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Model for evaluation of the maneuvering object's trajectory using Kalman filter[1]

by

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Abstract: A new approach for construction of an object maneuvering model for evaluation of movement trajectory using Kalman filter is proposed. The approach proposed is based on application of the object's dynamic equations. Such approach is better for obtaining adequate models of object maneuvering in comparison with the known ones. The state equations of Kalman filter are derived for describing the movement of a ship maneuvering by the heading.
Keywords: state equation, Kalman filter.

1. Introduction

The main disadvantage of the current approaches for construction of the object maneuvering models is the following: the equations of object dynamics are not used in them (Singer, 1970, Bar-Shalom and Birmival, 1982, Bogler, 1987, Moose et al., 1979, Ramahandra, 1987). Therefore, the parameters of the maneuvering movement model are treated as values not related to the parameters of object dynamics. Such assumption causes difficulties in reasonable selection of the above-mentioned parameters. In this paper, state equations of the Kalman filter are derived starting from the ship dynamics equations. The equations proposed are applicable for evaluation of trajectory and parameters of movement of a ship maneuvering by the heading, using Kalman filter.

2. Problem formulation and its solution

We start from the differential equation of the ship velocity vector angle $\psi(t)$:

$$\psi''(t) + c\psi'(t) = d\beta(t),$$

where $c = v/(aL)$, $d = rv^2/(aL^2)$, v — ship velocity vector modulus, L — length of ship, a, r — parameters describing the dynamic performance of the concrete ship (Basin, 1968); $\beta(t)$ — process of shifting the helm.

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Assume that ship maneuvering (change of the velocity vector angle $\psi(t)$) occurs because of random shifting of the helm $\beta(t)$. The helm is shifted in random moments t_i at a random angle β_i, $|\beta_i| \leq \beta_{max}$. Each shifting of the helm implies reversing the sign of the shift angle. The number of reversals of the angle sign during every interval t is the Poisson random value with parameter α_1; β_i is a uniform random value distributed on the interval $[-\beta_{max}, \beta_{max}]$. It is possible to show that in such conditions the random process $\beta(t)$ is stationary in the broad sense; its mean value is zero, and its covariance function is

$$R_\beta(\tau) = \sigma_\beta^2 e^{-\alpha_1|\tau|}, \ \sigma_\beta^2 = \frac{1}{3}\beta_{max}^2.$$

Consider the case when the ship executes maneuvers with some preset value of relative bearing ψ_0. Such movement may be considered as equivalent to the movement of the ship in a pre-determined direction with automatic steering device turned on, in conditions of random disturbance $\beta(t)$. The flowchart of the ship control system is shown in Fig. 1.

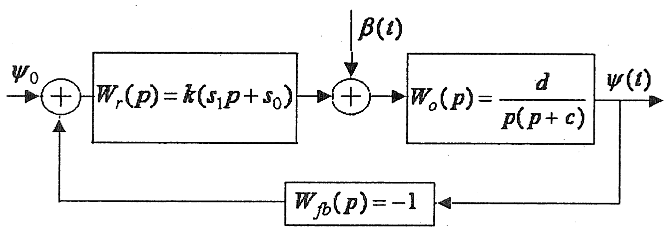

Figure 1

For the above flowchart, the following expressions can be derived for the covariance function and the mean value of the $\psi(t)$ process:

$$R_\psi(\tau) = \sum_{i=1}^{3} A_i e^{-\alpha_i|\tau|}, \ E(\psi(t)) = \psi_0, \tag{1}$$

where

$$A_1 = \frac{\sigma_\beta^2 d^2}{2\alpha_1(\alpha_1^2-\alpha_2^2)(\alpha_1^2-\alpha_3^2)}, \ A_2 = \frac{\alpha_1\sigma_\beta^2 d^2}{2\alpha_2(\alpha_2^2-\alpha_3^2)(\alpha_2^2-\alpha_1^2)},$$

$$A_3 = \frac{\alpha_1\sigma_\beta^2 d^2}{2\alpha_3(\alpha_3^2-\alpha_2^2)(\alpha_3^2-\alpha_1^2)},$$

$$\alpha_{2,3} = -\frac{c+kds_1}{2} \pm \sqrt{\left(\frac{c+kds_1}{2}\right)^2 - kds_0}.$$

