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A three level integrated inventory model with time dependent demand and production rate under a trade credit policy for both distributor and retailer*

by

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Abstract: This paper develops a three echelon supply chain inventory model with permissible delay in payment, in which distributor and retailer's demand is time dependent and production rate for manufacturer is also time sensitive. The models consider the two level trade credit policy in manufacturer-distributor and distributor-retailer relationship in this supply chain model. A simple solution algorithm is presented to determine the optimal order quantity and optimal cycle time of the total cost function and the number of shipments for distributor and retailer. The results are discussed with numerical examples and the particular cases of the model are also discussed in brief.

Keywords: time dependent demand, variable production rate, three echelon supply chain, trade credit

1. Introduction

A supply chain is a network of retailers, distributors, manufacturers, and suppliers, cooperating so as to satisfy the customers' demand. In traditional business environment, each business tries to minimize its costs based on its own cost structure, regardless of other supply chain participants. However, most decisions that are made by each supply chain member have direct and indirect impacts on the profitability of other chain members. For instance, a suitable order size for a buyer, minimizing her costs, can be unsuitable for the supplier and lead to respective extra costs. In another instance, a buyer's decision about the customer service level is based on the buyer's perspective, while shortages at the retailer's site have direct impact on the upstream profitability by changing the sales amount. Therefore, it appears to be essential to make decisions that are based on the profitability of the entire supply chain.

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Traditionally, inventory models considered the different subsystem in the supply chain independently. With the recent advances in communication and information technologies, the integration of these function is common phenomenon. Moreover, due to limited resources, increasing competition and market globalization, enterprises are forced to develop supply chains that can respond quickly to customer needs with minimum stock and minimum service level. Regarding the cooperation between manufacturers and retailers, Ishii et al. (1988) considered a three echelon system with one manufacturer, one wholesaler and one retailer. Then, Haq et al. (1991) considered a three echelon system with one production facility, several warehouses and several retailers. Goyal and Nebebe (2000) considered a problem of determining economic production level and shipment policy of a product from a vendor to a buyer. Woo et al. (2001) investigated an integrated inventory system, where a vendor purchases and processes raw materials and delivers the finished items to multiple buyers. Rau et al. (2003) developed a multi-echelon inventory model for a deteriorating item and derived an optimal joint total cost from an integrated perspective involving the suppliers, the producers and the buyers. Hangs et al. (2006) presented a new methodology for obtaining the joint economic lot size in a distribution system with multiple shipment policy. Singh et al. (2010) developed an EOQ model with Pareto distribution for deterioration, trapezoidal type demand and backlogging under trade credit policy. Singh (2011) analyzed an optimal replenishment policy for ameliorating item with shortage under inflation and time value of money using genetic algorithm.

We address in this paper a three echelon supply chain with linearly increasing time dependent demand rate, production rate, and permissible delay in payments. Jaber and Goyal (2008) considered channel coordination in a three-level supply chain. They assumed that both demand and supply are certain. He et al. (2009) analyzed a condition, in which the stochastic market demand is sensitive to both retail price and sales effort. Singh (2010) discussed supply chain models with imperfect production process and volume flexibility under inflation. Singh and Singh (2010) developed a supply chain inventory model with stochastic lead time under imprecise partial backlogging and fuzzy ramp type demand for expiring items, while Chen and Bell (2011) investigated a channel that consists of a manufacturer and a retailer, where the retailer simultaneously determines the retail price and order quantity, while experiencing customer returns and price dependent stochastic demand. They proposed an agreement that includes two buyback prices, one for unsold inventory and one for customer returns and show that this revised returns policy can achieve perfect supply-chain coordination and lead to a win–win situation. Singh and Vishnoi (2013) developed a supply chain inventory model with price-dependent consumption rate, with ameliorating and deteriorating items and two levels of storage.

In today's competitive business transactions, it is common for the manufacturer to offer a certain fixed credit period to the distributor/retailer for stimulating his/her demand. During this credit period the distributor can accumulate the revenue and earn interest on that revenue. However, beyond this period the manufacturer charges interest on the unpaid balance. Hence, a permissible delay indirectly reduces the cost of holding stock. On the other hand, trade credit offered by the manufacturer encourages the distributors, as well as retailers, to buy more. Thus, it is also a powerful promotional tool that attracts new customers, who consider it as an alternative incentive policy to quantity discounts. Goyal (1985) first developed mathematical model with permissible delay in payments to determine order quantity. Afterwards, several studies were proposed to improve Goyal's (1985) model. Aggarwal and Jaggi (1995) extended Goyal's model to determine the EOQ for deteriorating items. Chung (1998) also presented a simple procedure to determine the EOQ in the Goyal's model. Abad and Jaggi (2003) developed a seller-buyer model with a permissible delay in payments, using game theory to determine the optimal unit price and the credit period, considering that the demand rate is a function of retail price. Ouyang et al. (2006) performed a study on an inventory model for the non-instantaneously deteriorating items with permissible delay in payments. Goyal et al. (2007) established optimal ordering policies for the case when the supplier provides a progressive interest-payable scheme. Liao (2008) studied the case of the deteriorating items under the two-level trade credit. Thangam and Uthaykumar (2009) developed the two-echelon trade credit financing model for perishable items in a supply chain, when demand influences both the selling price and the credit period. Teng et al. (2011) obtained the retailer's optimal ordering policy when the supplier offers a progressive permissible delay in payments. Liu and Cruz (2012) proposed supply chain networks with corporate financial risks and trade credits under economic uncertainty. Soni (2013) discussed optimal replenishment policies for deteriorating items with stock sensitive demand under two-level trade credit and limited capacity. Chung and Cardenas-Barron (2013) simplified the solution procedure for deteriorating items under stock dependent demand and two level trade credits in the supply chain management. Lou and Wang (2013) developed a comprehensive extension of an integrated inventory model with ordering cost reduction and permissible delay in payments. Ouyang et al. (2013) have discussed a comprehensive extension of the optimal replenishment decision policies under two levels of trade credit policy, depending on the order quantity. Wu et al. (2014) have developed a supplier-retailer-buyer supply chain inventory model with optimal credit period and lot size for deteriorating items, with expiration dates, under two-level trade credit financing. Chung et al. (2014) have established a new economic production quantity (EPQ) inventory model for deteriorating items under two levels of trade credit, in which the supplier offers to the retailer a permissible delay period and, simultaneously, the retailer, in turn, provides a maximal trade credit period to its customers in a supply chain system comprised of three stages.

