any additional excitation of the process. The discussion on how to apply the suggested model as a basis for the chosen model-based control techniques is presented, and for the processes of the higher relative order, the practical simplification of this approach is shown. The experimental results show the practical applicability of the considered approach for the synthesis of the open loop Internal Model Controller (IMC) and of the Balance-Based Adaptive Controller (B-BAC).

Keywords: first principle process modeling, modeling inaccuracies compensation, model-based control, adaptive control

1. Introduction

The idea of applying the process model for the synthesis of the advanced model-based controller is very simple and promising – if an accurate model of a process is known, it can be somehow incorporated in the control law to ensure better control performance, because the nonlinearities described by a model can be directly compensated in the resulting control law. The unified tools for such techniques are still intensively investigated and expected from the side of industry (e.g. Seborg, 1999; Klatt and Marquardt, 2009; Rhinehart et al., 2011),

*Submitted: August 2012; Accepted: April 2014

even though there is already a large number of control techniques that are based on the simple input-output models, e.g. Model Algorithmic Control (MAC) and Predictive Funtional Control (PFC) from Richalet et al. (1978), Richalet, (1993), Dynamic Matrix Control (DMC) from Cutler and Ramaker (1980), and Internal Model Control (IMC) from Garcia and Morari (1982). In these cases, the strong process nonlinearities can be considered by nonstationary modeling, which leads to the adaptive control law.

Another possibility is to apply the first principle nonlinear models as a basis for the controller synthesis. They include the basic information about the phenomena taking place in a process and their parameters usually have strictly physical meaning, so sometimes their values can be known a priori. At the same time, such models are usually more accurate and allow for easy inclusion of the process nonlinearities and of the influence of the disturbing inputs (e.g. Meadows and Rawlings, 1997; Murray-Smith and Johansen, 1997). The significant difficulty in applying the first principle models as a basis for the controller synthesis is their relatively large complexity. However, there are some techniques for simplifying the complex physical nonlinear models to the desired simplified form - see, e.g., Kokotovic et al. (1986), Bastin and Dochain (1990). Additionally, in recent years, the tools for model reduction have been developed (Donida et al., 2010, Falk, 2010) to support the preparation of the simplified model for control synthesis on the basis of the complex nonlinear dynamical one.

For both groups of models, the problem of modeling inaccuracies stands as the major difficulty in their application for practical synthesis of the model-based controllers. Thus, there is a need to implement one of techniques to decrease the influence of the modeling inaccuracies on the overall control performance. One possibility is the direct inclusion of the integral action in the model-based controller (e.g. Lee and Sullivan, 1988; Metzger, 2001), which is very efficient for the offset-free control, but introduces inconvenient dynamics into the control system. The other possibility is to lump all modeling inaccuracies with the unmeasured disturbance, which was suggested for Nonlinear Inferential Control (NIC) (Parrish and Brosilow, 1988; Brosilow and Joseph, 2002) or is well known in the applications of Model Predictive Control (MPC) as the additive disturbance estimate (e.g. Maciejowski, 2002; Tatjewski, 2007). It is also possible to use the mismatch between the process and the model to bias the set point as it is suggested in Internal Model Control (IMC) by Garcia and Morari (1982). The satisfying modeling accuracy for a strongly nonlinear processes can be also ensured by the nonstationary modeling. In this case, it is necessary to apply the on-line multiparameter identification for unknown parameters and the observer design methodology for some non measurable process variables to update their values on-line (e.g. Bastin and Dochain, 1990, Henson and Seborg, 1997). These techniques usually require the additional external excitation signals (Dasgupta et al., 1991; Nelles, 2001; Kravaris et al., 2012), and this approach results in the adaptive form of the model-based control law, which in the majority of cases can improve the control performance (Ästrom and Wittenmark, 1989).

In this paper, the general and simple additive modeling of the inaccuracies for the first principle models is presented. This unified input-affine form of such a model describes directly the dynamics of the controlled variable and it can be derived on the basis of the general heat or mass conservation law. It includes only one additive and unknown parameter that lumps all the modeling inaccuracies with the not measurable disturbances, which can be easily estimated on-line based only on the measurable data collected from the process. For the processes of the higher relative order, the practical simplification of this approach is also shown. The discussion on how to apply the suggested model as a basis for the chosen model-based control techniques is also presented and the experimental results show the practical applicability of the considered approach.

2. Problem statement - controller synthesis based on the first principle models

In this paper, the control of the output Y of the hypothetical nonlinear SISO system of the relative order $r \geqslant 1$ is considered. The system can be controlled by its manipulated variable u and the control goal is to track the variations of the set-point Y_{sp} and to reject the influence of the disturbances $\underline{\tilde{d}} \in R^D$. The dynamics is represented by the state vector $\underline{\tilde{x}} \in R^N$. The complete first principle nonlinear description for this system is given in the following nonlinear standard form:

$$\begin{cases} \frac{d\underline{\tilde{x}}}{dt} = F\left(\underline{\tilde{x}}, \underline{\tilde{d}}, u\right) \\ Y = h\left(\underline{\tilde{x}}\right) \end{cases} . \tag{1}$$

For the controller synthesis, the model (1) should be rearranged into the dynamical equation describing directly the dynamics of the controlled variable Y, e.g. by applying the input-output linearizing technique based on the Lie algebra (e.g., Isidori, 1989; Bastin and Dochain, 1990; Henson and Seborg, 1997):

$$\frac{d^r Y}{dt^r} = \tilde{H}_1\left(Y, \underline{\tilde{x}}, \underline{\tilde{d}}\right) + \tilde{H}_2\left(Y, \underline{\tilde{x}}, \underline{\tilde{d}}\right)\ u. \tag{2}$$

