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Book review:

STABILIZATION OF LINEAR SYSTEMS

by

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At the end of the sixties, the state space theory of linear control systems, both for time-invariant and time-varying systems, had essentially been worked out. The basic concepts of controllability and observability and the weaker notions of stabilizability and detectability play an essential, fundamental role in the solutions of many important different optimal control problems. The primary concern of the reviewed monograph is the problem of stabilizability for linear, finite-dimensional, time-invariant dynamical control systems.

Stabilization is one of the most important problems in modern mathematical control theory. Roughly speaking, stabilization problem is to find a feedback control law ensuring stability of a closed-loop dynamical system. A feedback control consists of the construction of the input by using information on measurable output. Such construction leads to a new closed-loop dynamical system which no longer contains the control and whose dynamic is described by a new differential state equation.

On the other hand, let us observe that the well-known pole-assignment theorem motivates the introduction of the concepts of stabilizability and detectability. One of the most remarkable facts in modern mathematical control theory is the connection between stabilizability by the linear state feedback and controllability property of dynamical system. Moreover, it should be pointed out that the stabilizability of the linear control system is an essentially weaker concept than controllability, since stabilizability only requires that the uncontrollable part of the dynamical system be asymptotically stable.

In recent years, the stabilization problem has been studied in detail in the modern mathematical systems theory and has grown and expanded in many different directions. A particularly fruitful line of development has been that of putting the new tools to use to solve various stabilization problems for different types of dynamical systems both linear and nonlinear. The main purpose of the volume here considered is to present a review of recent results in this direction for the linear control systems, and of the techniques used to derive them. The results are presented in a series of six chapters that provide an overview of a broad spectrum of tools, which are used to study the different stabilization problems in dynamical systems. At the end of each chapter several comments and indications with respect to the references are given.

The main purpose of the short introductory chapter is to give the main idea of the stabilizability problem for linear, time-invariant, finite-dimensional control systems and to explain the reasons for the necessary mathematical developments in linear algebra, matrix theory and differential equations, which are presented in the book. A short overview of the stabilizability problems for linear finite-dimensional control systems, existing in the literature, is also presented.

The chapter entitled "Stabilization of Linear Systems" presents the stabilization problem for linear, finite-dimensional, continuous-time dynamical systems with constant coefficients. The concept of stabilization is explained in detail in terms of the differential state equation and the algebraic output equation. Moreover, certain connections between controllability, observability, detectability and the stabilization problem are pointed out. Next, Lyapunov equation and Kalman–Lurie–Yakubovich–Popov equation are defined and their role in the stabilization problem is explained. Several special cases are also discussed. Optimal stabilization problem for quadratic performance index is formulated and solved using known optimization methods in infinite-dimensional Hilbert spaces. Finally, stabilization problem with disturbances is considered and solved. Problems of optimal stabilization are considered in connection with frequency domain conditions.

In many practical situations mathematical models of control systems contain two sets of state variables: "slow" and "fast". The mathematical description of such a situation is realized by using the so called "singularly perturbed systems", named also the dynamical systems with a small parameter. The chapter entitled "Stabilization of Linear Systems with Two Time Scales" contains the results for linear, continuous-time, time-invariant and finite-dimensional dynamical systems with small parameter. In this case it is possible to express the differential state equation as the set of two coupled differential equations (fast and slow) with quite different properties. The special cases of such dynamical control systems are the so called singular or descriptor dynamical systems containing both linear differential equations and linear algebraic equations. In this chapter optimal and suboptimal stabilization problems for the two time scales systems and quadratic performance indexes are formulated and solved using mainly pure algebraic methods. Generally, the stabilization problem of a dynamical system with two time scales is reduced to the stabilization of the fast subsystem often named boundary layer system, and to the stabilization of the reduced model. The stabilizing procedure is based on a separate treatment of the fast and slow parts of the dynamical system.

One of the most important features of the book is the treatment of the stabilization problem under incomplete information about the control system. In the next chapter entitled "High-Gain Feedback Stabilization of Linear Systems" stabilization problems for dynamical systems with incomplete information concerning the coefficients are discussed in detail. Using singular perturbation theory the stabilization problem is solved for certain classes of linear dynamical systems. The special attention is paid to linear dynamical systems with minimum phase. Dynamical systems with minimum phase have invariant zeros with strictly negative real parts. Using the controllability results and the concept of invariant zeros high-gain feedback stabilization problem is discussed.

The chapter entitled "Adaptive Stabilization and Identification" is mainly devoted to the study of the stabilization problem for linear dynamical systems with uncertain parameters. This chapter explores several methods for constructing robust stabilizers via output feedback. Using the methods taken directly from the identification theory and the adaptive control theory, certain robust stabilization algorithms are proposed and discussed. Moreover, the adaptive stabilization problem for the two time scales dynamical systems is also considered and the asymptotic structure of the invariant zeros of these systems is investigated for some particular cases.

The last chapter entitled "Discrete Implementation of Stabilization Procedures" is devoted to a study of the stabilization problem for linear, continuous-time dynamical system, but with linear pure discrete-time feedback gain. In this case the closed-loop dynamical system has a hybrid structure i.e., it contains both a continuous and a discrete parts. The state variables describing the plant are continuous, but the feedback gain is a linear finite-dimensional discrete system. The main problem considered in this chapter is the discrete-time implementation of a stabilizing dynamic controller for continuous-time dynamical system. It is shown how to choose the coefficients of a discrete-time controller starting with those of a continuous-time controller in order to achieve the desired stabilization performances. The performance of the discrete-time controllers is evaluated by estimating the difference between solutions of the closed-loop systems corresponding to the associated discrete-time controller and the solutions corresponding to a continuous-time controller which has the desired properties. Moreover, several special discrete-time stabilization problems like, e.g., linear discrete-time feedback control for two time scales dynamical systems with small parameter and high-gain discrete-time feedback control, are also discussed. Particularly, certain adaptive stabilization scheme for systems with small parameter is proposed.

The monograph contains many recent far-reaching theorems and interesting results from different areas of stabilization problems for linear finite-dimensional, continuous-time control systems with constant coefficients. The theorems are often included with complete and detailed proofs or with references to the literature for details. In addition, challenging open problems are described and explained, and promising new research directions are indicated. Most of the results in this monograph belong to the Authors and some of them have been recently published in several research papers. Many of the results presented in the this book can be extended to linear systems with time-varying parameters, to linear discrete-time systems and also to general linear infinite-dimensional systems. The monograph contains an extensive list of references, and most of them have been published in recent years. Moreover, many results given in the monograph may serve as tools for the study of different problems connected with stabilization theory.

This volume has something to offer a broad spectrum of readers. The book should be a valuable reference for graduate students, scientists, and professional researchers in the area of mathematical control theory and control engineering and for mathematicians with an interest in the analysis and design of engineering control systems.

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