In the present paper, we discuss a three-echelon supply chain inventory model with time dependent production and demand rate under permissible delay in payments. We consider here a manufacturer, a distributor, and a retailer, and in order to encourage the sales, the distributor provides a delay period to the retailer. We determine the optimal cycle time, optimal order quantity and optimal payment time. Finally, numerical examples are given to illustrate the result and the managerial insights are also obtained.

2. Assumptions and notations

The following assumptions and notations are used for the single channel multi-echelon supply chain system with trade credit consideration.

2.1. Assumptions

1. The retailer's ordering quantity from distributor has to be on JIT basis that may require small and frequent replenishment basis and all shipments are of equal basis.
2. Demand and production rate are time dependent as $D = a+bt$ and $P = kD$, where a, $b \geqslant 0$, $a > b$ and $k > 1$.
3. Lead time is zero.
4. The two-level trade credit policy is adopted. The manufacturer provides trade credit period to the distributor, distributor provides a credit period to the retailer. Here, the distributor and the retailers are subject to pay the interest on the purchased amount if the account is not settled before the delay period expires. During the time the account is not settled, the generated sales revenue is deposited in an interest-bearing account.
5. The integrated model deals with a single manufacturer, single distributor and single retailers for a single product.
6. The time horizon is infinite, so as to reflect the long-term cooperative relationship.
7. Trading partners in supply chain operate based on the elements of collaboration and specifically trust mutuality, information exchange, openness and communication, in order to establish the long term cooperative relationship and remove the risk of opportunistic behaviour.
8. $S_d \geqslant S_m$, $S_r \geqslant S_d$, $I_e \geqslant I_p$, $N \geqslant M$, where S_m, $S_d S_r$ are the unit selling prices for manufacturer, distributor and retailer, I_e, I_p are interest earned and interest payable, and N, M are permissible delay period.

2.2. Notations

Manufacturer's parameters

D annual demand rate such that $D = a + bt$, where $a, b \geq 0$
P annual production rate of manufacturer, given as $P = kD$, where $k > 1$
A_m fixed production setup cost per lot size
h_m stock holding cost per unit per year (\$ / unit/ year)
τ_m transportation cost of a shipment from manufacturer to supplier
$I_{m_1}(t)$ inventory level, which changes with time t during production period
$I_{m_2}(t)$ inventory level, which changes with time t during non-production period
T common cycle time of production/ordering cycle
S_m unit selling price per item of good quality

I_m annual interest rate for calculating the manufacturers opportunity interest loss due to the delay in payment
TAC_m annual total relevant cost of the manufacturer

Distributor's parameters
A_d distributor's ordering cost per shipment
h_d stock holding cost per unit per year (dollar/unit/year)
τ_{d1} transportation cost of receiving a shipment from the manufacturer
τ_{d2} transportation cost of the distributor of delivering a shipment to the retailer
N distributor's permissible delay period offered by the manufacturer to the distributor in a year
n number of shipments per order from the manufacturer to the distributor, $n \geqslant 1$
$I_d(t)$ inventory level, which changes with time t during the period T_3
T_3 replenishment time interval, such that $T_3 = T/n$
I_0 annual interest rate for calculating the interest relevant for the distributor
S_d unit selling price per item of good quality
Q_d shipment quantity from manufacturer to distributor in each shipment (units)
TAC_d annual total relevant cost of the distributor

Retailer's parameters
A_r retailer's ordering cost per contract
h_r stock holding cost per unit per year ($/unit/year)
τ_r fixed transportation cost of receiving a shipment from distributor ($/shipment)
Q_r shipment size from the distributor to the retailer in each shipment (units)
S_r unit selling price per item of good quality
m number of shipments per order from the distributor to the retailer, $m \geqslant 1$
M retailer's permissible delay period offered by the distributor in a year
I_e interest earned per dollar per year
I_p interest payable per dollar per year
$I_r(t)$ inventory level, which changes with time t during the period T_4
T_4 replenishment time interval, with $T_4 = T_3/m = T/mn$
TAC_r annual total relevant cost of the retailer.

3. Model formulation

In order not to allow for any shortage to appear, the production rate P is assumed to be higher than the time dependent demand rate for the product, through the parameter k ($k > 1$, $P = kD$). Given that in each ordering cycle, the manufacturer delivers n shipments to the distributor, with each shipment having Q_d units of the product; the manufacturer uses a policy of producing nQ_d units with time dependent production rate in time T_1 shown in Figs. 1 and 2.