Generally, the controller synthesis is based on inverting the model (2), which is possible due to its affinity with respect to the manipulated variable u. One possibility is to assume the r-th order reference model that describes the desired stable closed-loop dynamics of the control error $e = Y_{sp} - Y$:

$$\frac{d^r e}{dt^r} + \sum_{k=0}^{r-1} \lambda_k \frac{d^k e}{dt^k} = 0. \tag{3}$$

Then, assuming the constant set-point Y_{sp}, after rearrangements, the following equation is obtained:

$$\frac{d^r Y}{dt^r} = \lambda_0 \left(Y_{sp} - Y\right) - \sum_{k=1}^{r-1} \lambda_k \frac{d^k Y}{dt^k}, \tag{4}$$

and it can be combined with the model (2) and solved for the manipulated variable u giving the final form of the linearizing controller (e.g., Isidori, 1989; Bastin and Dochain, 1990; Henson and Seborg, 1997):

$$u = \frac{\lambda_0 \left(Y_{sp} - Y\right) - \sum_{k=1}^{r-1} \lambda_k \frac{d^k Y}{dt^k} - \tilde{H}_1 \left(Y, \underline{\tilde{x}}, \underline{\tilde{d}}\right)}{\tilde{H}_2 \left(Y, \underline{\tilde{x}}, \underline{\tilde{d}}\right)}. \tag{5}$$

Readers should note that there are some limitations that must be faced when the model (2) is to be applied as a basis for the synthesis of the model-based controller (5):

- In practice, this model describes the real system only partially - there may be some phenomena taking place, which are not recognizable or for which the description cannot be suggested for different reasons. At the same time, some model parameters can be known inaccurately.
- All the states $\underline{\tilde{x}}$ and the disturbances $\underline{\tilde{d}}$ should be measurable on-line and so the suitable additional sensors are required in the control system. If some states are not measurable, they can be computed by any observer technique, but this requires complex calculations based on the complete form of the model (1). If this model is inaccurate, the practical applicability of such a technique is questionable.

These limitations are restrictive, but at the same time they are realistic, because the model (2), even if very detailed, is only an approximation of a much more complex reality. Consequently, due to potential modeling inaccuracies, the controller based on this model cannot ensure satisfying offset-free control performance without additional application of any adaptability or integral action. This difficulty can be solved by applying the nonstationary modeling, in which all the modeling inaccuracies are compensated by on-line updating of one or more model parameters. Potentially, in the modern control systems, many additional sensors are applied to collect the measurement data for the feedforward action from some of the system states and disturbances. This measurement data can be also applied as a source of information for the nonstationary modeling. However, for a multiparameter identification, some complex methods must be applied jointly with the intentional process upsets to ensure the desired level of excitation (Dasgupta et al., 1991; Nelles, 2001; Kravaris et al., 2012). If the simplicity is the priority and the additional process upsets are impossible, the solution is to update only a single model parameter to compensate for all modeling inaccuracies. In this case, only the single-parameter identification is used and the estimation complexity is significantly lower. This approach was already presented and discussed. Rhinehart and Riggs (1991) suggested to choose one of the model parameters, with respect to which the model is the most sensitive and which potentially represents major modeling inaccuracy. Bastin and Dochain (1990) proposed a relatively simple procedure, in which a single parameter representing intensity of the biological reaction is updated on-line. Bequette (1989) suggested to lump several uncertain model parameters that normally need to be estimated separately into a single unknown parameter and presented the procedure for its on-line updating.

The second possibility of providing the offset-free control on the basis of the model (2) is to derive the controller by the application of the IMC (Internal Model Control) framework. Its conventional linear form was proposed by Garcia and Morari (1982), but for the model (2), the nonlinear IMC approach from Economou et al. (1986) is relevant. The IMController is based on the direct inverse of the model (2) and all modeling inaccuracies are compensated by the closed loop system, in which the process model (1) must be computed on-line and the modeling error is applied for on-line adjusting of the set-point Y_{sp} to ensure the offset-free control. The practical difficulties result from the fact that this technique requires the application of the additional filter of the relevant order to provide robust control, whose synthesis, in the nonlinear case, is far from being trivial (Henson and Seborg, 1991). There is also a need to solve the model (1) on-line, numerically.

3. Additive inaccuracy modeling for first principle models

In this section, it is shown how the simple additive modeling of the model inaccuracies can be applied for the model (2) and, consequently, it is discussed, how this approach can be used for deriving the model-based controllers providing the offset-free control.

3.1. Additive compensation for modeling inaccuracies

For practical implementation, the model (2) can be simplified by removing the terms including not measurable states and/or disturbances and by compensating all modeling inaccuracies by lumping them into a single additive parameter R_Y estimated on-line:

$$\frac{d^r Y}{dt^r} = \underbrace{H_1\left(Y, \underline{x}, \underline{d}\right) + H_2\left(Y, \underline{x}, \underline{d}\right)\ u}_{\text{known part of the model}} - R_Y. \tag{6}$$

The functions $H_1(.)$ and $H_2(.)$ form the *known part* of this model because $\underline{x}$ and $\underline{d}$ denote only measurable states and disturbances, respectively. Apart from all the modeling inaccuracies, resulting from simplifications, the inclusion of the time-varying parameter R_Y additionally compensates for the measurement errors for measurable disturbances resulting from sensor inaccuracies and for the potential mismatch between the real relative order of the process dynamics and the relative order of the process model (6).

