Control and Cybernetics

vol. **49** (2020) No. 1

LMI based stability criterion for uncertain neutral-type neural networks with discrete and distributed delays [*†]

by

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Abstract: This paper studies the problem of robust stability analysis for uncertain neutral-type neural networks with discrete and distributed delays. By constructing an appropriate Lyapunov-Krasovskii functional, new delay-dependent criteria are obtained. We utilized the free-weighting matrices approach and bounding lemmas to estimate the derivative of the Lyapunov-Krasovskii functional. The stability criterion are established in terms of linear matrix inequalities (LMIs). Finally, a numerical example is presented to illustrate the effectiveness of the proposed method.

Keywords: stability analysis, neutral networks, Lyapunov method, Linear Matrix Inequality

1. Introduction

For the past few decades neural networks have remained a hot research topic because of their wide applications in various fields, like image processing, pattern recognition, fixed-point computation, associative memory and combinatorial optimization, see Arik (2000, 2014), Liu (1997), Syed Ali et al. (2017, 2018), Saravanakumar et al. (2016), Syed Ali (2011, 2014), Ge, Hu and Guan (2014), Tian, Xiong and Xu (2014). The existence of time delays is usually one of the primary sources of instability and oscillations. Thus, the stability problem of neural networks with time delays has been widely considered by many researchers (see Tian and Zhong, 2011; Song, 2008; Sun et al., 2009; He et al., 2007; Chen et al., 2010; Zhang, Yue and Tian, 2009; Wang, Yang and Zuo, 2012; Thuan, Trinh and Hien, 2016; or Kwon et al., 2012). Generally, stability criteria of neural networks with time delays are classified into two categories: delay-independent stability

[*]The work was supported by CSIR. 25(0274)/17/EMR-II dated 27/04/2017

[†]Submitted: June 2018; Accepted: April 2020

criteria and delay-dependent stability criteria. Delay-dependent stability criteria are less conservative than delay-independent ones. Therefore, researchers virtually always consider the delay-dependent stability criteria. Neural networks usually have a spatial extent due to the presence of many parallel pathways of a variety of axon sizes and lengths. Thus, there will be distribution of conduction velocities along these pathways and distribution of propagation delays and both discrete and distributed delays should be considered in the neural network model (see Syed Ali, Arik and Saravanakumar, 2005; Feng, Xu and Zou, 2009; Shi et al., 2015). Therefore, much effort has been devoted to delay-dependent stability analysis of delayed neural networks since, as indicated, delay-dependent stability criteria are generally less conservative than delay-independent ones, especially when the size of the time delay is small. Recently, more attention is paid towards robustness analysis for uncertain neural networks because of the existence of modeling errors, external disturbance and parameter fluctuations (Li et al., 2016; Syed Ali and Saravanan, 2016; Liu, 2009; Syed Ali, Saravanakumar and Zhu, 2015).

Also, owing to the complicated dynamic properties of the neural cells in the real world, the existing neural network models in many cases cannot characterize the properties of the neural reaction process precisely. It is natural and essential that systems will contain some information about the derivative of the past state to describe further and model the dynamics for such complex neural reactions. This new type of neural network is called a neutral neural network or neural network of neutral type. The past state of the network affects the current state in neutral type systems and they are investigated also under this respect. Due to the existence of parameter variations, modeling errors, and process uncertainties, stability analysis of neutral uncertainties systems has gained a lot of attention (see Ren, Feng and Sun, 2016; Petersen, 1987; Feng, Xu and Zou, 2009; Lee, Kwon and Park, 2010; Nagamani and Balasubramanian, 2012; or Balasubramanian, Nagamani and Rakkiyappan, 2010).

This paper investigates the problem of asymptotic stability analysis for neutral-type uncertain neural networks with Markovian jumping parameters and time-varying delays. Some new delay-dependent sufficient conditions ensuring the stability for uncertain neutral type networks with discrete and distributed delays are obtained in terms of linear matrix inequalities (LMIs) by constructing an appropriate Lyapunov-Krasovskii functional, which contains two triple integral terms. By using some free weighting matrices, the proposed results are obtained. Numerical results are provided to show the effectiveness of the given results.

The main contribution of the paper lies in the following aspects:

- A novel Lyapunov-Krasovskii functional that involves double and triple integrals terms is constructed to obtain stability conditions.

- A new integral inequality is applied in terms of second-order reciprocally convex inequality and Jensen's inequality.
- New delay-dependent sufficient condition ensuring the stability is obtained in terms of linear matrix inequalities (LMIs).

Notation: The notation used in this paper is standard. R^n denotes n dimensional Euclidean space, the superscript "T" denotes the transpose, and the notation $P > 0$ (≥ 0) means P is real symmetric positive (semi-positive) definite, max(P) and min(P) denote the maximum and minimum eigenvalues of matrix P, respectively. I is an identity matrix with appropriate dimension. diag$\{a_i\}$ denotes the diagonal matrix with the diagonal elements a_i, (i = 1,2,. . .). The asterisk $*$ in a matrix is used to denote a term that is induced by symmetry.

2. Problem formulation and main results

Consider the following uncertain neutral-type neural networks with discrete and distributed delay:

$$\begin{cases} \dot{x}(t) = -A(t)x(t) + W_0(t)f(x(t)) + W_1(t)f(x(t-\tau)) + C(t)\dot{x}(t-h) + \\ \qquad W_2(t)\int_{t-r}^{t} f(x(s))ds, t \geq 0, \\ x(\theta) = \phi(\theta),\ \forall\theta \in [-\eta, 0],\ \eta = max\{h, \tau, r\} \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state vector; h, τ, t represent the neutral delay, discrete delay and distributed delay, respectively. The initial condition $\phi(t)$ denotes a continuous vector-valued initial function on the interval $[-\eta, 0]$. In (1),

$$A(t) = A + \Delta A(t),$$

$$W_0(t) = W_0 + \Delta W_0(t),$$
$$W_1(t) = W_1 + \Delta W_1(t),$$
$$C(t) = C + \Delta C(t),$$
$$W_2(t) = W_2 + \Delta W_2(t),$$

where $A,\ W_0,\ W_1,\ C$ and $W_2 \in R^{n\times n}$ are constant matrices, and $\Delta A(t)$, $\Delta W_0(t)$, $\Delta W_1(t)$, $\Delta C(t)$ and $\Delta W_2(t)$ are the unknown matrices, denoting the uncertainties of the concerned system and satisfying the following:

$$\begin{aligned} &[\Delta A(t)\ \Delta W_0(t)\ \Delta W_1(t)\ \Delta C(t)\ \Delta W_2(t)] \\ &= MF(t)\left[E_{-A}\ E_{W_0}\ E_{W_1}\ E_C\ E_{W_2}\right], \end{aligned} \tag{2}$$

where $M, E_{-A},\ E_{W_0},\ E_{W_1},\ E_C$ and E_{W_2} are known matrices, $F(t)$ is an unknown, real and possibly time-varying matrix with Lebesgue measurable elements, which satisfies

$$F^T(t)F(t) \leq I. \tag{3}$$

For system (1), the nominal form is given as follows:

$$\begin{cases} \dot{x}(t) = -Ax(t) + W_0 f(x(t)) + W_1 f(x(t-\tau)) + C\dot{x}(t-h) + \\ \qquad W_2 \int_{t-r}^{t} f(x(s))ds, t \leq 0, \\ x(\theta) = \phi(\theta), \ \forall\theta \in [-\eta, 0], \ \eta = max\{h, \tau, r\}. \end{cases} \tag{4}$$

LEMMA 2.1 *(Gu, 1994): For the symmetric matrices $R > 0$, $\mathcal{X}$ and matrix $\mathcal{Y}$ the following statements are equivalent:*
(1) $\mathcal{X} - \mathcal{Y}^T R\mathcal{Y} < 0$,
(2) There exists an appropriate dimensional matrix $\mathcal{Z}$ such that

$$\begin{bmatrix} \mathcal{X} + \mathcal{Y}^T\mathcal{Z} + \mathcal{Z}^T\mathcal{Y} & \mathcal{Z}^T \\ * & -R \end{bmatrix} < 0.$$

LEMMA 2.2 *(Petersen, 1987): Given matrices J, E and $\Theta = \Theta^T$, inequality $\Theta + EF(t)G + G^T F^T(t)E^T < 0$ holds for any $F(t)$ satisfying $F^T(t)F(t) \leq I$, if there exists a scalar $\gamma > 0$ such that $\Theta + \gamma^{-1}EE^T + \gamma G^T G < 0$.*

3. Main results

THEOREM 3.1 *For three given scalars $h > 0$, $\tau > 0$, and $r > 0$, if there exist some positive definite symmetric matrices: P_{11}, P_{22}, P_{33}, P_{44}, Q_i ($i = 1, 2, ..., 10) \in R^{n\times n}$ and some appropriately dimensional matrices: $(P_{ij})_{1\leq i<j\leq 4}$, $K = [K_1^T, K_2^T]^T$, $L = [L_1^T, L_2^T]^T$, $M = [M_1^T, M_2^T]^T$, $N = [N_1^T, N_2^T]^T$, such that the following linear matrix inequalities (LMIS) hold:*

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ * & P_{22} & P_{23} & P_{24} \\ * & * & P_{33} & P_{34} \\ * & * & * & P_{44} \end{bmatrix} \geq 0, \tag{5}$$

and

$$\Xi^1 = \begin{bmatrix} \Omega_1 & (A_c^1)^T Q_2 & h(A_c)^T Q_6 & h(A_c)^T Q_9 \\ * & -l_1 Q_2 & 0 & 0 \\ * & * & -hQ_6 & 0 \\ * & * & * & -2Q_9 \end{bmatrix} < 0, \tag{6}$$

$$\Xi^2 = \begin{bmatrix} \Omega_2 & (A_c^2)^T Q_2 & \tau(A_c)^T Q_7 & \tau(A_c)^T Q_{10} \\ * & -l_2 Q_2 & 0 & 0 \\ * & * & -\tau Q_7 & 0 \\ * & * & * & -2Q_{10} \end{bmatrix} < 0, \tag{7}$$

where

$$\begin{aligned}
&\Omega^k = (\Omega^k_{ij})_{9\times 9},\\
&\Omega^k_{11} = l_k[-P_{11}A - A^T P_{11}^T + P_{13}^T + P_{13} + P_{14}^T + P_{14}\\
&\qquad -Q_1 + Q_3 + hQ_4 + \tau Q_5 + K_1^T + K_1 + hL_1^T\\
&\qquad +hL_1 + M_1 + M_1^T + \tau N_1^T + \tau N_1],\\
&\Omega^k_{12} = l_k[-A^T P_{12} + P_{23}^T - P_{13} + P_{24}^T - K_1^T + K_2 + hL_2],\\
&\Omega^k_{13} = l_k[P_{12} + P_{11}C],\\
&\Omega^k_{14} = l_k[-P_{14} - M_1^T + M_2 + \tau N_2],\\
&\Omega^k_{15} = l_k P_{11} W_0,\\
&\Omega^k_{16} = l_k P_{11} W_1,\\
&\Omega^k_{17} = l_k P_{11} W0._2,\\
&\Omega^1_{18} = h[-A^T P_{13} + P_{33} + P_{34}^T - L_1^T],\\
&\Omega^2_{18} = \tau[-A^T P_{14} + P_{34} + P_{44}^T - N_1^T],\\
&\Omega^1_{19} = -hk_1^T - \frac{h^2}{2} L_1^T,\\
&\Omega^2_{19} = -\tau M_1^T - \frac{\tau^2}{2} N_1^T,\\
&\Omega^k_{22} = l_k[-P_{23}^T - P_{23} - K_2^T - K_1 - Q_1],\\
&\Omega^k_{23} = l_k[P_{22} + P_{12}^T C],\\
&\Omega^k_{24} = l_k[-P_{24}],\\
&\Omega^k_{25} = l_k[P_{12}^T W_0],\\
&\Omega^k_{26} = l_k[P_{12}^T W_1],\\
&\Omega^k_{27} = l_k[P_{12}^T W_2],\\
&\Omega^1_{28} = h[-P_{33} - L_2^T],\\
&\Omega^2_{28} = \tau P_{34},\\
&\Omega^1_{29} = -hk_2^T - \frac{h^2}{2} L_2^T,\\
&\Omega^2_{29} = 0,
\end{aligned}$$