E is here the mean value operator. Indeed, the transfer function for the closed-loop system (see Fig. 1) is described by the following expression:

$$W_{\psi\beta}(p) = \frac{W_o(p)}{1 + W_o(p)W_{fb}(p)W_r(p)}$$
$$= \frac{d}{p^2 + (c + kds_1)p + kds_o} = \frac{d}{(p - \alpha_1)(p - \alpha_2)},$$

where α_1, α_2 are the roots of the following equation

$$p^2 + (c + kds_1)p + kds_o = 0.$$

The spectral density of the input process $\beta(t)$ is the following:

$$s_\beta(\omega) = \int_{-\infty}^{+\infty} e^{-j\omega\tau} R_\beta(\tau) d\tau = \frac{\sigma_\beta^2}{\omega^2 + \alpha_1^2},$$

therefore, the spectral density of the output process $\psi(t)$ is described by the following expression:

$$s_\psi(\omega) = s_\beta(\omega)|W_{\psi\beta}(j\omega)|^2 = \frac{\sigma_\beta^2 d^2}{(\omega^2 + \alpha_1^2)(\omega^2 + \alpha_2^2)(\omega^2 + \alpha_3^2)}.$$

Applying backward Fourier transformation

$$R_\psi(\tau) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{j\omega\tau}\, s_\psi(\omega)\, d\omega,$$

we can derive the covariance function (1).

The projections of velocity vector on the axes of Cartesian coordinate system are described by the following expressions:

$$x'(t) = v_x(t) = v cos(\overset{o}{\psi}(t) + \psi_0),$$
$$y'(t) = v_y(t) = v sin(\overset{o}{\psi}(t) + \psi_0),$$

where $\overset{o}{\psi}(t) = \psi(t) - \psi_0$. Assume that $\psi(t)$ is a normal process (the output process of a linear dynamic system) with the covariance function $R_\psi(t_1, t_2)$. Using the following table integrals (Prudnikov et al., 1981):

$$\int_{-\infty}^{+\infty} exp(-q^2x^2) sin x\, dx = 0,$$
$$\int_{-\infty}^{+\infty} exp(-q^2x^2) cos x\, dx = \frac{\sqrt{\pi}}{q} exp\left(-\frac{1}{4q^2}\right),$$

and taking into consideration that one-dimensional and two-dimensional distributions of process $\psi(t)$ are normal, we can derive the following auxiliary formulas:

$$E(sin\overset{o}{\psi}(t)) = 0,\ E(sin\overset{o}{\psi}(t_1)cos\overset{o}{\psi}(t_2)) = 0,\ E(cos\overset{o}{\psi}(t)) = exp\left(-\frac{\sigma_\psi^2(t)}{2}\right),$$

$$E(cos\overset{o}{\psi}(t_1)cos\overset{o}{\psi}(t_2)) = exp\left(-\frac{1}{2}(R_\psi(t_1,t_1) + R_\psi(t_2,t_2))\right)chR_\psi(t_1,t_2),$$

$$E(sin\overset{o}{\psi}(t_1)sin\overset{o}{\psi}(t_2)) = exp\left(-\frac{1}{2}(R_\psi(t_1,t_1) + R_\psi(t_2,t_2))\right)shR_\psi(t_1,t_2),$$

where sh and ch are, respectively, hyperbolic sine and cosine. Using these formulas, we can derive the covariance functions and the mean values for the $x'(t), y'(t)$ processes:

$$R_{x'}(\tau) = \frac{v^2}{2}e^{-\sigma_\psi^2}(e^{R_\psi(\tau)} - 1) + \frac{v^2}{2}cos2\psi_0 e^{-\sigma_\psi^2}(e^{-R_\psi(\tau)} - 1), \quad (2)$$

$$R_{y'}(\tau) = \frac{v^2}{2}e^{-\sigma_\psi^2}(e^{R_\psi(\tau)} - 1) - \frac{v^2}{2}cos2\psi_0 e^{-\sigma_\psi^2}(e^{-R_\psi(\tau)} - 1), \quad (3)$$

$$E(x'(t)) = v\, cos\psi_0 e^{-\sigma_\psi^2/2} = c_x, \quad (4)$$

$$E(y'(t)) = v\, sin\psi_0 e^{-\sigma_\psi^2/2} = c_y, \quad (5)$$

where $\sigma_\psi^2 = A_1 + A_2 + A_3$ — variance of $\psi(t)$ process. In this paper, we neglect the mutual covariance function of the processes $x'(t), y'(t)$, i.e., we assume that the processes $x'(t), y'(t)$ are independent. For next transformations, instead of the exact expression (1), the following approximating expression will be used:

$$R_\psi(\tau) = \sigma_\psi^2 e^{-\alpha|\tau|}, \quad (6)$$

where

$$\alpha = max\{\alpha_1, \alpha_2, \alpha_3\}.$$

After expanding the exponential functions in (2), (3) into Taylor series in the vicinity of $R_\psi(\tau) = 0$, using only linear terms and taking into consideration the expression (6), the following expressions will be obtained instead of (2), (3):

$$R_{x'}(\tau) = \sigma_{x'}^2 e^{-\alpha|\tau|}, \quad (7)$$

$$R_{y'}(\tau) = \sigma_{y'}^2 e^{-\alpha|\tau|}, \quad (8)$$

where

$$\sigma_{x'}^2 = v^2\sigma_\psi^2 sin^2\psi_0 e^{-\sigma_\psi^2},$$

$$\sigma_{y'}^2 = v^2\sigma_\psi^2 cos^2\psi_0 e^{-\sigma_\psi^2}.$$

The processes with the covariance functions (7), (8) and mean values (4), (5) are generated by the following differential equations:

$$x''(t) + \alpha x'(t) = c_x + \omega_x(t), \quad (9)$$

$$y''(t) + \alpha y'(t) = c_y + \omega_y(t), \quad (10)$$

where $\omega_x(t), \omega_y(t)$, are white noises with spectral densities $2\alpha\sigma^2_{x'}$ and $2\alpha\sigma^2_{y'}$ respectively.

From the continuous time equations (9), (10), the discrete time state equations (Åström, 1970) can be obtained. After such transformation, the following state equations of Kalman filter will be obtained:

$$X(k+1) = FX(k) + W_x(k), \tag{11}$$
$$Y(k+1) = FY(k) + W_y(k), \tag{12}$$

where $X^T = (x, x', c_x)$, $Y^T = (y, y', c_y)$, T — transposition symbol, x, y — ship coordinates in the Cartesian coordinate system, x', y' – projections of ship velocity vector on axes X and Y, c_x, c_y — mean values of ship velocity on axes X and Y, F — transition matrix described by the following expression:

$$F = \begin{pmatrix} 1 & (1-exp(-\alpha T))/\alpha & (exp(-\alpha T)+\alpha T - 1)/\alpha^2 \\ 0 & exp(-\alpha T) & (1-exp(-\alpha T))/\alpha \\ 0 & 0 & 1 \end{pmatrix}. \tag{13}$$

Covariance matrices of noise vectors $W_x(k)$ and $W_y(k)$ are described by the following expressions:

$$Q_x = \begin{pmatrix} 2\alpha\sigma^2_{x'}Q & 0 \\ 0 & 0 \end{pmatrix}, \quad Q_y = \begin{pmatrix} 2\alpha\sigma^2_{y'}Q & 0 \\ 0 & 0 \end{pmatrix}, \tag{14}$$

$$Q = \begin{pmatrix} (4e^{-\alpha T} - 3 - e^{-2\alpha T} + 2\alpha T)/2\alpha^3 & (1-2e^{-\alpha T}+e^{-2\alpha T})/2\alpha^2 \\ (1-2e^{-\alpha T}+e^{-2\alpha T})/2\alpha^2 & (1-e^{-2\alpha T})/2\alpha \end{pmatrix},$$

where T — sampling time period. Taking into consideration that the processes $x'(t), y'(t)$ are assumed independent, equations (11), (12) are also independent.