Application of the additive parameter R_Y in the model (6) has very similar functionality to the idea of the additive disturbance estimate used in the MPC technique (e.g. Maciejowski, 2002; Tatjewski, 2007). Readers should note that the suggested modeling simplifications are fully justified from the practical viewpoint because only the unknown terms are lumped into the parameter R_Y.

All the known terms that can be computed on-line on the basis of the measurement data, are included in the *known part* of the simplified model (6) and thus there is no possibility of any oversimplification. The user decides on the level of simplification, basing on the knowledge and on the technological conditions.

3.2. Estimation procedure

For the unified form of the model (6), the additive parameter R_Y, lumping all modeling inaccuracies, must be estimated on-line to ensure the robust offset-free control if the controller is derived on the basis of Eq. (6). Potentially, there are few possibilities for computing the estimate of R_Y considering the limitation of the upset–less model parameterization. Following the suggestions of Isaacs et al. (1992) or, more generally, the idea of the additive disturbance estimate given for MPC (e.g. Maciejowski, 2002; Tatjewski, 2007), the value of this unknown parameter can be computed at each time instant by the direct rearrangement of Eq. (6) as $R_Y = \frac{d^rY}{dt^r} - H_1(Y, \underline{x}, \underline{d}) - H_2(Y, \underline{x}, \underline{d})\ u$. However, in practice, such computation is based on the noisy measurement data, so the results would be very sensitive to this noise. Rhinehart and Riggs (1991) suggest two simple techniques for on-line updating of the chosen model parameters by applying the incremental Newton's method that can be directly used for the suggested form of the model (6). Another possibility is the application of the extended observer technique (e.g. Henson and Seborg, 1997) based on the models (2) and (6), in which the unknown parameter can be considered as the unmeasurable state variable with very simple state equation $\frac{dR_Y}{dt} = 0$. However, the significant drawback of this technique is its mathematical complexity. Van Lith et al. (2001) suggest the possibility of estimating the values of the unknown model parameters by the application of the PI controller, for which the set-point is the measured process output, the controlled variable is the model output and the manipulating variable is the unknown parameter. This technique is very simple and intuitive, and it can be directly applied for the suggested model (6), but due to its potential nonlinearities, the difficulties with proper PI tuning can be encountered.

The estimation procedure presented in this paper benefits from the general form of the simplified model (6), which always ensures the affinity with respect to the unknown parameter R_Y. After discretization of Eq. (6), the estimate $\hat{R}_Y$ of the unknown parameter R_Y is described by the following equation:

$$-T_R^r \hat{R}_{Y,i} = \underbrace{\nabla_{T_R}^r[Y] - T_R^r(H_{1,i} + H_{2,i}\, u_i) + \varepsilon_i}_{w_i} = -T_R^r R_{Y,i} + \varepsilon_i, \tag{7}$$

where i denotes the i-th sampling, T_R is the discretization instant, $H_{1,i} = H_1(Y_i, \underline{x}_i, \underline{d}_i)$, $H_{2,i} = H_2(Y_i, \underline{x}_i, \underline{d}_i)$ and $\nabla_{T_R}^r[Y]$ represents the r-th order finite backward difference operator. Due to the presence of the measurement noise, represented by the additive error ε, Eq. (7) is not recommended for the direct calculation of the estimate $\hat{R}_Y$, and thus the estimation procedure based on the WRLS (Weighted Recursive Least-Squares) method is applied to minimize the influence of this noise on the estimation accuracy (Czeczot, 1998). Eq. (7) defines the measurable auxiliary variable w and it represents the linear equation affine to the unknown parameter $\hat{R}_Y$ with the constant regressor $(-T_R^r)$. Consequently, it allows for the application of the simplified scalar discrete-time form of the well known WRLS equations, where $\alpha \in (0,1)$ denotes the forgetting factor:

$$P_i = \frac{P_{i-1}}{\alpha + T_R^{2r} P_{i-1}}, \tag{8}$$

$$\hat{R}_{Y,i} = \hat{R}_{Y,i-1} - T_R^r P_i \left(w_i + T_R^r \hat{R}_{Y,i-1} \right), \tag{9}$$

with the initial values: $P_0 > 0$ and a freely but reasonably chosen $\hat{R}_{Y,0}$. The properties of the estimation procedure (8), (9) are shown and discussed below.

THEOREM 1 *The value of P_i described by the recursive formula (8) converges to the stable equilibrium point $P_\infty = (1-\alpha)/T_R^{2r}$ if only the initial value $P_0 \neq 0$.*

Proof. Eq. (8) does not depend on the form of the model (6) in any way. The value of P_i is computed iteratively at each time instant, starting from the initial value P_0. Application of the mathematical induction shows that Eq. (8) can be written as:

$$P_i = \frac{P_0}{\alpha^i + T_R^{2r} P_0 \sum\limits_{k=0}^{i-1} \alpha^k}. \tag{10}$$

For $\alpha < 1$, which is requested for WRLS method, for $i \to \infty$ the following formulas hold:

$$\lim_{i\to\infty} \alpha^i = 0, \tag{11}$$

$$\lim_{i\to\infty} \sum_{k=0}^{i-1} \alpha^k = \sum_{k=0}^{\infty} \alpha^k = \frac{1}{1-\alpha}. \tag{12}$$