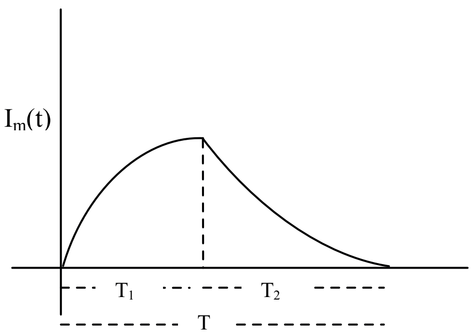

Figure 1. Manufacturer's inventory level with respect to time

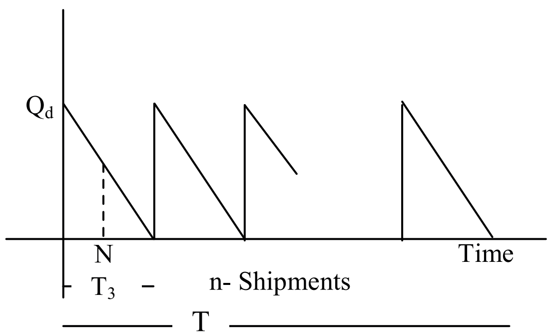

Figure 2. For the distributor

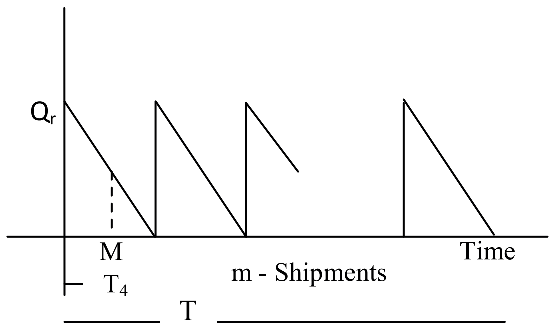

Figure 3. For the retailer

Again, the manufacturer allows the distributor a trade credit without interest during a permissible delay period. Distributor, again, splits the quantity Q_d

into m shipments and delivers Q_r units of the product to the m retailers in each shipment. So, the inventory of the distribution center resembles a step function, each step having the height of quantity Q_r (Q_d/m), shown in Fig. 3.

3.1. Manufacturer's model

The variable production rate starts at $t = 0$ and continues up to $t = T_1$, when the inventory level reaches its maximum. Production then stops at $t = T_1$ and the inventory gradually depletes to zero at the end of the cycle time $t = T$, due to consumption, as shown in Fig. 1. Therefore, during the time interval $(0,\ T_1)$, the system is subject to the effect of production and demand, and during the time interval $(0,\ T_2)$, the system is subject to the effect of demand only. Then, the change in inventory level can be described by the following differential equations

$$\frac{dI_{m1}(t)}{dt} = (k-1)(a+bt), \quad \text{where} \quad 0 \leqslant t \leqslant T_1$$

and

$$\frac{dI_{m2}(t)}{dt} = -(a+bt) \quad \text{where} \quad 0 \leqslant t \leqslant T_2,$$

with conditions

$$I_{m1}(0) = 0 \quad \text{and} \quad I_{m2}(T_2) = 0 \quad \text{holding.}$$

Solutions to the above equations are

$$I_{m1}(t) = (k-1)\left(at + \frac{1}{2}bt^2\right), \quad \text{where} \quad 0 \leqslant t \leqslant T_1,$$

and

$$I_{m2}(t) = a(T_2 - t) + \frac{1}{2}b\left(T_2^2 - t^2\right), \quad \text{where} \quad 0 \leqslant t \leqslant T_2.$$

In addition, from the boundary condition $I_{m1}(T_1) = I_{m2}(0)$, we can derive the following equation:

$$(k-1)\left(aT_1 + \frac{1}{2}bT_1^2\right) = aT_2 + \frac{1}{2}bT_2^2. \tag{1}$$

The individual costs are now evaluated before they are grouped together:

1. Annual set-up cost $SC_m = A_m/T$.
2. Annual transportation cost $TC_m = \tau_m n/T$.
3. Annual stockholding cost

$$HC_m = \frac{h_m}{T}\left[\int\limits_0^{T_1} I_{m1}(t)\,dt + \int\limits_0^{T2} I_{m2}(t)\,dt\right] =$$

$$\frac{h_m}{T}\left[(k-1)\left(\frac{1}{2}aT_1^2 + \frac{1}{6}bT_1^3\right) + \left(\frac{1}{2}aT_2^2 + \frac{1}{6}bT_2^3\right)\right].$$

4. Opportunity interest loss per unit time in n shipments

$$IL_m = \frac{I_m S_m n}{T}\int_0^N D(t)\,dt = \frac{I_m S_m n}{T}\left(aN + \frac{1}{2}bN^2\right).$$

The annual total relevant cost of the manufacturer

$$TAC_m = SC_m + TC_m + HC_m + IL_m. \tag{2}$$

Determination of values of $\mathbf{T_1}$ and $\mathbf{T_2}$

In this point, we shall determine the values of T_1, and T_2.