$$\Omega_{33}^k = -l_k Q_2,$$
$$\Omega_{34}^k = 0,$$
$$\Omega_{35}^k = 0,$$
$$\Omega_{36}^k = 0,$$
$$\Omega_{37}^k = 0,$$
$$\Omega_{38}^1 = h[C^T P_{13} + P_{23}],$$
$$\Omega_{38}^2 = \tau[C^T P_{14} + P_{24}],$$
$$\Omega_{39}^k = 0,\ \Omega_{44}^k = l_k[-Q_3 - M_2^T - M_2],\ \Omega_{45}^k = 0,$$
$$\Omega_{46}^k = 0,$$
$$\Omega_{47}^k = 0,$$
$$\Omega_{48}^1 = h[-P_{34}],$$
$$\Omega_{48}^2 = \tau[-P_{44}^T - N_2^T],$$
$$\Omega_{49}^1 = 0,$$
$$\Omega_{49}^2 = -\tau M_2^T - \frac{\tau^2}{2} N_2^T,$$
$$\Omega_{55}^k = l_k[r^2 Q_8 + S],$$
$$\Omega_{56}^k = 0,$$
$$\Omega_{57}^k = 0,$$
$$\Omega_{58}^1 = h[W_0^T P_{13}],$$
$$\Omega_{58}^2 = \tau[W_0^T P_{14}],$$
$$\Omega_{59}^k = 0,$$
$$\Omega_{66}^k = l_k[-S],$$
$$\Omega_{67}^k = 0,$$
$$\Omega_{68}^1 = h[W_1^T P_{13}],$$
$$\Omega_{68}^2 = \tau[W_1^T P_{14}],$$
$$\Omega_{69}^k = 0, \Omega_{77}^k = l_k[-Q_8],$$
$$\Omega_{78}^1 = h[W_2^T P_{13}],$$
$$\Omega_{78}^2 = \tau[W_2^T P_{14}],$$
$$\Omega_{79}^k = 0,$$
$$\Omega_{88}^1 = l_k[-h Q_4],$$
$$\Omega_{88}^2 = l_k[-\tau Q_5],$$
$$\Omega_{89}^k = 0,$$

$$\Omega_{99}^1 = l_k[-hQ_6 - \frac{h^2}{2}Q_9],$$
$$\Omega_{99}^2 = l_k[-\tau Q_7 - \frac{\tau^2}{2}Q_{10}],$$

$$A_C^k = \begin{bmatrix} -l_kA & 0 & l_kC & 0 & l_kW_0 & l_kW_1 & l_kW_2 & 0 & 0 \end{bmatrix},$$

$k = 1, 2, \ldots\ldots\ldots\ldots,$

$$A_C = \begin{bmatrix} -A & 0 & C & 0 & W_0 & W_1 & W_2 & 0 & 0 \end{bmatrix}.$$

with $l_1 = \frac{h}{h+\tau}$, $l_2 = \frac{\tau}{h+\tau}$ *and * meaning the symmetric terms,*
then the nominal system (4) *is asymptotically stable.*

Proof: Define a Lyapunov-Krasovskii functional candidate for system (4) as

$$V(t,x) = V_1(t,x) + V_2(t,x) + V_3(t,x), \tag{8}$$

where

$$\begin{aligned}
V_1(t,x) &= \xi^T(t)P\xi(t),\\
V_2(t,x) &= \int_{t-h}^{t} x^T(s)Q_1x(s)ds + \int_{t-h}^{t} \dot{x}^T(s)Q_2\dot{x}(s)ds\\
&\quad + \int_{t-\tau}^{t} x^T(s)Q_3x(s)ds + \int_{t-\tau}^{t} f^T(x(s))Sf(x(s))ds,\\
V_3(t,x) &= \int_{-h}^{0}\int_{t+\theta}^{t} x^T(s)Q_4x(s)dsd\theta + \int_{-\tau}^{0}\int_{t+\theta}^{t} x^T(s)Q_5x(s)dsd\theta\\
&\quad + \int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)Q_6\dot{x}(s)dsd\theta + \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{x}(s)Q_7\dot{x}(s)dsd\theta\\
&\quad + r\int_{-r}^{0}\int_{t+\theta}^{t} f^T(x(s))Q_8f(x(s))dsd\theta\\
&\quad + \int_{-h}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t} \dot{x}^T(s)Q_9\dot{x}(s)dsd\lambda d\theta\\
&\quad + \int_{-\tau}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t} \dot{x}^T(s)Q_{10}\dot{x}(s)dsd\lambda d\theta,
\end{aligned}$$

with

$$\xi(t) = \begin{bmatrix} x^T(t) & x^T(t-h) & \int_{t-h}^{t} x^T(s)ds & \int_{t-\tau}^{t} x^T(s)ds \end{bmatrix}.$$

Then, the time derivative of $V(t,x)$ with respect to t along the system (4) is

$$\dot{V}(t,x) = \dot{V}_1(t,x) + \dot{V}_2(t,x) + \dot{V}_3(t,x), \tag{9}$$

where

$$\begin{aligned}
&\dot{V}_1(t,x) = \\
&2[x^T(t)P_{11} + x^T(t-h)P_{12}^T + \int_{t-h}^{t} x^T(s)dsP_{13}^T + \int_{t-\tau}^{t} x^T(s)dsP_{14}^T] \\
&[-Ax(t) + W_0 f(x(t)) + W_1 f(x(t-\tau)) + C\dot{x}(t-h) + W_2 \int_{t-r}^{t} f(x(s))ds] \\
&+2[x^T(t)P_{12} + x^T(t-h)P_{22} + \int_{t-h}^{t} x^T(s)dsP_{23}^T + \int_{t-\tau}^{t} x^T(s)dsP_{24}^T]\dot{x}(t-h) \\
&+2[x^T(t)P_{13} + x^T(t-h)P_{23} + \int_{t-h}^{t} x^T(s)dsP_{33} \\
&\quad + \int_{t_\tau}^{t} x^T(s)dsP_{34}^T][x(t) - x(t-h)] \\
&+2[x^T(t)P_{14} + x^T(t-h)P_{24} + \int_{t-h}^{t} x^T(s)dsP_{34} \\
&\quad + \int_{t-\tau}^{t} x^T(s)dsP_{44}][x(t) - x(t-\tau)],
\end{aligned} \tag{10}$$