Consequently, if only $P_0 \neq 0$, the value of P_∞ can be calculated as follows:

$$P_\infty = \lim_{i\to\infty} P_i = \lim_{i\to\infty} \frac{P_0}{\alpha^i + T_R^{2r} P_0 \sum\limits_{k=0}^{i-1} \alpha^k} = \frac{1-\alpha}{T_R^{2r}}, \tag{13}$$

which shows that the value of P_i always converges to the value of P_∞, described by Eq. (13). In fact, in the practical applications, the value of P_i converges very fast to P_∞ and remains constant during the whole operating time. □

Theorem 2 *Regardless of the initial value of $\hat{R}_{Y,0}$, the estimate $\hat{R}_{Y,i}$ always converges to the stable equilibrium point $\hat{R}_{Y,\infty} \to R_{Y,\infty} = H_{1,\infty}(.) + H_{2,\infty}(.)\,u_\infty$ without any additional excitation signal, where subscript ∞ denotes the steady state values.*

Proof. For $i >>1$, Eq. (9) can be rearranged into the following form by substituting for P_i its limit value P_∞, given by Eq. (13):

$$\hat{R}_{Y,i} = \hat{R}_{Y,i-1} - T_R^r P_\infty \left(w_i + T_R^r \hat{R}_{Y,i-1} \right) = \hat{R}_{Y,i-1} - \frac{1-\alpha}{T_R^r} \left(w_i + T_R^r \hat{R}_{Y,i-1} \right). \tag{14}$$

Then, after simple rearrangements, Eq. (14) can be written as:

$$\hat{R}_{Y,i} = \alpha\, \hat{R}_{Y,i-1} - \frac{1-\alpha}{T_R^r} w_i. \tag{15}$$

Based on Eq. (7), the measurable value $w_i = -T_R R_{Y,i} + \varepsilon_i$, so Eq. (15) can be rearranged as:

$$\hat{R}_{Y,i} = \alpha\, \hat{R}_{Y,i-1} + (1-\alpha) \left(R_{Y,i} - \frac{\varepsilon_i}{T_R^r} \right), \tag{16}$$

which clearly shows that for the suggested scalar form (8),(9), the WRLS procedure has the time invariant linear dynamics of the first order with the unitary gain and the time constant depending on the forgetting factor α. The estimate $\hat{R}_Y$ tracks its true value R_Y and, at the same time, it ensures the filtering of the additive measurement error ε. For the stability of Eq. (16) it is only required that $\alpha \in (0,1)$, which is always requested for the WRLS method. □

Therefore, the convergence of the estimation procedure (8), (9) was proved. It ensures accurate estimation without the necessity of applying any additional excitation input signals. In fact, even at the steady state, the estimate $\hat{R}_Y$ always converges to its true value R_Y with the rate of convergence depending only on the value of the forgetting factor α. The significant practical difficulty results from the necessity of the on-line calculation of the backward finite differences $\nabla_{T_R}^r [Y]$ in Eq. (7), based on the noisy measurement data.

The estimation procedure (8), (9), requires adjusting initial values for P_0 and $\hat{R}_{Y,0}$, but it was shown that these values do not influence the estimation convergence if only $P_0 \neq 0$. Thus, $P_0 = 1$ can be adjusted because P_i converges to P_∞ very fast. The choice of $\hat{R}_{Y,0}$ is not very crucial and it can be adjusted arbitrarily at any reasonable value, e.g. as $\hat{R}_{Y,0} = 0$. Then, the estimate $\hat{R}_Y$ converges to its true value with the rate that can be adjusted by quantifying the value of the forgetting factor α (if $\alpha \to 0$, the convergence is very fast). Thus, the problem of adjusting the value of $\hat{R}_{Y,0}$ is not significant if the preliminary transient degrading the accuracy of the initial stage of the estimation is acceptable. However, in the closed loop systems with a controller based on the simplified model (6), first, the estimation procedure should be run in the open loop to allow for its convergence, and then the loop can be closed.

3.3. Application for the offset-free control

The potential application of the model (6) for synthesis of the multistep predictive controllers is very limited, because this methodology must be based on a model that predicts the process output only on the basis of the assumed variations of some current and past process inputs. For the model (6), the compensation of all modeling inaccuracies is made by the on-line update of the estimate $\hat{R}_Y$ on the basis of the current measurement data, which is somehow similar to the additive disturbance estimate suggested for the Model Predictive Control technique. However, the model (6) could be used for the prediction of the future process behavior only assuming constant value of the estimate $\hat{R}_Y$ over the whole prediction horizon, which is equivalent to the case when only its *known part* were applied for this prediction, without any compensation of the modeling inaccuracies.

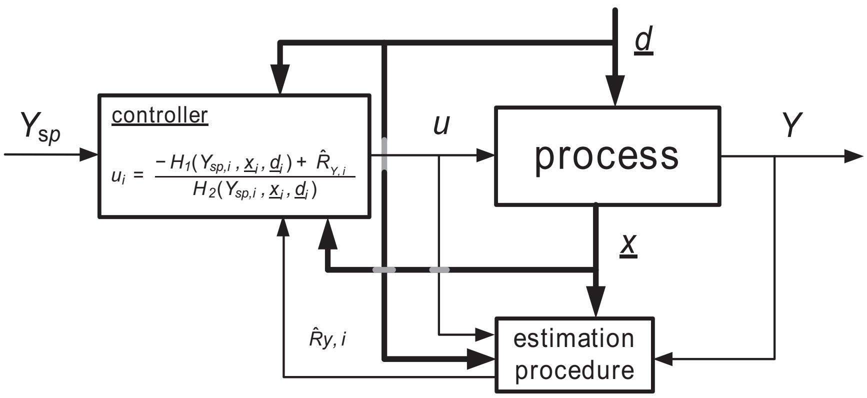

Figure 1. Open loop IMC structure based on the model (6)

The model (6) benefits from the on-line compensation of the modeling inaccuracies and for this case, the open loop IMC-based control system presented in Fig. 1 is proposed in this paper. The IMController is derived basing on the complete form of the model (6) but, contrary to the conventional IMC framework, the estimation procedure provides the feedback from the modeling inaccuracy.