By solving eq. (1), with $T_1 + T_2 = T$, we find

$$T_1 = \frac{-(bT + ka) + \sqrt{(k-1)\,b^2T^2 + 4(k-1)\,abT + k^2a^2}}{(k-2)\,b} \tag{3}$$

$$T_2 = \frac{[(k-1)\,bT + ka] + \sqrt{(k-1)\,b^2T^2 + 4(k-1)\,abT + k^2a^2}}{(k-2)\,b}, \quad \text{where } k > 2. \tag{4}$$

It should be noted that when k =1 that is, production and demand rate are same, there is no accumulation of products and stock will finish at the end of T_1. It would mean that $T_2 = 0$. If k =2, the values of T_1 and T_2 are undetermined (0/0 form), that is, production rate equal twice the demand rate is not admissible in this situation, with the given rate of change (a+bt). Therefore, all the further discussion will be conducted for the value of k>2.

Lemma 1 *Both T_1 and T_2 are positive numbers.*

Proof Let us suppose that T_1 >0, then

$$\frac{-(bT + ka) + \sqrt{(k-1)\,b^2T^2 + 4(k-1)\,abT + k^2a^2}}{(k-2)\,b} > 0$$

$$\sqrt{(k-1)\,b^2T^2 + 4(k-1)\,abT + k^2a^2} > (bT + ka)$$

$$(k-1)\,b^2T^2 + 4(k-1)\,abT + k^2a^2 > b^2T^2 + k^2a^2 + 2kabT$$

$$(k-2)\,b^2T^2 + 2(k-2)\,abT > 0$$

as k >2, so (k-2) is a positive number;

$$bT\,(bT + 2a) > 0.$$

Here, according to the assumptions, a, b, k and T, are all positive, so that $(bT + 2a) > 0$ is true. Therefore, T_1 is a positive number.

In the same way we can show that T_2 is also a positive number. □

3.2. Distributor's model

The level of inventory$I_d(t)$ gradually decreases to meet the demands from the retailers, which is shown in Fig. 2. Hence, the variation of inventory with respect to time t can be described by the following differential equations:

$$\frac{dI_d(t)}{dt} = -(a+bt), \quad \text{where} \quad 0 \leqslant t \leqslant T_3 \quad \text{and} \quad I_d(T_3) = 0,$$

consequently, the solution is given by

$$I_d(t) = a(T_3 - t) + \frac{1}{2}b\left(T_3^2 - t^2\right), \quad \text{where } 0 \leqslant t \leqslant T_3 \text{and } T_3 = T/n$$

and the order quantity is

$$Q_d = I_d(0) = aT_3 + \frac{1}{2}bT_3^2. \tag{5}$$

The individual costs are now evaluated before they are grouped together:

1. Annual ordering cost $(OC_d) = nA_d/T$.
2. Annual stockholding cost (excluding interest charges)

$$HC_d = \frac{nh_d}{T}\int_0^{T_3} I_d(t)\,dt = \frac{nh_d}{T}\left(\frac{1}{2}aT_3^2 + \frac{1}{6}bT_3^3\right).$$

3. The distributor incurs two annual shipment cost elements, one for receiving shipments from manufacturer and the other for delivering shipments to the retailers:
   The shipment cost for receiving $(TC_{d_1}) = \tau_{d_1}nT$;
   The shipment cost for delivering $(TC_{d_2}) = \tau_{d_2}mnT$.
4. Opportunity interest loss per unit time in mn shipments

$$IL_d = \frac{I_o.S_d.mn}{T}\int_0^{M} D(t)\,dt = \frac{I_o.S_d.mn}{T}\left(aM + \frac{1}{2}bM^2\right).$$

5. Regarding interest earned and payable, we have two following possible cases, based on the value of T_3 and N:

Case I: when $N \leqslant T_3$

1. Interest earned per year in n shipments

$$IE_{d1} = \frac{nI_eS_d}{T}\int_0^{T_3}(T_3 - t)\,D(t)\,dt = \frac{nI_eS_d}{T}\left(\frac{1}{2}aT_3^2 + \frac{1}{6}bT_3^3\right)$$

2. Interest payable per year in n shipments

$$IP_{d1} = \frac{nI_pS_m}{T}\int_N^{T_3} I_d\left(t\right)dt = \frac{nI_pS_m}{T}\left[\frac{1}{2}a\left(T_3 - N\right)^2 + \frac{1}{6}b\left(2T_3^3 - 3T_3^2N + N^3\right)\right].$$

Case II: when $N \geqslant T_3$

1. Interest earned per year in n shipments:

$$\begin{aligned} IE_{d2} &= \frac{nI_eS_d}{T}\left[\int_0^{T_3}\left(T_3 - t\right)D\left(t\right)dt + \left(N - T_3\right)\int_0^{T_3}D\left(t\right)dt\right] \\ &= \frac{nI_eS_d}{T}\left[N\left(aT_3 + \frac{1}{2}bT_3^2\right) - \left(\frac{1}{2}aT_3^2 + \frac{1}{3}bT_3^3\right)\right] \end{aligned}$$

2. In this case, no interest charges are paid for the items kept in stock, i.e. $IP_{d2} = 0$.

Therefore, the annual total relevant cost of the distributor is

$$TAC_d = \begin{cases} TAC_{d1} & if \quad N \leqslant T_3 \\ TAC_{d2} & if \quad N \geqslant T_3 \end{cases}, \quad \text{where} \tag{6}$$

$$TAC_{d_1} = OC_d + HC_d + TC_{d_1} + TC_{d_2} + IL_d + IP_{d_1} - IE_{d_1} \tag{7}$$

and

$$TAC_{d_2} = OC_d + HC_d + TC_{d_1} + TC_{d_2} + IL_d + IP_{d_2} - IE_{d_2}. \tag{8}$$