$$\begin{aligned}
\dot{V}_2(t,x) = {}& x^T(t)[Q_1 + Q_3]x(t) + x^T(t-h)[-Q_1]x(t-h) + \dot{x}^T(t)[Q_2]\dot{x}(t) \\
&+ \dot{x}^T(t-h)[-Q_2]\dot{x}(t-h) + x^T(t-\tau)[-Q_3]x(t-\tau) \\
&+ f^T(x(t-\tau))[-S]f(x(t-\tau)) + f^T(x(t))[S]f(x(t)),
\end{aligned} \tag{11}$$

$$\begin{aligned}
&\dot{V}_3(t,x) = x^T(t)[hQ_4 + \tau Q_5]x(t) + f^T(x(t))[r^2 Q_8]f(x(s)) \\
&\quad +\dot{x}^T(t)[hQ_6 + \tau Q_7 + \frac{h^2}{2}Q_9 + \frac{\tau^2}{2}Q_{10}]\dot{x}(t) \\
&\quad - \int_{t-h}^{t} \dot{x}^T(s)[Q_4]x(s)ds - \int_{t-\tau}^{t} x^T(s)[Q_5]x(s)ds - \int_{t-h}^{t} \dot{x}^T(s)[Q_6]\dot{x}(s)ds \\
&\quad - \int_{t-\tau}^{t} \dot{x}^T(s)[Q_7]\dot{x}(s)ds - r\int_{t-r}^{t} f^T(x(s))[Q_8]f(x(s))ds \\
&\quad - \int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)[Q_9]\dot{x}(s)dsd\theta - \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)[Q_{10}]\dot{x}(s)dsd\theta.
\end{aligned} \tag{12}$$

By using Jensen's inequality

$$[\int_0^r w(s)ds]^T M[\int_0^r w(s)ds] \leq r\int_0^r w^T(s)Mw(s)ds$$

we obtain,

$$\begin{aligned}
\dot{V}_3(t,x) &= x^T(t)[hQ_4+\tau Q_5]x(t)+f^T(x(t))[r^2Q_8]f(x(t))\\
&+\dot{x}^T(t)[hQ_6+\tau Q_7+\frac{h^2}{2}Q_9+\frac{\tau^2}{2}Q_{10}]\dot{x}(t)\\
&-[\int_{t-h}^t x(s)ds]^T[Q_4]x(s)-[\int_{t-\tau}^t x(s)ds]^T[Q_5]x(s)-[\int_{t-h}^t \dot{x}(s)ds]^T[Q_6]\dot{x}(s)\\
&-[\int_{t-\tau}^t \dot{x}(s)ds]^T[Q_7]\dot{x}(s)-[\int_{t-r}^t f(x(s))ds]^T[Q_8][\int_{t-r}^t f(x(s))ds]\\
&-[\int_{-h}^0\int_{t+\theta}^t \dot{x}(s)dsd\theta]^T[Q_9]\dot{x}(s)-[\int_{-\tau}^0\int_{t+\theta}^t \dot{x}(s)dsd\theta]^T[Q_{10}]\dot{x}(s). \qquad (13)
\end{aligned}$$

Now, from the Leibinz-Newton formula the following equations are true for real matrices K, L, M, N with appropriate dimensions,

$$\alpha_1(t) := 2\xi_1^T(t)K^T[x(t)-x(t-h)-\int_{t-h}^t \dot{x}(s)ds]=0, \qquad (14)$$

$$\alpha_2(t) := 2\xi_1^T(t)L^T[hx(t)-\int_{t-h}^t x(s)ds-\int_{-h}^0 \dot{x}(s)dsd\theta]=0, \qquad (15)$$

$$\alpha_3(t) := 2\xi_2^T(t)M^T[x(t)-x(t-\tau)-\int_{t-\tau}^t \dot{x}(s)]=0, \qquad (16)$$

$$\alpha_4(t) := 2\xi_2^T(t)N^T[\tau x(t)-\int_{t-\tau}^t x(s)ds-\int_{-\tau}^0\int_{t+\theta}^t \dot{x}(s)dsd\theta]=0, \qquad (17)$$

where

$$\xi_1(t)=\begin{bmatrix} x^T(t) & x^T(t-h)\end{bmatrix}^T, \xi_2(t)=\begin{bmatrix} x^T(t) & x^T(t-\tau)\end{bmatrix}^T.$$

Suppose that

$$
\begin{aligned}
\sum_{i=1}^{4}\alpha_i(t) &= x^T(t)\big[K_1^T + K_1 + hL_1^T + hL_1 + M_1^T + M_1 + \tau N_1^T + \tau N_1\big]x(t)\\
&+2x^T(t)\big[-K_1^T + K_2 + hL_2\big]x(t-h) + 2x^T(t)\big[-M_1^T + M_2 - \tau N_2\big]x(t-\tau)\\
&+x^T(t-h)\big[-K_2^T - K_2\big]x(t-h) + x^T(t-\tau)\big[-M_2^T - M_2\big]x(t-\tau)\\
&+x^T(t)\big[-hL_1^T\big]x(s) + 2x^T(t)\big[-\tau N_1^T\big]x(s) + 2x^T(t)\big[-hK_1^T - \frac{h^2}{2}L_1^T\big]\dot{x}(s)\\
&+x^T(t)\big[-\tau M_1^T - \frac{\tau^2}{2}N_1^T\big]\dot{x}(s)\\
&+2x^T(t-h)\big[-hL_1^T\big]x(s) + 2x^T(t-\tau)\big[-\tau N_2^T\big]\dot{x}(s)\\
&+2x^T(t-h)\big[-hK_2^T - \frac{h^2}{2}L_2^T\big]\dot{x}(s) + 2x^T(t-\tau)\big[-\tau M_2^T - \frac{\tau^2}{2}N_2^T\big]\dot{x}(s).
\end{aligned} \tag{18}
$$