The third possibility is to apply the linearizing technique, presented in Section 2, to the model (6). After replacing the unknown parameter R_Y by its estimate $\hat{R}_Y$, computed on-line by the procedure (8),(9), and discretization, the following linearizing controller for the nonlinear processes of the dynamics of the r-th order can be derived:

$$u = \frac{\lambda_0 \left(Y_{sp} - Y\right) - \sum_{k=1}^{r-1} \lambda_k \frac{\nabla_{T_R}^k[Y]}{T_R^k} - H_{1,i} + \hat{R}_{Y,i}}{H_{2,i}}\,. \tag{17}$$

Both latter approaches ensure the offset-free control when the controllers are implemented jointly with the estimation procedure (8). However, there are still few difficulties to cope with for the practical implementation:

- mathematical complexity of deriving the model (6) based on the model (1) can be too high for industrial engineers,
- implementation of the controller (17) requires computing the backward finite differences $\nabla_{T_R}^{k}[Y]$ on the basis of the noisy measurement data.

Additionally, for the linearizing controller (17), its tuning for the processes of higher relative order $r>1$ is difficult - it requires adjusting r tuning parameters: λ_0 .. λ_{r-1} for the controller itself and the forgetting factor α for the estimation procedure (8).

4. Practical model simplification

The difficulties in the practical application of the suggested IMController (Fig. 1) and of the linearizing controller (17) result from the high relative r-th order of the model (6). They can be overcome by simplifying the model (6) according to Eq. (18) that assumes the reference model of the unitary relative order:

$$\frac{dY}{dt} = \underbrace{f\left(Y, \underline{x}, \underline{d}\right) + g\left(Y, \underline{x}, \underline{d}\right)\, u}_{\text{known part of the model}} - R_Y. \tag{18}$$

Now, the additive parameter R_Y additionally compensates for the difference between the unitary relative order of the model (18) and the relative order of the real process.

The form of the model (18) can be considered as the extension of the approach based on the so-called substrate consumption rate (Czeczot, 1998, 2007) and, potentially, there are two possibilities of its derivation. If the model (6) is known, the functions $f(.)$ and $g(.)$ can be directly defined as $f(.)=H_1(.)$ and $g(.)=H_2(.)$. However, if deriving the model (6) is impossible or too complex, the simplified model (18) can be derived as the simple input-output nonlinear model, based on the very general first principle considerations written for the controlled system, e.g. Czeczot (1998, 2001, 2006, 2006a). Then, after simple rearrangements, this equation can be rewritten in the form of Eq. (18) with all the unknown terms lumped into the single parameter R_Y.

Compared to other methods of updating the model parameters, discussed above, this approach has the following significant advantages:

- Applying the unknown parameter R_Y as the rate representation is more natural as the complement for the first principle based synthesis of the *known part* of the model (6), even if it is included only in order to represent the potential modeling uncertainties.
- The sensitivity of the model output to the variations of the unknown parameter R_Y is always constant, which protects from the potentially ill-conditioned numerical calculations during on-line estimation.

- Representing the modeling inaccuracies as the single additive unknown rate parameter R_Y always allows for the very clear distinction between the known and unknown parts of the model (6). Consequently, this model always can be defined in this general form, which allows for the synthesis of the general form of the procedure for updating the value of the parameter R_Y on-line. This feature is especially important for the practical PLC-based implementation of any controller based on this model in the form of the general purpose encapsulated function block (Klopot et al., 2012).

For the model (18), the estimate $\hat{R}_Y$ can be calculated by the same estimation procedure (8), (9), but readers should note the simplification - only the first order backward difference $\nabla^1_{T_R}[Y]$ must be computed, due to the unitary relative order of the model (18). Consequently, it is possible to derive the offset-free controllers based on the model (18) in the same way as it was presented in Section 3.3 for the model (6). For the IMC-based open loop control, the only difference results from simplifying the complexity of the estimation procedure (8), (9). The application of the linearizing technique to the model (18), in the form dedicated to control the systems of the unitary relative order (Bastin and Dochain, 1990), leads to the Balance-Based Adaptive Controller (B-BAC) proposed by Czeczot, e.g. Czeczot (2001, 2006), whose dynamical properties are discussed in details in Stebel et al. (2014). This methodology benefits from the linearizing technique, but very similar form of the controller can be obtained by applying the one-step ahead prediction control from Bequette (1989), or the Process Model-Based Control (PMBC) technique from Rhinehart and Riggs (1990).

