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Objectives of an enterprise. Bi-criteria analysis and negotiation problems*

by

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Abstract: A decision-making process is considered for a firm, in which two coexisting groups of interests pursue different goals. An original model based on a non-neoclassical production function is proposed. The function satisfies the conditions formulated by R. Frisch, which makes it possible to investigate firms operating in the environment far from the perfect competition and pursuing goals other than profit maximization. A two-criteria optimization problem is formulated with the two criteria representing the goals of the groups: maximization of profit and maximization of income generated by the firm with respect to capital and labor. The problem is considered in two variants of the product market, namely the perfect and the imperfect competition. Solutions of the problem are analyzed including the derived Pareto sets. The importance of knowledge about the Pareto set in negotiations between the groups of interests in the firm is illustrated and discussed.

Keywords: economic modelling, production function, cost function, bi-criteria optimization, negotiations

1. Introduction

In this paper, the problem is considered of how to reconcile conflicting interests, coexisting within a firm. The goals representing the interests differ not only with respect to the time horizon, but also due to different internal interests of those participating in the functioning of a firm. Separate analyses of the particular goals can lead to different, often conflicting solutions of the decision-making problems.

The paper presents the potential of the multicriteria analysis, which makes it possible to jointly consider those goals and determine the Pareto optimal solutions, which help approaching a compromise between different interests within the firm.

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This study is based on a mathematical model of the firm this model consisting of two sub-models: the first one describes the interdependence between production and inputs, while the other deals with the short-term relation between production and particular categories of costs. Main production factors, i.e. capital and labor, are being accounted for, as well as intermediate inputs. Our considerations include also the condition of the product market, i.e. the perfect competition treated as a reference, and its alternative: the imperfect competition. We assume that in the perfect competition the volume of production of a single considered firm does not affect the level of the market price, while in the imperfect competition decisions of the firm concerning the output volume significantly affect the resulting price.

The model described above has been adopted to formulate the two-criteria optimization problem with the two simultaneously maximized goals: profit and income of the firm, with the decision variables being the capital and labor. Such an approach is motivated by the fact that both owners and managers of the firm strive to maximize profit, while employees are usually interested in maximizing the volume of production, which is commonly related to the system of wages. Solutions of this problem are presented, and in particular the Pareto sets in the cases of the perfect and imperfect competition. In a negotiation process, one typically looks for solutions satisfactory to both groups of interests. The importance of familiarity with the Pareto set in negotiation is discussed. It is illustrated by the example of a single negotiation text procedure, see Raiffa (1982), applied to the considered problem. We conclude that the familiarity with the Pareto set can provide grounds for possible negotiations between the groups of interests mentioned. In general, the negotiation problems are discussed in a wide bibliography. The exemplary papers of Kersten (1988), Kersten and Lai (2007), Kruś (2001), de Almeida and Wachowicz (2017), Wachowicz, Kersten and Roszkowska (2019), Wierzbicki, Kruś and Makowski (1993) present different aspects of negotiations, including analysis, procedures, decision support and e-negotiation systems.

We would like to emphasize the outstanding contribution of Gregory Kersten to the scientific and experimental research on the negotiation problems. His great impact on the progress in this research encompasses original personal achievements as well as inspirations for many other researchers, engaged in the area of negotiations.

In the present paper, this Introduction is followed by sections devoted to Model of supply (2); Production factors: capital and labor (3); Production function (4); Short-term production costs (5); Conditions of the product market (6); Goals of the firm (7); Bi-criterion analysis (8); Summary (9); and References that are included in the last section of the paper.

2. Model of supply

In the considerations, presented in this paper, we were inspired, *inter alia*, by the non-classical approaches to the economics of the firm presented, for example, in Furubotn (1999, 2001) and Pejovich (2008, 2011).

In modeling supply, the following issues are generally addressed: relationship between the production capacity and the amounts of the inputs of the main production factors, the capital and labor; utilization of the production capacity and the production structure in the case of production of multiple products. In the paper, only the first issue is considered.