By combining relations from (10) to (18), we obtain

$$
\dot{V}(t,x) = \dot{V}_1(t,x) + \dot{V}_2(t,x) + \dot{V}_3(t,x) + \sum_{i=1}^{4}\alpha_i(t),
$$

$$
\begin{aligned}
\dot{V}(t,x) \le\ & x^T(t)\big[-P_{11}^T A - P_{11}A^T + P_{13}^T + P_{13} + P_{14} + Q_1 + Q_3 + hQ_4 + \tau Q_5\\
&+K_1^T + K_1\\
&+hL_1^T + hL_1 + M_1^T + M_1 + \tau N_1^T + \tau N_1\big]x(t)\\
&+2x^T(t)\big[-A^T P_{12} + P_{23}^T - P_{13} + P_{24}^T - K_1^T + K_2 + hL_2\big]x(t-h)\\
&+2x^T(t)\big[P_{12} + P_{11}C\big]\dot{x}(t-h) + 2x^T(t)\big[-P_{14} - M_1^T + M_2 + \tau N_2\big]x(t-\tau)\\
&+2x^T(t)\big[P_{11}W_0\big]f(x(t)) + 2x^T(t)\big[P_{11}W_1\big]f(x(t-\tau))\\
&+2x^T(t)\big[P_{11}W_2\big]\int_{t-r}^{t} f(x(s))ds + 2x^T(t)h\big[-A^T P_{13} + P_{33} + P_{34}^T - L_1^T\big]x(s)\\
&\quad [0.2em] +2x^T(t)\tau\big[-A^T P_{14} + P_{34} + P_{44}^T - N_1^T\big]x(s)\\
&+2x^T\big[-hK_1^T - \frac{h^2}{2}L_1^T\big]\dot{x}(s) + 2x^T(t)\big[-\tau M_1^T - \frac{\tau^2}{2}N_1^T\big]\dot{x}(s)
\end{aligned}
$$

$$+2x^T(t-h)\big[-P_{23}^T-P_{23}-K_2^T-K_1-Q_1\big]x(t-h)$$
$$+2x^T(t-h)\big[P_{22}+P_{12}^TC\big]\dot{x}(t-h)+2x^T(t-h)\big[P_{24}\big]x(t-\tau)$$
$$+2x^T(t-h)\big[P_{12}^TW_0\big]f(x(t))+2x^T(t-h)\big[P_{12}^TW_1\big]f(x(t-\tau))$$
$$+2x^T(t-h)\big[P_{12}^TW_2\big]\int_{t-r}^{t}f(x(s))ds$$
$$+2x^T(t-h)h\big[-P_{33}-L_1^T\big]x(s)+2x^T(t-h)\big[\tau P_{34}]x(s)$$
$$+2x^T(t-h)\big[-hK_2^T-\frac{h^2}{2}L_2^T\big]\dot{x}(s)+2\dot{x}^T(t-h)\big[-Q_2\big]\dot{x}(t-h)$$
$$+2\dot{x}^T(t-h)h\big[C^TP_{13}+P_{23}\big]x(s)+2\dot{x}^T(t-h)\tau\big[C^TP_{14}+P_{24}\big]x(s)$$
$$+x^T(t-\tau)\big[-Q_3-M_2^T-M_2]x(t-\tau)$$
$$+2x^T(t-\tau)h\big[-P_{34}^T\big]x(s)+2x^T(t-\tau)\tau\big[-P_{44}^T-N_2^T\big]x(s)$$
$$+2x^T(t-\tau)\big[-\tau M_2^T-\tau^22N_2^T\big]\dot{x}(s)$$
$$+f^T(x(t))\big[r^2Q_8+S\big]f(x(t))$$
$$+2f^T(x(t))h\big[W_0^TP_{13}\big]x(s)+2f^T(x(t))\tau\big[W_0^TP_{14}\big]x(s)$$
$$+2f^T(x(t-\tau))\big[-S\big]f(x(t-\tau))+2f^T(x(t-\tau))h\big[W_1^TP_{13}\big]x(s)+2f^T(x(t-\tau))\tau\big[W_1^TP_{14}\big]x(s)$$
$$+2\int_{t-r}^{t}f^T(x(s))ds\big[-Q_8\big]\int_{t-r}^{t}f(x(s))ds+\int_{t-r}^{t}f^T(x(s))h\big[W_2^TP_{13}\big]x(s)$$
$$+2\int_{t-r}^{t}f^T(x(s))ds\tau\big[W_2^TP_{14}\big]x(s)$$
$$+x^T(s)\big[-hQ_4\big]x(s)+x^T(s)\big[-\tau Q_5\big]x(s)$$
$$+\dot{x}^T(s)\big[-hQ_6-\frac{h^2}{2}Q_9]\dot{x}(s)+\dot{x}^T(s)\big[-\tau Q_7-\frac{\tau^2}{2}Q_{10}]\dot{x}(s)$$
$$+\dot{x}^T(t)\big[hQ_6+\tau Q_7+\frac{h^2}{2}Q_9+\frac{\tau^2}{2}Q_{10}+Q_2\big]\dot{x}(t). \qquad (19)$$

Then,

$$\dot{V}(t,x)\leq\eta^T(t,s,u,\theta)\ \Xi^1\ \eta(t,s,u,\theta)+\eta^T(t,s,u,\theta)\ \Xi^2\ \eta(t,s,u,\theta). \qquad (20)$$

Now, using the fact that

$$1=\frac{2}{h^3}\int_{-h}^{0}\int_{t+\theta}^{t}\int_{t-h}^{t}dsdud\theta=\frac{2}{\tau^3}\int_{-\tau}^{0}\int_{t+\theta}^{t}\int_{t-\tau}^{t}dsdud\theta,$$

we have

$$\begin{aligned}\dot{V}(t,x) &= \frac{2}{h^3}\int_{-h}^{0}\int_{t+\theta}^{t}\int_{t-h}^{t}\eta^T(t,s,u,\theta)\ \Xi^1\ \eta(t,s,u,\theta)dsdud\theta \\ &+ \frac{2}{\tau^3}\int_{-\tau}^{0}\int_{t+\theta}^{t}\int_{t-\tau}^{t}\eta^T(t,s,u,\theta)\ \Xi^2\ \eta(t,s,u,\theta)dsdud\theta < 0, \\ \dot{V}(t,x) &\leq 0, \end{aligned} \tag{21}$$

where

$$\eta = \left[x^T(t)x^T(t-h)\dot{x}^T(t-h)x^T(t-\tau)f^T(x(t))f_1\int_{t-r}^{t}f(x(s))dsx^T(s)\dot{x}^T(s)\right]^T,$$

with $f_1 = f^T(x(t-\tau))$. And thus, according to Lyapunov stability theory, the nominal system (4) is asymptotically stable. □