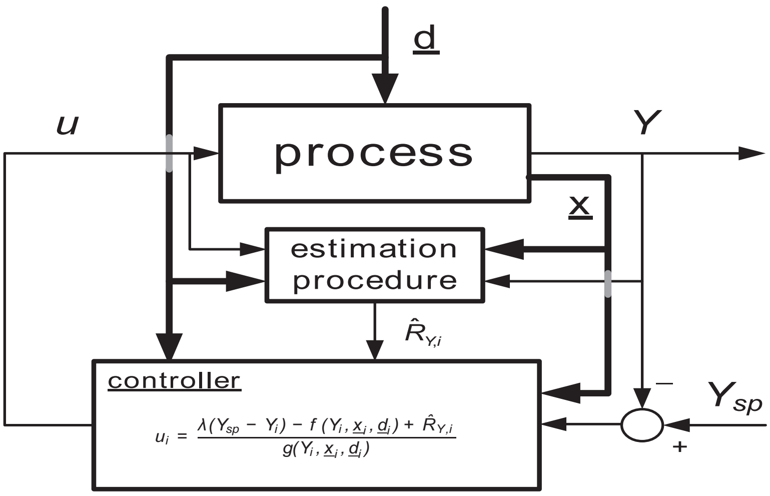

Figure 2. Control system with the B-BAController

The respective closed loop control system with the B-BAController is presented in Fig. 2. In comparison with the linearizing controller (17), its tuning is much simpler because there are only two tuning parameters: λ for the controller and the forgetting factor α for the estimation procedure.

5. Practical experiments

In this section, the practical validation of two considered controllers derived on the basis of the model (18) is presented. The investigations are shown in the application to the practical illustrative example: the electric flow heater presented in Fig. 3. The supplying water flows through its vessel with adjustable and measurable flow rate $F = 0.5$ [L/min] and measurable inlet temperature T_{in} [^{o}C]. The power supply P_h can be adjusted in the range between 0% and 100% of the nominal power $P_{nom} = 5500$ [W]. The heated water flows out with the same flow rate F and with the measurable outlet temperature T_{out} [^{o}C]. The volume $V = 0.25$ [L] is constant and the heater is well insulated while the pipe at its outlet is not insulated so the heat loss takes place in this part of the system.

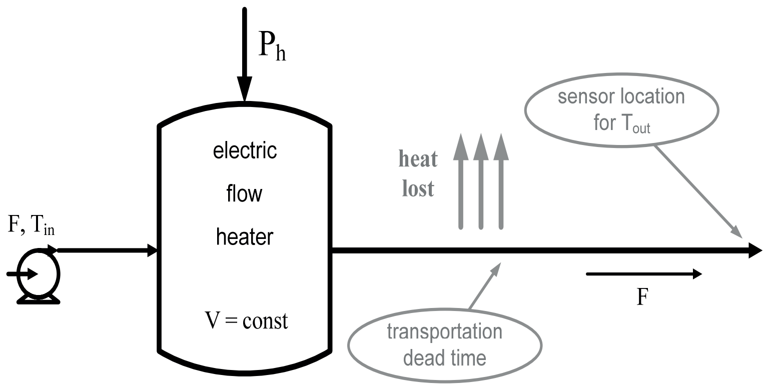

Figure 3. Simplified diagram of the electric flow heater

In this system, the controlled output is defined as $Y = T_{out}$ and the manipulated variable as $u = P_h$. The disturbances are the inlet temperature and the flow rate $\underline{d} = [T_{in}F]^T$. Apart from imperfect mixing in the heater and from the existing heat loss in the pipe, a significant transportation dead time appears due to the location of the sensor for the outlet temperature T_{out} at the end of the pipe of the diameter 5 [cm], approximately at the distance of 1.5 [m] from the heater outlet.

For simplified modeling, it is assumed that the whole system is perfectly insulated and mixed. Then, the following simplified model in the form of Eq.

(18) is derived by the basic heat conservation law:

$$\frac{dY}{dt} = \underbrace{\overbrace{\frac{F}{V}(T_{in} - Y)}^{f(.)} + \overbrace{\frac{P_{nom}}{V\rho c_S\,100\%}}^{g(.)} P_h}_{\text{known part of the model}} - R_Y, \tag{19}$$

where $\rho = 1$ [kg/L] and $c_S = 4200$ [J/kg ^{o}C] denote, respectively, the density and the specific heat of the flowing water. The unknown parameter R_Y represents the modeling inaccuracy resulting from the unknown description for the heat lost, from the possible imperfect mixing and from the presence of the transportation dead time, not included in the simplified model (19) and varying according to the variations of the disturbing flow rate F.

The experiments were carried out under the same scenario. Starting from the operating point defined by the flow rate $F = 2$ and the set-point $Y_{sp} = 30$, the indicated successive step changes of the set point Y_{sp} and of the disturbing flow rate F were applied to the system.

Fig.4 shows the control performance of the IMC-based control system (IMC_1) based on the model (19) and presented generally in Fig.1. It is compared with the conventional closed loop nonlinear IMController (IMC_2) based only on the inaccurate *known part* of the model (19). Due to the IMC closed loop structure, the latter ensures offset-free tracking and disturbance rejection. However, its performance is more oscillatory compared to (IMC_1), which also ensures offset-free tracking and disturbance rejection but, at the same time, the transients are significantly smoother.

Fig. 5 shows the performance of the B-BAController with the tunings $\lambda = 0.1$ and $\alpha = 0.1$, derived on the basis of the suggested model (19) and implemented in the closed loop structure presented in Fig. 2. For comparison, the control performance of the conventional linearizing controller based only on the *known part* of the model (19) is also presented. As it was expected, the linearizing controller fails with the tracking due to the significant modeling inaccuracies. These inaccuracies are successfully compensated by the on-line estimation of the parameter R_Y and the B-BAController provides satisfying offset-free tracking and disturbances rejection with smooth transients.