3.3. Retailer's model

The level of inventory $I_r\left(t\right)$ gradually decreases to meet demands to customers. This is shown in Fig. 3. Hence, the variation of inventory with respect to time t can be described by the following differential equation:

$$\frac{dI_r\left(t\right)}{dt} = -\left(a + bt\right), \quad \text{where} \quad 0 \leqslant t \leqslant T_4 \quad \text{and} \quad I_r\left(T_4\right) = 0,$$

consequently, the solution is given by $I_r\left(t\right) = a\left(T_4 - t\right) + \frac{1}{2}b\left(T_4^2 - t^2\right)$, where $0 \leqslant t \leqslant T_4$, $T_4 = T_3/m$ and $T_4 = T/mn$
and the order quantity is

$$Q_r = I_r\left(0\right) = aT_4 + \frac{1}{2}bT_4^2. \tag{9}$$

The individual costs are now evaluated before they are grouped together:

1. Annual ordering cost $\left(OC_r\right) = mnA_r/T$.

2. Annual stock-holding cost (excluding interest charges):

$$HC_r = \frac{mn\ h_r}{T} \int_0^{T_4} I_r(t)\,dt = \frac{mn\ h_r}{T}\left(\frac{1}{2}a_4^2 + \frac{1}{6}bT_4^3\right).$$

3. The transportation cost for receiving shipments from the distributor:

$$TC_r = \tau_r {}^{mn}\!/_{T}.$$

4. Regarding interest earned and payable, we have two following possible cases, based on the values of T_4 and M:

Case 1: when $M \leqslant T_4$

1. Interest earned per year in mn shipments:

$$IE_{r1} = \frac{mnI_eS_r}{T}\int_0^{T_4}(T_4 - t)\,D(t)\,dt = \frac{mnI_eS_r}{T}\left(\frac{1}{2}aT_4^2 + \frac{1}{6}bT_4^3\right).$$

2. Interest payable per year in mn shipments:

$$IP_{r1} = \frac{mnI_pS_d}{T}\int_M^{T_4} I_r(t)\,dt = \frac{mnI_pS_d}{T}\left[\frac{1}{2}a\,(T_4 - M)^2 + \frac{1}{6}b\left(2T_4^3 - 3T_4^2M + M^3\right)\right].$$

Case 2: when $M \geqslant T_4$

1. Interest earned per year in mn shipments

$$IE_{r2} = \frac{mnI_eS_r}{T}\left[\int_0^{T_4}(T_4 - t)\,D(t)\,dt + (M - T_4)\int_0^{T_4} D(t)\,dt\right] = \frac{mnI_eS_r}{T}\left[M\left(aT_4 + \frac{1}{2}bT_4^2\right) - \left(\frac{1}{2}aT_4^2 + \frac{1}{3}bT_4^3\right)\right].$$

2. In this case, no interest charges are paid for the items kept in stock, i.e.

$$IP_{r2} = 0.$$

Therefore, the annual total relevant cost of the retailers is

$$TAC_r = \begin{cases} TAC_{r1} & if \quad M \leqslant T_4 \\ TAC_{r2} & if \quad M \geqslant T_4 \end{cases}, \tag{10}$$

where

$$TAC_{r_1} = OC_r + HC_r + TC_r + IP_{r_1} - IE_{r_1} \tag{11}$$

and

$$TAC_{r_2} = OC_r + HC_r + TC_r + IP_{r_2} - IE_{r_2}. \tag{12}$$

Finally, the annual total cost of the entire supply chain TCS is composed of the manufacturer's annual cost TAC_m, the distributor's annual cost TAC_d and retailer's annual cost TAC_r. It is important to note that by differentiating the cycle time T and the permissible delay periods N and M one incurs different annual costs to the distributor and retailer. Hence, the annual total relevant cost of the entire system will also be different for different cases

Case I: when $N \leqslant T_3$

The annual total cost of the system can be written as

$$TCS_\alpha = \begin{cases} TCS_1 & if \quad M \leqslant T_4 \\ TCS_2 & if \quad M \geqslant T_4 \end{cases}, \text{ where} \tag{13}$$

$TCS_1 = TAC_m + TAC_{d1} + TAC_{r1}$ and $TCS_2 = TAC_m + TAC_{d1} + TAC_{r2}$.

Case II: when $N \geqslant T_3$

$$TCS_\beta = \begin{cases} TCS_3 & if \quad M \leqslant T_4 \\ TCS_4 & if \quad M \geqslant T_4 \end{cases}, \text{ where} \tag{14}$$

$TCS_3 = TAC_m + TAC_{d2} + TAC_{r1}$ and $TCS_4 = TAC_m + TAC_{d2} + TAC_{r2}$.

This study develops an integrated production-inventory model with a certain permissible delay in payment for distributor and retailers. An approximate model with a single manufacturer, single distributor, and single retailer is developed to derive the optimal production policy and lot size. Since $T_4 = T/mn$, $T_3 = T/n$ and the values of T_1 and T_2 are determined from equations (3) and (4), the problem can be stated as an optimization problem and formulated as follows:

Minimize:

$$TCS(m, n, T) = TAC_m + TAC_d + TAC_r \tag{15}$$

Subject to:

$$0 \leqslant T, 0 \leqslant m, 0 \leqslant n. \tag{16}$$

4. The algorithm

Input: Delivery per order m and n, where $m, n \in I^+$.
Output: minimum value$TCS\,(m^*, n^*, T^*)$of $TCS\,(m, n, T)$ given in equation (15).