Theorem 3.2 *For the given scalars $h > 0,\ \tau > 0$ and $r > 0$, if there exist some positive definite symmetric matrices : $P_{11},\ P_{22},\ P_{33},\ P_{44},\ Q_i\ (i = 1, 2, 3..., 10) \in R^{n\times n}$, some appropriate matrices: $(P_{ij})_{1\leq i<j\leq 4}$, $K = [K_1^T, K_2^T]^T, L = [L_1^T, L_2^T]^T$, $M = [M_1^T, M_2^T]^T$, $N = [N_1^T, N_2^T]^T$, and two positive scalars: $\varepsilon_1 > 0,\ \varepsilon_2 > 0$ such that the following linear matrix inequalities (LMIS) hold:*

$$\begin{bmatrix} \Omega^1 & (A_c^1)^TQ_2 & h(A_c)^TQ_6 & h(A_c)^TQ_9 & \Xi_{15}^1 \\ * & -l_1Q_2 & 0 & 0 & l_1Q_2M \\ * & * & -hQ_6 & 0 & hQ_6M \\ * & * & * & -2Q_9 & hQ_9M \\ * & * & * & * & \varepsilon I \end{bmatrix} < 0, \tag{22}$$

and

$$\begin{bmatrix} \Omega^2 & (A_c^2)^TQ_2 & \tau(A_c)^TQ_7 & \tau(A_c)^TQ_{10} & \Xi_{15}^2 \\ * & -l_2Q_2 & 0 & 0 & l_2Q_2M \\ * & * & -\tau Q_7 & 0 & \tau Q_7M \\ * & * & * & -2Q_{10} & \tau Q_{10}M \\ * & * & * & * & -\varepsilon I \end{bmatrix} < 0, \tag{23}$$

where

$$
\begin{aligned}
&\Omega^k = (\Omega^k_{ij})_{9\times 9},\\
&\Omega^1_{11} = l_1[-P_{11}A - A^T P_{11}^T + P_{13}^T + P_{13} + P_{14}^T + P_{14} - Q_1 + Q_3 + hQ_4 + \tau Q_5\\
&\qquad +K_1^T + K_1 + hL_1^T + hL_1 + M_1 + M_1^T + \tau N_1^T + \tau N_1] + \varepsilon_1 E_{-A}^T E_{-A},\\
&\Omega^1_{12} = l_1[-A^T P_{12} + P_{23}^T - P_{13} + P_{24}^T - K_1^T + K_2 + hL_2] + \varepsilon_k E_C E_{-A}^T,\\
&\Omega^1_{13} = l_1[P_{12} + P_{11}C] + \varepsilon_1 E_{-A}^T E_C,\\
&\Omega^1_{14} = l_1[-P_{14} - M_1^T + M_2 + \tau N_2],\\
&\Omega^1_{15} = l_1 P_{11} W_0 + \varepsilon_1 E_{-A}^T E_{W_0},\\
&\Omega^1_{16} = l_1 P_{11} W_1 + \varepsilon_1 E_{-A}^T E_{W_1},\\
&\Omega^1_{17} = l_1 P_{11} W_2 + \varepsilon_1 E_{-A}^T E_{W_2},\\
&\Omega^1_{18} = h[-A^T P_{13} + P_{33} + P_{34}^T - L_1^T] + \varepsilon_1 E_{W_0} E_{-A}^T,\\
&\Omega^1_{19} = -hK_1^T - \frac{h^2}{2} L_1^T,\\
&\Omega^1_{22} = l_1[-P_{23}^T - P_{23} - K_2^T - K_1 - Q_1],\\
&\Omega^1_{23} = l_1[P_{22} + P_{12}^T C] + \varepsilon_1 E_C^T E_C,\\
&\Omega^1_{24} = l_1[-P_{24}],\\
&\Omega^1_{25} = l_1[P_{12}^T W_0] + \varepsilon_1 E_C^T E_{W_0},\\
&\Omega^1_{26} = l_1[P_{12}^T W_1] + \varepsilon_1 E_C^T E_{W_1},\\
&\Omega^1_{27} = l_1[P_{12}^T W_2] + \varepsilon_1 E_C^T E_{W_2},\\
&\Omega^1_{28} = h[-P_{33} - L_2^T],\\
&\Omega^1_{29} = -hK_2^T - \frac{h^2}{2} L_2^T,\\
&\Omega^1_{33} = -L_1 Q_2,\\
&\Omega^1_{34} = 0,\\
&\Omega^1_{35} = 0,\\
&\Omega^1_{36} = 0,\\
&\Omega^1_{37} = 0,\\
&\Omega^1_{38} = h[C^T P_{13} + P_{23}] + \varepsilon_1 E_C^T E_{W_0},\\
&\Omega^1_{39} = 0,\\
&\Omega^1_{44} = l_1[-Q_3 - M_2^T - M_2],\\
&\Omega^1_{45} = 0,\\
&\Omega^1_{46} = 0,\\
&\Omega^1_{47} = 0,
\end{aligned}
$$

$$\Omega^1_{48} = h[-P_{34}],$$
$$\Omega^1_{49} = 0,$$
$$\Omega^1_{55} = l_1[r^2 Q_8 + S],$$
$$\Omega^1_{56} = 0,$$
$$\Omega^1_{57} = 0,$$
$$\Omega^1_{58} = h[W_0^T P_{13}] + \varepsilon_1 E^T_{W_0} E_{W_0},$$
$$\Omega^1_{59} = 0,$$
$$\Omega^1_{66} = l_1[-S],$$
$$\Omega^1_{67} = 0,$$
$$\Omega^1_{68} = h[W_1^T P_{13}] + \varepsilon_1 E^T_{W_1} E_{W_0},$$
$$\Omega^1_{69} = 0,$$
$$\Omega^1_{77} = l_1[-Q_8],$$
$$\Omega^1_{78} = h[W_2^T P_{13}] + \varepsilon_1 E^T_{W_2} E_{W_0},$$
$$\Omega^1_{79} = 0,$$
$$\Omega^1_{88} = l_1[-hQ_4],$$
$$\Omega^1_{89} = 0,$$
$$\Omega^1_{99} = l_1[-hQ_6 - \frac{h^2}{2} Q_9],$$
$$\Omega^2_{11} = l_2[-P_{11}A - A^T P^T_{11} + P^T_{13} + P_{13} + P^T_{14} + P_{14} - Q_1 + Q_3 + hQ_4 + \tau Q_5 + K^T_1 + K_1 + hL^T_1 + hL_1 + M_1 + M^T_1 + \tau N^T_1 + \tau N_1] + \varepsilon_2 E^T_{-A} E_{-A},$$
$$\Omega^2_{12} = l_2[-A^T P_{12} + P^T_{23} - P_{13} + P^T_{24} - K^T_1 + K_2 + hL_2] + \varepsilon_2 E_C E^T_{-A},$$
$$\Omega^2_{13} = l_2[P_{12} + P_{11}C] + \varepsilon_2 E^T_{-A} E_C,$$
$$\Omega^2_{14} = l_2[-P_{14} - M^T_1 + M_2 + \tau N_2],$$
$$\Omega^2_{15} = l_2 P_{11} W_0 + \varepsilon_2 E^T_{-A} E_{W_0},$$
$$\Omega^2_{16} = l_2 P_{11} W_1 + \varepsilon_2 E^T_{-A} E_{W_1},$$
$$\Omega^2_{17} = l_2 P_{11} W_2 + \varepsilon_2 E^T_{-A} E_{W_2},$$
$$\Omega^2_{18} = \tau[-A^T P_{14} + P_{34} + P^T_{44} - N^T_1] + \varepsilon_2 E_{W_1} E^T_{-A},$$
$$\Omega^2_{19} = -\tau M^T_1 - \frac{\tau^2}{2} N^T_1,$$
$$\Omega^2_{22} = l_2[-P^T_{23} - P_{23} - K^T_2 - K_1 - Q_1],$$
$$\Omega^2_{23} = l_2[P_{22} + P^T_{12} C] + \varepsilon_2 E^T_C E_C,$$
$$\Omega^2_{24} = l_2[-P_{24}],$$