In the short-term analysis it is assumed that the decision variable is the labor employed, while the amount of capital, management system and technology remain constant. Hence, production function used in the short-term analysis has one decision variable and constant parameters.

In the long-term analysis, production capacity is determined by the variable amounts of capital and labor, while the management system and production technology remain constant.

In this paper, production factors are divided into two categories: the main and the intermediate ones. Main production factors are the capital and labor, while the intermediate ones consist of energy, materials and services, which are assumed to be proportionally consumed in the production process. The main factors determine the production capacity via the production function.

3. Production factors: capital and labor

The amount of capital representing given technology is described by the following relationship:

$$K_{t+1} = K_t + I_t - \delta K_t \,, \tag{1}$$

where:

K_t – stock of capital at the beginning of the period t,

I_t – investment in the period t;

δ – depreciation coefficient, $0 < \delta < 1$;

δK_t – depreciation of capital.

Labor is divided into two categories: one is related to production, and the other to the management of the firm. In the further part of this paper the cost associated with the former is ascribed to the variable cost, while the cost of the latter is a part of the fixed costs.

As for the category of labor, it will be assumed that it is a monotonously increasing, albeit non-linear function of the volume of production. Details of this relationship will be presented further on. It will also be assumed that the labor involved in the management is constant.

4. Production function

Two-factor production function F with two variables representing labor L and capital K determines output Q of the firm:

$$Q = F(K, L). \tag{2}$$

Both variables are non-negative and the following conditions are met:

$$F(K, 0) = 0,\ F(0, L) = 0, \tag{3}$$

It follows from the expressions (3) that in order to obtain a non-zero result of production it is necessary for both production factors to be greater than zero. The above conditions are accepted by all schools of production modeling. However, as we intend to examine optima of the firm functioning not only in the environment close to the model of perfect competition, we employ the production function having the properties postulated by Frisch (1965). This approach excludes the application of the neoclassical production functions, such as CES, see Arrow et al. (1961), or Cobb-Douglas, see Cobb and Douglas (1928), which are assumed to be restricted to the profit maximizing firms.

In further considerations we assume constant returns to scale, which means that any increase of the input of the production factors by the proportion of λ causes an increase of the output by the proportion of λ. Accordingly, under such assumption the production function can be presented in the following way:

$$Q = F(K, L) = L \times H(U, 1) = L \times P_L(U), \tag{4}$$

where $H(U, 1)$ denotes the average productivity of labor $P_L(U)$ with respect to the capital-to-labor ratio U, $U = K/L$. Expression (4) implies that the average productivity of labor can be expressed as a function of only one argument: the capital-to-labor ratio. Under the above assumption the productivities of the production factors (the average productivity of labor $P_L = \frac{Q}{L}$ and the average productivity of capital $P_K = \frac{Q}{K}$) do not change, if the capital-to-labor ratio remains constant.

Ragnar Frisch formulated the postulates concerning the properties of the production function in his book *Theory of production* (Frisch, 1965). These postulates were based on empirical analyses. However, the book does not contain specification of any function. This paper includes a proposal of the function satisfying these postulates.

In Fig. 1 the curve representing the short-terms relationship (i.e. based on the assumption that the capital, technology and management system remain constant and the intermediary factors are abundant) between the output and the employment of labor is shown, see, for example: Begg et al. (2008), Jehle and Reny (2011), Mansfield (1985). The shape of this relationship is consistent with the above-mentioned postulates of R. Frisch.

Note that in Fig. 1 there are four points distinguished on the horizontal axis, which correspond, respectively (looking from the left to the right), to the following statements: zero employment causes zero output, for employment equal L_1 the output is equal to the vertical coordinate of the inflexion point, then, point L_2 corresponds to the tangency point between the curve and the auxiliary line starting at the beginning of the coordinate system, and at the point L_3, the analyzed curve attains its maximum. Note that in the point L_1 the marginal productivity of labor $\frac{\partial Q}{\partial L}$ attains maximum, in the point L_2 the average productivity of labor $\frac{Q}{L}$ attains the maximum, and in the point L_3 the productivity of capital $\frac{Q}{K_0}$ attains the maximum. It is also necessary to note that employment of labor above the level of L_3 creates surplus of labor, because the same result can be obtained with lower employment, not mentioning the money cost, which will be discussed later on.