begin

choose m, n such that $m \geqslant 1$ and $n \geqslant 1$
$TCS\,(m^*, n^*, T^*) = 100^{100}$ // initially we take a very large quantity
repeat
find $\frac{\partial}{\partial T} TCS\,(m, n, T)$
put $\frac{\partial}{\partial T} TCS\,(m, n, T) = 0$ and find all the values of T;
// let$T_1, T_2 .... T_n$ be all such values of T
for (i = 1 to n) do
if $\left(\frac{\partial^2}{\partial T^2}\,(TCS\,(m, n, T))\right) \geqslant 0$ then
calculate $TCS\,(m, n, T_i)$;
if $(TCS\,(m, n, T_i) \leqslant TCS\,(m^*, n^*, T_i^*))$
$TCS\,(m^*, n^*, T_i^*) = TCS\,(m, n, T_i)\,$;
end if
end if
end for
until (minimum value $TCS\,(m^*, n^*, T_i^*)$ of $TCS\,(m, n, T_i)$ is found for all possible values of m and n)
derive the $T_1^*, T_2^*........$
end

5. Numerical examples

To illustrate the performance of the proposed coordination model, several examples with a variety of essential data have been investigated. Optimal production and replenishment policy meant to minimize the total system cost may be obtained by using the methodology proposed in the preceding section.

Example 5.1 for $N \leqslant T_3$ and $M \leqslant T_4$

Thus, the following numerical example illustrates the model. Consider a three-echelon supply chain with single manufacturer, single distributor and single retailer for single item, the values of the parameters adopted in this study are shown in Table 1.

Table 1. Input for TCS

$k=3$	$A_m=500$	$I_m=0.1$	$I_e=0.2$	$I_0=0.15$	$S_d=10$	$S_r=12$	$\tau_r=50$
$a=10$	$h_m=2$	$S_m=8$	$I_p=0.3$	$h_d=3$	$A_d=80$	$A_r=90$	
$b=5$	$\tau_m=300$	$\tau_{d_1}=70$	$\tau_{d_2}=150$	$N=2$	$M=1$	$h_r=5$	

The computational results are shown in Table 2. The costs of the manufacturer, distributor and retailer are presented in Table 3. The major conclusions and the special condition are drawn from numerical are as follows:

1. In this example, TCS_1^* is \$ 702, while the optimal values of n^*, m^*, T_1^*, T_2^*, T_3^* and T_4^* are 2, 2, 2.52, 3.78, 3.15 and 1.57, respectively.

2. The optimal total cost for manufacturer, distributor and retailer from coordinated model (from Table 3) are $TAC_m = 247$, $TAC_d = 287$ and $TAC_r = 168$ for n =2 and m =2.
3. The optimal total cost for the manufacturer, distributor and retailer from uncoordinated model (from Table 4) are $TAC_m = 235$, $TAC_d = 566$ and $TAC_r = 666$ for n =2 and m =2.
4. The total system cost from Table 4 is \$ 1467 and the total system cost from Table 3 is \$ 702. This means the increase of \$ 765 per unit time. That is, if all parties work independently, the total system cost increases significantly.
5. Since TCS is the function of T, an optimization technique as shown in Section 4 (the algorithm) is used to find the optimal solution. A graphical representation and numerical analysis are also presented to show the convexity of the TCS. Based on the above analysis and graphical representation of Figs. 4 and 5, one can postulate that TCS is a convex function. When $n = 2$ and $m = 2$, the sufficient condition is $d^2\,TCS_1/dT^2 = 29.66>0$.
6. In Figs. 6 and 7 we have presented the discrete graphs of TCS with respect to the values of n and m that show the minimum value of total system cost at $n = 2$ and $m = 2$. This means that the lower or bigger numbers of shipments for distributor and retailer increase the total system cost.

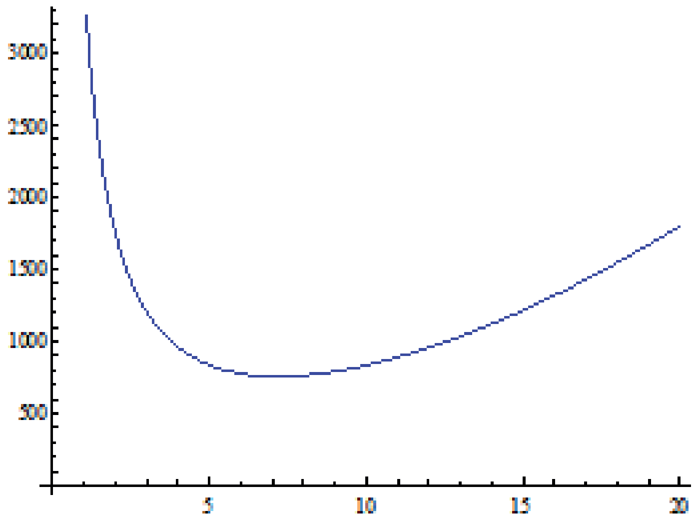

Figure 4. TCS with respect to cycle time T (0, 20)

Example 5.2 for $N \leqslant T_3$ and $M \geqslant T_4$

We take $k = 3$, $a = 10$, $b = 5$, A_m = \$ 500/order, h_m = \$ 2/unit, τ_m = \$ 300/order, $I_m = 0.1$, S_m = \$ 8/unit, $I_e = 0.2$, $I_p = 0.3$, $I_0 = 0.15$, h_d = \$