$$\Omega^2_{25} = l_2[P^T_{12}W_0] + \varepsilon_2 E^T_C E_{W_0},$$
$$\Omega^2_{26} = l_2[P^T_{12}W_1] + \varepsilon_2 E^T_C E_{W_1},$$
$$\Omega^2_{27} = l_2[P^T_{12}W_2] + \varepsilon_2 E^T_C E_{W_2},$$
$$\Omega^2_{28} = \tau[P_{34}],$$
$$\Omega^2_{29} = 0,$$
$$\Omega^2_{33} = -l_2 Q_2,$$
$$\Omega^2_{34} = 0,$$
$$\Omega^2_{35} = 0,$$
$$\Omega^2_{36} = 0,$$
$$\Omega^2_{37} = 0,$$
$$\Omega^2_{38} = \tau[C^T P_{14} + P_{24}] + \varepsilon_2 E^T_C E_{W_0},$$
$$\Omega^2_{39} = 0,$$
$$\Omega^2_{44} = l_2[-Q_3 - M^T_2 - M_2],$$
$$\Omega^2_{45} = 0,$$
$$\Omega^2_{46} = 0,$$
$$\Omega^2_{47} = 0,$$
$$\Omega^2_{48} = \tau[-P^T_{34} - N^T_2], \Omega^2_{49} = -\tau M^T_2 - \frac{\tau^2}{2} N^T_2,$$
$$\Omega^2_{55} = l_2[r^2 Q_8 + S],$$
$$\Omega^2_{56} = 0,$$
$$\Omega^2_{57} = 0,$$
$$\Omega^2_{58} = \tau[W^T_0 P_{14}] + \varepsilon_2 E^T_{W_0} E_{W_2},$$
$$\Omega^2_{59} = 0,$$
$$\Omega^2_{66} = l_2[-S],$$
$$\Omega^2_{67} = 0,$$
$$\Omega^2_{68} = \tau[W^T_1 P_{14}] + \varepsilon_2 E^T_{W_1} E_{W_2},$$
$$\Omega^2_{69} = 0,$$
$$\Omega^2_{77} = l_2[-Q_8],$$
$$\Omega^2_{78} = \tau[W^T_2 P_{14}] + \varepsilon_2 E^T_{W_2} E_{W_2},$$
$$\Omega^2_{79} = 0,$$
$$\Omega^2_{88} = l_2[-\tau Q_5],$$
$$\Omega^2_{89} = 0,$$
$$\Omega^2_{99} = l_2[-\tau Q_7 - \frac{\tau^2}{2} Q_{10}],$$

$$\Xi^1 = \begin{bmatrix} l_1 M^T P_{11} & l_1 M^T P_{12} & 0 & 0 & 0 & 0 & 0 & hM^T P_{13} & 0 \end{bmatrix}^T,$$

$$\Xi^2 = \begin{bmatrix} l_2 M^T P_{11} & l_2 M^T P_{12} & 0 & 0 & 0 & 0 & 0 & \tau M^T P_{14} & 0 \end{bmatrix}^T,$$

$k = 1, 2.....$, $l_1 = \frac{h}{h+\tau}$, $l_2 = \frac{\tau}{h+\tau}$, and other items of Ω^k, A_C^k $(k = 1, 2, ...,)$, A_C are given as in Theorem 3.1, then the uncertain system (1) *is robustly asymptotically stable.*

Proof: If $-A, W_0, W_1, C$ and W_2 in (5) to (7) are replaced with $-A+MF(t)E_{-A}$, $W_0 + MF(t)E_{W_0}$, $W_1 + MF(t)E_{W_1}$, $C + MF(t)E_C$ and $W_2 + MF(t)E_{W_2}$, respectively, then (6),(7) for the uncertain neutral-type neural networks (1) are equivalent to the following conditions:

$$\Xi^1 + \Gamma_{1d}F(t)\Gamma_e^T + \Gamma_e F^T(t)\Gamma_{1d}^T < 0, \tag{24}$$

$$\Xi^2 + \Gamma_{2d}F(t)\Gamma_e^T + \Gamma_e F^T(t)\Gamma_{2d}^T < 0. \tag{25}$$

with

$$\Gamma_{1d} = \begin{bmatrix} l_1 M^T P_{11} & l_1 M^T P_{12} & 0 & 0 & 0 & hM^T P_{13} & 0 & 0 & 0 \\ & & & & & l_1 M^T Q_2 & hM^T Q_6 & hM^T Q_9 & \\ & & & & & & & & \end{bmatrix}^T,$$

$$\Gamma_{2d} = \begin{bmatrix} l_2 M^T P_{11} & l_2 M^T P_{12} & 0 & 0 & 0 & \tau M^T P_{14} & 0 & 0 & 0 \\ & & & & & l_2 M^T Q_2 & \tau M^T Q_7 & \tau M^T Q_{10} & \\ & & & & & & & & \end{bmatrix}^T,$$

$$\Gamma_e = \begin{bmatrix} E_{-A}^T & 0 & E_C^T & E_{W_0}^T & E_{W_1}^T & E_{W_2}^T & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Now, let D, E and $F(t)$ be real matrices of appropriate dimensions and $F(t)$ satisfying $F^T(t)F(t) \leq I$. Then, the following inequality hods for any constant $\varepsilon > 0$,

$$DF(t)E + E^T F^T(t) D^T \leq DD^T + \varepsilon^{-1} E^T E,$$

the necessary and sufficient condition for (24) and (25) being that there exist two positive scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ such that

$$\Xi^1 + \varepsilon_1 \Gamma_{1d}\Gamma_{1d}^T + \varepsilon_1^{-1}\Gamma_e^T \Gamma_e < 0, \tag{26}$$

$$\Xi^2 + \varepsilon_2 \Gamma_{2d}\Gamma_{2d}^T + \varepsilon_2^{-1}\Gamma_e^T \Gamma_e < 0. \tag{27}$$