6. Concluding remarks

In this paper, it is shown how the simple additive modeling of the model inaccuracies can be applied to derive the model-based controllers that provide the offset-free regulation without the direct integral action for the systems of higher relative order. The *known part* of the simplified model is complemented with the single additive parameter R_Y, which lumps all modeling inaccuracies. This approach requires the on-line feedback from the measurement data of the process output and of the measurable process states and disturbances. The unknown parameter R_Y is estimated on-line by the unified scalar form of the

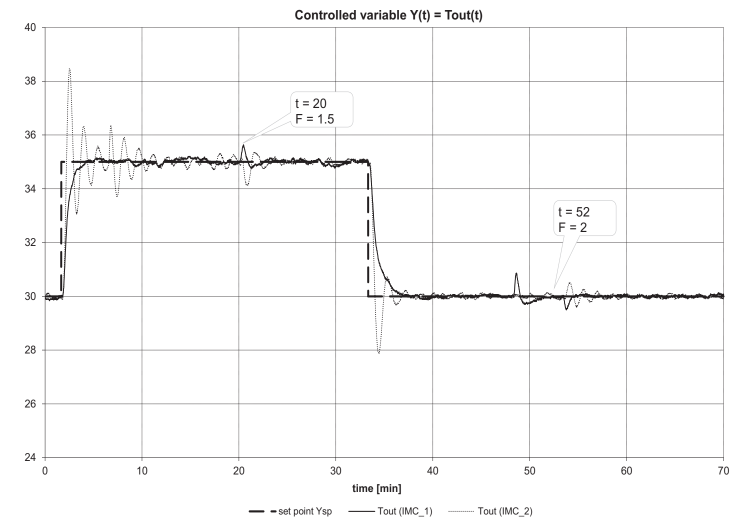

Figure 4. Control performance of the open loop IMController (IMC_1) for the electric flow heater, in comparison with the conventional nonlinear IMC closed loop controller (IMC_2) based only on the *known part* of the model (19)

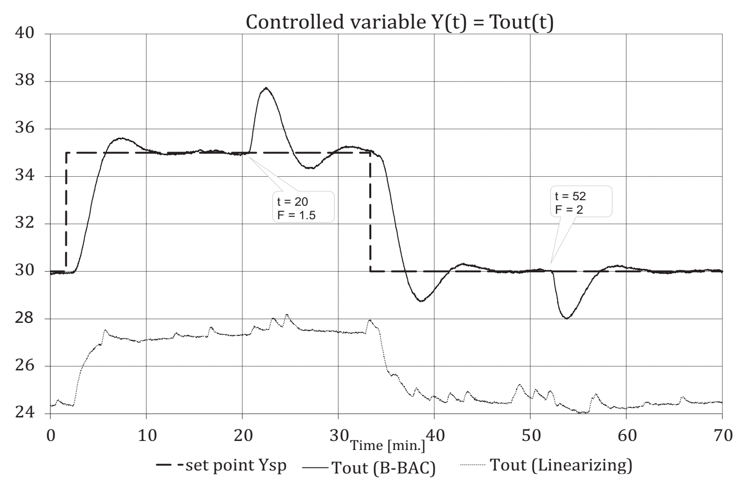

Figure 5. Control performance of the B-BAController for the electric flow heater, in comparison with the conventional linearizing controller based only on the *known part* of the model (19)

WRLS estimation procedure, which ensures high modeling accuracy irrespective of the level of the model simplifications.

The relevance of this approach for the controller synthesis is presented in the paper for two model-based techniques with the model of the process being additionally simplified to the unitary relative order: the open loop IMC-based controller and the B-BAController. In both cases, the experimental results show that the control systems based on the respective approaches provide very good control performance, which results from the compensation properties of the estimation procedure and from the fact that the suggested model introduces the feedforward action to the final form of the control law without any additional effort (Stebel et al., 2014).

Acknowledgement

This work was supported: partially by the National Science Center, project no. N N514 146438, 2010-2012 and partially by the Ministry of Science and Higher Education under grant BK-UiUA.

Referenes

ÄSTROM, K.J. and WITTENMARK, B. (1989) *Adaptive Control*. Addison–Wesley Publishing Company, Reading.

BASTIN, G. and DOCHAIN, D. (1990) *On-line Estimation and Adaptive Control of Bioreactors.* Elsevier Science Publishers, Amsterdam.

BEQUETTE, B.W. (1989) A one-step-ahead approach to nonlinear process control. In: *Proc. of ISA/89 International Conference*. ISA - International Society for Measurment and Control, Philadelphia, 711–717.

BROSILOW, C. and JOSEPH, B. (2002) *Techniques of Model-Based Control*. Prentice Hall.

CUTLER, C.R. and RAMAKER, B.C. (1980) Dynamic Matrix Control – A Computer Control Algorithm. *American Control Conference*. American Institute of Chemical Engineers, San Francisco, USA, paper WP5–BJACC.

CZECZOT, J. (1998) Model-based adaptive control of fed-batch fermentation process with the substrate consumption rate application. In: R.R. Bitmead, M.A. Johnson and M.J. Grimble, eds., *Adaptive Systems in Control and Signal Processing 1998: Proc. of the IFAC Workshop, Glasgow, UK*. IFAC Proceedings Volumes, 357–362.

CZECZOT, J. (2001) Balance-Based Adaptive Control of the Heat Exchange Process. In: *Proc. of 7^{th} IEEE International Conference on Methods and Models in Automation and Robotics MMAR*, Międzyzdroje. Wydawnictwo Uczelniane Politechniki Szczecińskiej, 853–858.

CZECZOT, J. (2006) Balance-Based Adaptive Control Methodology and its Application to the Nonlinear CSTR. *Chemical Eng. and Processing* **45**(4), 359–371.