Let us consider the process of deriving model (11), (13), (14) from model (9). The equation (9) can be presented in the following vector normal form:

$$X'(t) = A\,X(t) + W_x(t),$$

where

$$X^T(t) = (x \quad v_x \quad c_x),\; W_x^T(t) = (0 \quad \omega_x \quad 0),\; A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & -\alpha & 1 \\ 0 & 0 & 0 \end{pmatrix}.$$

The following discrete time state equation corresponds to the above equation (Åström, 1970):

$$X(t_{i+1}) = F(t_{i+1}, t_i)\,X(t_i) + W_x(t_i),$$

where $F(t_{i+1}, t_i)$ matrix is a solution of the following differential equation:

$$\frac{dF(t,t_i)}{dt} = A\,F(t,t_i), \; t_i \le t \le t_{i+1}, \tag{15}$$
$$F(t_i, t_i) = I, \tag{16}$$

I — identity matrix, $W_x(t_i)$ — series of independent random vectors with zero mean value and the following covariance matrix:

$$Q_x = E(W_x(t_i)W_x^T(t_i)) = \int_{t_i}^{t_{i+1}} F(t_{i+1}, s)\, R(s) F^T(t_{i+1}, s)\, ds, \tag{17}$$

$R(s)$ — the intensity of white noise $W_x(t)$,

$$R(s) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2\alpha\sigma_{x'}^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{18}$$

By solving equation (15) with initial conditions (16) under $t_{i+1} - t_i = T$, we can obtain the following matrix exponential expression:

$$F(T) = F = exp(AT),$$

from which expression (13) is obtained. Substituting the matrices (13), (18) in integral (17) and taking this integral, we obtain the expression for matrix Q_x in (14).

Similarly the model (12), (13), (14) is derived from the model (10).

Assume that the values of the distance between the observer and the object D and the angle of direction to the object (bearing) φ are measured. The results of measurements can contain errors. The observed values z_x, z_y of the object coordinates x, y are defined by the following formulas:

$$z_x = Dcos\varphi, \; z_y = Dsin\varphi. \tag{19}$$

Starting from these formulas, the following expressions for observation equations are obtained:

$$Z_x(k) = H_x X(k) + v_x(k), \; Z_y(k) = H_y Y(k) + v_y(k), \tag{20}$$

where z_x, z_y are the observed values of the x, y coordinates,

$$H_x = (1 \quad 0 \quad 0), \; H_y = (1 \quad 0 \quad 0),$$

$v_x(k), v_y(k)$ are the series of independent random values (measurement errors) with mean values equal to zero, and variances defined by the following expressions:

$$\sigma_x^2 = \sigma_D^2 cos^2\varphi + D^2\sigma_\varphi^2 sin^2\varphi, \; \sigma_y^2 = \sigma_D^2 sin^2\varphi + D^2\sigma_\varphi^2 cos^2\varphi, \tag{21}$$

σ_D^2 is an error of distance measurement, σ_φ^2 is an error of angle measurement. The expressions of variances (21) are obtained by linearization of functions (19) in the vicinity of exact values of distance and angle, by calculation of variances for linear forms obtained, and by substitution of the observed values in place of the exact ones.

As a result, the state equations (11), (12) and the observation equations (20) are obtained. The Kalman filter for them is described by the following expressions (Kalman, 1960):

$$X_e(k) = FX_f(k-1),$$
$$P_{xe}(k) = FP_{xf}(k-1)F^T + Q_x,$$
$$\Gamma_x(k) = P_{xe}(k)H_x(H_xP_{xe}(k)H_x^T + \sigma_x^2)^{-1},$$
$$X_f(k) = X_e(k) + \Gamma(k)(z_x(k) - H_xX_e(k)),$$
$$P_{xf}(k) = P_{xe}(k) - \Gamma_x(k)H_xP_{xe}(k),$$
$$Y_e(k) = FY_f(k-1),$$
$$P_{ye}(k) = FP_{yf}(k-1)F^T + Q_y,$$
$$\Gamma_y(k) = P_{ye}(k)H_y(H_yP_{ye}(k)H_y^T + \sigma_y^2)^{-1},$$
$$Y_f(k) = Y_e(k) + \Gamma(k)(z_y(k) - H_yY_e(k)),$$
$$P_{yf}(k) = P_{ye}(k) - \Gamma_y(k)H_yP_{ye}(k).$$

In these expressions $X_e(k), Y_e(k)$ are one-step predictors of the state vectors X, Y, $P_{xe}(k), P_{ye}(k)$ are error covariance matrices, $\Gamma_x(k), \Gamma_y(k)$ are filter gains, $X_f(k), Y_f(k)$ are corrected state vectors X, Y, $P_{xf}(k), P_{yf}(k)$ are error covariance matrices. The calculations in the above expressions are to be executed in the order in which these expressions are presented.