Table 2. Numerical results for the illustrative example

n	m	T	T_1	T_2	T_3	T_4	TCS_1	n	m	T	T_1	T_2	T_3	T_4	TCS_1
1	1	2.71	1.08	1.62	2.71	2.71	778	4	1	8.51	3.40	5.10	2.12	2.12	789
1	2	3.53	1.41	2.11	3.53	1.76	735	4	2	10.7	4.28	6.42	2.67	1.33	756
1	3	3.99	1.59	2.40	3.99	1.33	772	4	3	12.4	4.96	7.44	3.10	1.03	837
1	4	4.33	1.73	2.59	4.33	1.08	825	5	1	10.2	4.08	6.12	2.04	2.04	745
2	2	6.31	2.52	3.78	3.15	1.57	**702**	5	2	12.4	4.96	7.44	2.48	1.24	799
2	3	7.17	2.86	4.30	3.58	1.19	756	results not satisfying both conditions $N \leqslant T_3$ and $M \leqslant T_4$							
3	3	9.85	3.94	5.91	3.28	1.09	789	2	4	7.81	3.12	4.68	3.90	0.97	822
3	2	8.70	3.48	5.22	2.90	1.45	721	3	4	10.7	4.28	6.42	3.56	0.89	865
3	1	6.79	2.71	4.07	2.26	2.26	706	4	5	14.4	5.76	8.64	3.60	0.72	1009

Table 3. The costs of the manufacturer, distributor and retailer

	Manufactures's costs							Distributor's costs						Retailer's costs						
n	M	T_1	T_2	Q_m	TC_m	HC_m	IL_m	TAC_m	T_3	Q_d	TC_d	HC_d	IL_d	TAC_d	T_4	Q_r	TC_r	HC_r	TAC_r	TCS_1
2	2	2.5	3.8	49.8	95.2	65.5	7.6	247	3.1	55	117	72	12	287	1.6	22	31.7	49	168	702
3	1	2.7	4.0	54.5	134	71.2	11	291	2.2	34	98.5	46	8.3	220	2.2	34	22.3	76	196	706
3	2	3.5	5.2	74.9	103	102	8.2	272	2.9	50	127	64	13	284	1.4	18	34.7	45	167	721
1	2	1.4	2.1	24.9	85.7	30.8	6.8	267	3.5	65	105	83	11	298	1.7	24	28.5	56	172	735
2	3	2.8	4.3	58.4	84.5	77.1	6.7	239	3.6	68	146	84	16	347	1.2	15	42.2	35	172	756
1	3	1.6	2.4	27.9	75	35.2	6.3	243	4.0	80	130	100	14	363	1.3	17	37.5	39	168	772
1	1	1.0	1.6	18.3	111	22.5	8.8	328	2.7	45	81.4	58	7	221	2.7	45	18.5	97	230	778
3	3	3.9	5.9	87.6	91.8	121	7.3	272	3.3	60	160	75	17	342	1.0	13	45.9	32	177	789

Table 4. The costs of the manufacturer, distributor and retailer for the uncoordinated models (from sections 3.1, 3.2 and 3.3)

		Manufacturer's costs						Distributor's costs, $N<T_3$				Retailer's costs, $M<T_4$			
n	m	T	T_1	T_2	Q_m	HC_m	TAC_m	T_3	Q_d	HC_d	TAC_d	T_4	Q_r	HC_r	TAC_r
2	2	8.3	3.3	4.9	71	96	235	2.7	45	117	566	1.4	19	172	666
3	1	9.2	3.6	5.5	80	111	272	2.4	38	151	655	1.4	19	129	500
3	2	9.2	3.6	5.5	80	111	272	2.7	45	176	849	1.4	19	260	999
1	2	7.3	2.9	4.3	60	80	194	2.7	45	58	283	1.4	19	86	333
2	3	8.3	3.3	2.9	71	96	235	3.0	53	135	680	1.4	19	260	999

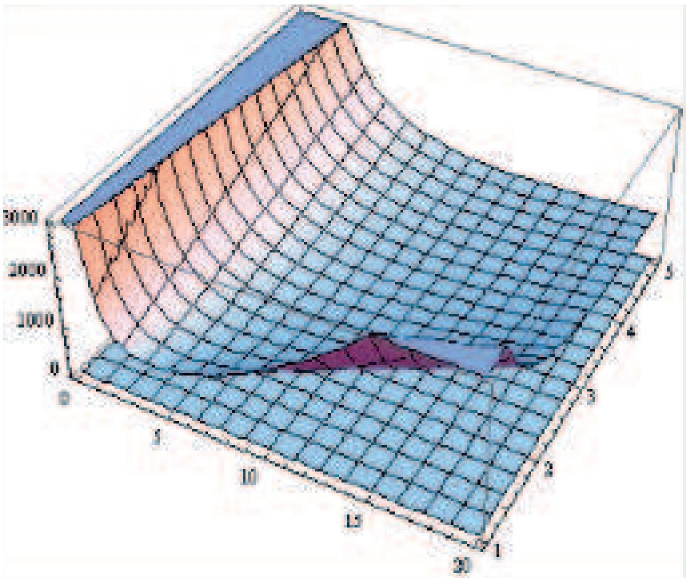

Figure 5. TCS with respect to T (0, 20) and n (1, 5)