CZECZOT, J. (2006a) Balance-Based Adaptive Control of a Neutralization Process. *International Journal of Control* **79**(12), 1581–1600.

CZECZOT, J. (2007) On the minimum form of the balance-based adaptive controller. In: *Proc. of 13th IEEE International Conference on Methods and Models in Automation and Robotics MMAR*. Przedsiębiorstwo Produkcyjno–Handlowe ZAPOL, Szczecin, Poland, 445–450.

DASGUPTA, S., SHRIVASTAVA, Y. and KRENZER G. (1991) Persistent excitation in bilinear systems. *IEEE Trans. on Automatic Control* 36, 305–313.

DONIDA, F., CASELLA, F. and FERRETTI, G. (2010) Model order reduction for object-oriented models: a control system perspective, *Math. Computer Model. Dyn. Syst.* **16**(3), 269–284.

ECONOMOU, C.G., MORARI, M. and PALSSON, B.O. (1986) Internal Model Control. Extension to Nonlinear Systems. *Ind. Eng. Chem. Process Des. Dev.*, 25, 403-411.

FALK, E. (2010) A note on POD model reduction methods for DAEs. *Math. Computer Model. Dyn. Syst.* **16**(2), 115–131.

GARCIA, C.E. and MORARI, M. (1982) Internal model control 1. A unifying review and some new results. *Ind. Engng. Chem. Process. Des. Dev.* 21, 308.

HENSON, M.A. and SEBORG, D.E. (1991) An internal model control strategy for nonlinear systems. *AIChE Journal* **37**(7), 1065-1081.

HENSON, M.A. and SEBORG, D.E. (1997) *Nonlinear Process Control*. Prentice Hall.

ISAACS, S.H., SOEBERG, H. and KUMMEL, M. (1992) Monitoring and Control of a Biological Nutrient Removal Processes: Rate Data as a Source of Information. In: *Proc. of IFAC Modelling and Control of Biotechnological Processes*. IFAC Publications, Colorado, USA, 239–242.

ISIDORI, A. (1989) *Nonlinear Control Systems: An Introduction.* 2nd edition. Springer Verlag.

KLATT, K.U. and MARQUARDT, W. (2009) Perspectives for process systems engineering - Personal views from academia and industry. *Computers and Chemical Engineering* 33, 563–550.

KLOPOT, T., CZECZOT, J. and KLOPOT, W. (2012) Flexible function block for PLC-based implementation of the Balance-Based Adaptive Controller. In: *Proc. American Control Conference ACC 2012*, Montreal, Canada. Institute of Electrical and Electronics Engineers, Piscataway, 6467–6472.

KOKOTOVIC, P., KHALIL, H.K. and O'REILLY, J. (1986) *Singular Perturbation Methods in Control: Analysis and Design.* Academic Press, London.

KRAVARIS, C., HAHN, J. and CHU, Y. (2012) Advances and selected recent developments in state and parameter estimation. *Comp. Chem. Engng.* 51, 111-123.

LEE, P.L. and SULLIVAN, G.R. (1988) Generic model control (GMC). *Comp. Chem. Engng.* **12**(6), 573–580.

MACIEJOWSKI, J.M. (2002) *Predictive Control with Constraints.* Prentice Hall.

METZGER, M. (2001) Easy programmable MAPI controller based on simplified process model. In: *Proc. of the IFAC Workshop on Programmable Devices and Systems*, Gliwice, Elsevier, 166-170.

MEADOWS, E.S. and RAWLING, J.B. (1997) *Model Predictive Control. Nonlinear Process Control.* Prentice Hall, Upper Saddle River, New Jersey.

MURRAY-SMITH, R. and JOHANSEN, T.A. (1997) *Multiple Model Approaches to Modeling and Control.* Taylor and Francis, New York.

NELLES, O. (2001) *Nonlinear System Identification: From Classical Approaches to Neural Networks and Fuzzy Models.* Springer Verlag, Berlin - Heidelberg - New York.

PARRISH, J.R. and BROSILOW, C.B. (1988) Nonlinear inferential control. *AIChE Journal* 34, 633-644.

RHINEHART, R.R. and RIGGS, J.B. (1990) Process Control through Nonlinear Modeling. *Control* **3**(7), 86.

RHINEHART, R.R. and RIGGS, J.B. (1991) Two simple methods for on-line incremental model parameterization. *Comp. Chem. Engng* **15**(3), 181–189.

RHINEHART, R.R., DARBY, M.L. and WADE, H.L. (2011) Editorial - Choosing advanced control. *ISA Transactions* 50, 2-10.

RICHALET, J. (1993) *Pratique de la commande predictive.* Hermes, Paris.

RICHALET, J., RAULT, A., TESTUD, J.L. and PAPON, J. (1978) Model Predictive Heuristic Control: Application to Industrial Processes. *Automatica* **14**(2), 413–428.

SEBORG, D.E. (1999) A perspective on advanced strategies for process control. *ATP* **41**(11), 13–31.

STEBEL, K., CZECZOT, J. and LASZCZYK, P. (2014) General tuning procedure for the nonlinear balance-based adaptive controller. *International Journal of Control* **87**(1), 76–89.

TATJEWSKI, P. (2007) *Advanced Control of Industrial Processes. Structures and Algorithms.* Springer Verlag, London.

VAN LITH, P.F., WITTEVEEN, H., BETLEM, B.H.L. and ROFFEL, B. (2001) Multiple nonlinear parameter estimation using PI feedback control. *Control Engineering Practice* 9, 517–531.