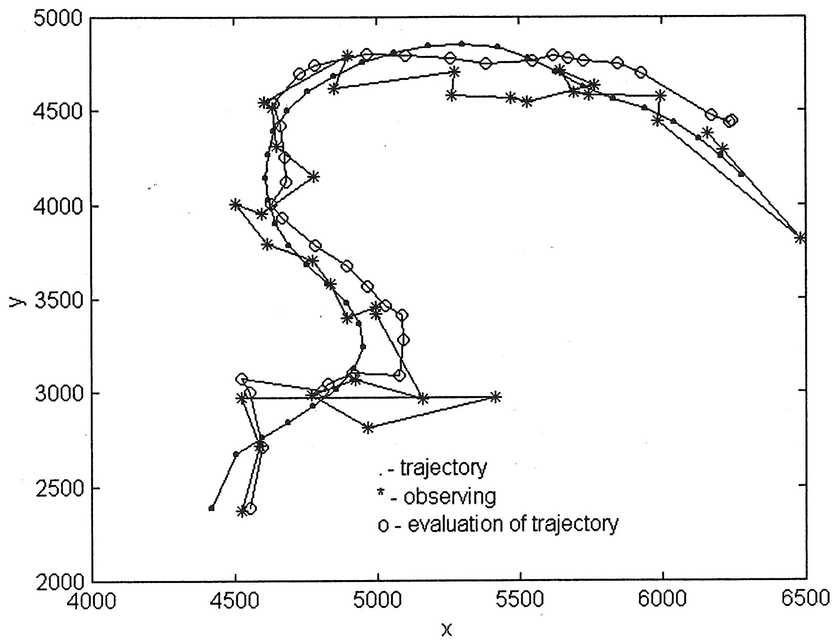

Figure 2

3. Simulation

For the problem discussed above, the computer simulation was executed with the following parameters: $v = 8.22$, $a = 1.3$, $r = 1.0$, $L = 130.91$, $k = 1.429$, $\alpha_1 = 0.033$, $\beta_{max} = 35^o$, $\psi_0 = 45^o$, $\sigma_D = 100$, $\sigma_\varphi = 1.7^o$, $T = 15$. The process of movement illustrated on the flow chart (Fig. 1) was simulated. The results of simulation are presented in Fig. 2, where the movement trajectory, its observations and filtration evaluation of trajectory are shown. The accuracy of filtration can be estimated by the value of total deviation from trajectory over all steps. For observations shown in Fig. 2 this value is equal to 5980, and for filtration evaluation it is equal to 5589. In spite of simplifications used in model construction (11), (12), the results of simulation confirm the applicability of the proposed model for evaluation of the maneuvering object's trajectory.

References

ÅSTRÖM, K.J. (1970) *Introduction to stochastic control theory.* Academic Press, New York.

BAR-SHALOM, Y. and BIRMIVAL, K. (1982) Variable dimension filter for maneuvering target tracking. *IEEE Trans. Aerospace and Electr. Syst.*, **AES-18**, 5, 621–629.

BASIN, A.M. (1968) *Propulsius quality and controllability of a ship. Controllability of a ship.* Transport Publ., Moscow (in Russian).

BOGLER, P.L. (1987) Tracking maneuvering target using input estimation. *IEEE Trans. Aerospace and Electr. Syst.*, **AES-23**, 3, 298–310.

KALMAN, R.E. (1960) A new approach to linear filtering and prediction problems. *Trans. ASME. Series D. Journal of basic engineering.* March 1960, 35–45.

MOOSE, R.L., VANLANDINGHAM, H.F. and MCCABE D.H. (1979) Modeling and estimation for tracking maneuvering targets. *IEEE Trans. Aerospace and Electr. Syst.*, **AES-15**, 3, 448–456.

PRUDNIKOV, A.P., BRYCHKOV, Y.A. and MARICHEV, O.I. (1981) *Integrals and Series. Elementary Functions.* Nauka Publ., Moscow (in Russian).

RAMAHANDRA, K.V. (1987) Optimum steady state position velocity and acceleration estimation using noisy sampled position data. *IEEE Trans. Aerospace and Electr. Syst.*, **AES-23**, 5, 705–708.

SINGER, R.A. (1970) Estimating optimal tracking filter performance for manned maneuvering targest. *IEEE Trans. Aerospace and Electr. Syst.*, **AES-6**, 4, 473–483.