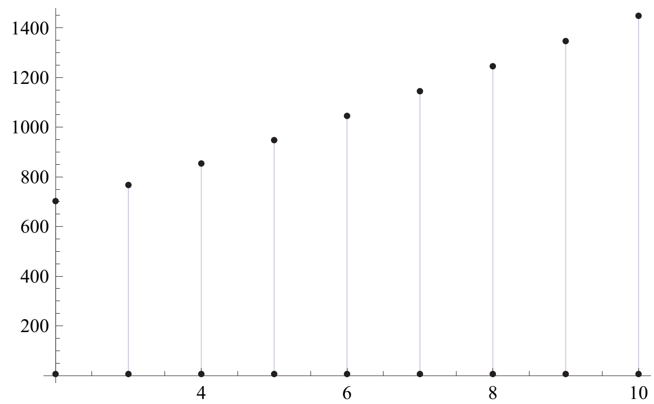

Figure 6. Discrete graphical representation for TCS_1 with respect to n (1, 10)

3/unit, S_d = \$ 10/unit, A_d = \$ 80/order, S_r = \$ 12/unit, A_r = \$ 90/order, h_r = \$ 5/unit, τ_r = \$ 50/order, τ_{d_1} = \$ 70/order, τ_{d_2} = \$ 150/order, $M = 1$ year N =2 years. Using the TCS_2 in the solution procedure, we get for $m = 5$ and $n = 3$, $T = 11.40$, $T_1 = 4.56$, $T_2 = 6.84$, $T_3 = 3.8$, $T_4 = 0.76$ and TCS_2 (T^*) = \$ 940.

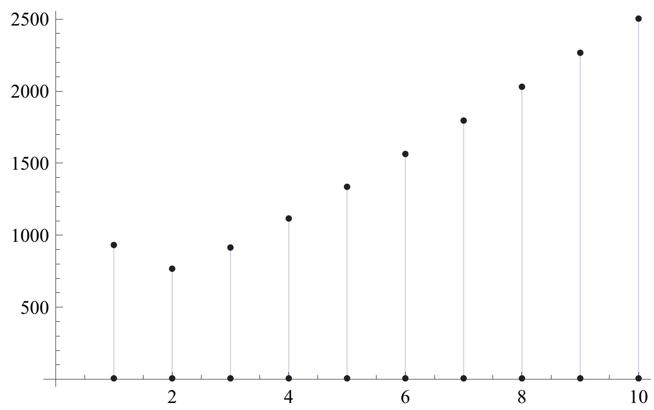

Figure 7. Discrete graphical representation for TCS_1 with respect to m (2, 10)

Example 5.3 for $N \geqslant T_3$ and $M \leqslant T_4$

We take $k = 3$, $a = 10$, $b = 5$, A_m = \$ 500/order, h_m = \$ 2/unit, τ_m = \$ 300/order, $I_m = 0.1$, S_m = \$ 8/unit, $I_e = 0.2$, $I_p = 0.3$, $I_0 = 0.15$, h_d = \$ 3/unit, S_d = \$ 10/unit, A_d = \$ 80/order, S_r = \$ 12/unit, A_r = \$ 90/order, h_r = \$ 5/unit, τ_r = \$ 50/order, τ_{d_1} = \$ 70/order, τ_{d_2} = \$ 150 /order, $M = 2$ years, $N = 3$ years. Using the TCS_2 in the solution procedure, we get for $m = 1$ and $n = 2$, $T = 4.60$, $T_1 = 1.84$, $T_2 = 2.76$, $T_3 = 2.30$, $T_4 = 2.30$ and TCS_3 (T^*) = \$ 833.

Example 5.4 for $N \geqslant T_3$ and $M \geqslant T_4$

We take $k = 3$, $a = 10$, $b = 5$, A_m = \$ 500/order, h_m = \$ 2/unit, τ_m = \$ 300/order, $I_m = 0.1$, S_m = \$ 8/unit, $I_e = 0.2$, $I_p = 0.3$, $I_0 = 0.15$, h_d = \$ 3/unit, S_d = \$ 10/unit, A_d = \$ 80/order, S_r = \$ 12/unit, A_r = \$ 90/order, h_r = \$ 5/unit, τ_r = \$ 50/order, τ_{d_1} = \$ 70/order, τ_{d_2} = \$ 150/order, M = 3 years, N = 4 years. Using the TCS_2 in the solution procedure, we get for $m = 3$ and $n = 2$, $T = 5.8$, $T_1 = 2.32$, $T_2 = 3.48$, $T_3 = 2.9$, $T_4 = 0.97$ and TCS_4 (T^*) = \$1155.

6. Conclusions

Using various methods for reducing costs has become the major focus for supply chain management. In order to decrease the joint total cost, the manufacturer, distributor and retailer are willing to invest in reducing the different costs. This leads to the necessity of developing integrated inventory models for attaining the win–win objectives for all sides. To achieve this purpose, this paper presents the integrated manufacturer-distributor-retailer models with two-level permissible delays in payments (manufacturer offers the distributor and distributor, analogously, provides to the retailer) to determine the optimal replenishment time interval and replenishment frequency, with the aim of reducing the total system costs to all the parties. It is shown that the total costs of the manufacturer, distributor and retailer, and other costs (given in Table 4), obtained without the integration model, are usually higher than the ones obtained with integration (given in Table 3). In this paper, we have considered a three-stage production-inventory system, under the just-in-time manufacturing environment with time dependent production and demand, where the manufacturer must deliver the products in small quantities to minimize the distributor's and retailer's holding cost and accept the supply of small quantities of raw material to minimize its own holding cost. We have used some realistic costs like transportation cost and cost for opportunity interest loss due to permissible delay. The proposed model can be extended in several ways. For example, one can generalize the model to allow for imperfect production process and deteriorating items.

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