By applying the complement we can obtain that (26) and (27) are equivalent to (22) and (23), respectively, and the proof is completed. □

Similarly, based on the theorem above we can obtain the robust asymptotical stability for system (1).

4. Numerical example

To illustrate the usefulness of the proposed approach, we present the following example.

Consider the following uncertain neutral-type neural networks (1) with the parameters given below:

$$\dot{x}(t) = -Ax(t) + W_0 f(x(t)) + W_1 f(x(t-\tau)) + C\dot{x}(t-h) + W_2 \int_{t-r}^{t} f(x(s))ds,$$

$$A = \begin{bmatrix} -0.05 & 0 \\ 0 & -1.8 \end{bmatrix}, \quad W_0 = \begin{bmatrix} -0.02 & 0 \\ -0.05 & 1 \end{bmatrix}, \quad W_1 = \begin{bmatrix} 4 & 0.1 \\ 0.2 & -0.03 \end{bmatrix},$$

$$C = \begin{bmatrix} -1 & 1 \\ 0.2 & 0.1 \end{bmatrix}, \quad W_2 = \begin{bmatrix} -6.5 & 1 \\ 0.2 & 0.1 \end{bmatrix}, \quad D = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad E_3 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad E_4 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix},$$

$$E_5 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad E_6 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix},$$

with $\varepsilon = 0.35,\ l_1 = 3,\ l_2 = 2,\ h = 0.01,\ \tau = 0.02,\ r = 0.01$. Then, upon applying Theorem (3.1) in MATLAB LMI Toolbox the feasible solutions obtained are

$$P_{11} = 10^{-6} \begin{bmatrix} 0.0466 & -0.1564 \\ -0.1564 & 1.0949 \end{bmatrix}, \quad P_{12} = 10^{-6} \begin{bmatrix} 0.0333 & -0.0595 \\ -0.0595 & 0.0542 \end{bmatrix},$$

$$P_{13} = 10^{-6} \begin{bmatrix} 0.1476 & -0.0014 \\ -0.0014 & 0.1363 \end{bmatrix}, \quad P_{14} = 10^{-6} \begin{bmatrix} 0.1562 & -0.0500 \\ -0.0500 & 1.8133 \end{bmatrix},$$

$$P_{22} = 10^{-6} \begin{bmatrix} 0.2785 & -0.3741 \\ -0.3741 & 0.3.2018 \end{bmatrix}, \quad P_{23} = \begin{bmatrix} -0.1477 & -0.0004 \\ -0.0004 & -0.1440 \end{bmatrix}$$

$$P_{24} = 10^{-6} \begin{bmatrix} 5.4204 & -0.1904 \\ -0.1904 & 5.7305 \end{bmatrix}, \quad P_{33} = 10^{-6} \begin{bmatrix} 0.3873 & -0.1851 \\ -0.1851 & 3.8283 \end{bmatrix}$$

$$P_{34} = 10^{-6} \begin{bmatrix} 0.7163 & -0.7828 \\ -0.7828 & 9.9407 \end{bmatrix}, \quad P_{44} = 10^{-6} \begin{bmatrix} 4.1666 & 0 \\ 0 & 4.1666 \end{bmatrix}$$

$$Q_1 = 10^{-6}\begin{bmatrix} 6.3798 & -0.0489 \\ -0.0489 & 5.7799 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 0.0030 & -0.0023 \\ -0.0023 & 0.0074 \end{bmatrix},$$

$$Q_3 = 10^{-6}\begin{bmatrix} 1.4845 & 0.0155 \\ 0.0155 & 1.5291 \end{bmatrix}, \quad Q_4 = \begin{bmatrix} 0.0138 & 0.0022 \\ 0.0022 & 0.0090 \end{bmatrix},$$

$$Q_5 = 10^{-6}\begin{bmatrix} 4.0398 & 0.0070 \\ 0.0070 & 4.0488 \end{bmatrix}, \quad Q_6 = \begin{bmatrix} 0.0040 & -0.0433 \\ -0.0433 & 0.4208 \end{bmatrix}$$

$$Q_7 = \begin{bmatrix} 0.1262 & -0.9209 \\ -0.9209 & 7.0205 \end{bmatrix}, \ Q_8 = \begin{bmatrix} 5.2884 & 0.0101 \\ 0.0101 & 6.8036 \end{bmatrix}$$

$$Q_9 = \begin{bmatrix} 0.0311 & -0.3239 \\ -0.3239 & 2.8042 \end{bmatrix}, \ Q_{10} = \begin{bmatrix} 0.2529 & -1.6616 \\ -1.6616 & 12.5428 \end{bmatrix}$$

$$S = \begin{bmatrix} -0.0010 & -0.0007 \\ -0.0007 & 0.0001 \end{bmatrix}.$$

Therefore the uncertain neutral-type neural networks is asymptotically stable.

5. Conclusion

In this paper, by constructing an appropriate Lyapunov-Krasovskii functional, and employing free-weighting matrices technique, some sufficient conditions ensuring the robust stability for uncertain neutral-type neural networks with discrete and distributed delays are derived. Numerical example is provided to illustrate the effectiveness of our results. In most of the papers on stability of neutral type neural networks Lyapunov functional is considered with double integral only. In this paper we considered the Lyapunov functional with triple integrals. Future investigations shall include delimiting the domain of attraction and the upper bound of perturbations. Since external disturbances inevitably appear in any real system, it is also interesting to explore stochastic disturbances.

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