It is necessary to note that for the labor belonging to the range (L_2, L_3) the productivity of labor is a decreasing function of labor while the productivity of capital is an increasing function of labor. Note that in the case of the neoclassical production function this property holds true for all the positive values of L.

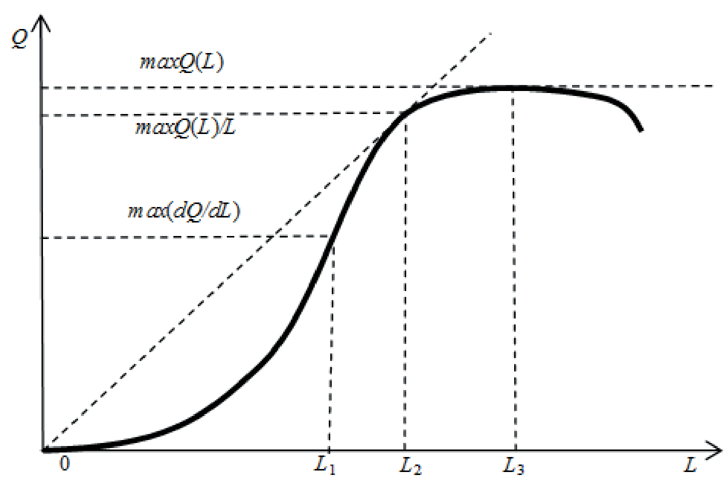

Figure 1: The short-term relationship between the output Q and employment L. Here and further on: all figures are the results of own calculations, if not otherwise indicated

The long-term analysis of production that uses the production function satisfying Frisch's postulates is illustrated in Fig. 2, where its contour line for the output Q_0, $Q_0 > 0$, is presented. The contour line, shown in Fig. 2, reveals the essential properties of the production function, satisfying Frisch's postulates. Producing quantity Q_0 requires at least a certain minimum input of labor $minL(Q_0)$ and at least a certain amount of the input of capital $minK(Q_0)$. According to the assumption that the production function is homogenous of order one, all points lying on the line starting from the beginning of the system of coordinates (the same capital-to-labor ratio) represent the same level of the productivities of the production factors.

Three points can be distinguished on the depicted contour line: Θ_1, Θ_2 and Θ_3. The first represents output of Q_0 with the maximum marginal productivity of labor applying the capital-to-labor ratio equal U_1 (in the short-term it would be equal to $U_1 = K_0/L_1$); the point Θ_2 represents the output of Q_0 with the maximum average productivity of labor applying the capital-to-labor ratio equal U_2 (in the short-term it would be equal to $U_2 = K_0/L_2$) and the point Θ_3 represents the output of Q_0 with the maximum productivity of capital applying the capital-to-labor ratio equal U_3 (in the short-term it would be equal to $U_3 = K_0/L_3$).

Note that in this type of production function the substitution between factors is limited to the range (U_2, U_3); an attempt to produce beyond that range causes excessive usage of the production factor, thus lowering the physical efficiency of production and increasing production costs, which will be discussed later on.

Authors of this paper found only one production function that fulfils R. Frisch's postulates and can be applied in both the short-term and long-term analysis. This function, Gadomski (1992), has the following form:

$$Q = F(K, L) = \begin{cases} 0, & K = 0,\ L > 0\ ; \\ 0, & K > 0,\ L = 0; \\ 0, & K = 0,\ L = 0\ ; \\ L \times P_L^* \left[\frac{\frac{K}{L}}{U_2} \exp\left(1 - \frac{\frac{K}{L}}{U_2}\right) \right]^{\beta}, & K > 0,\ L > 0. \end{cases} \tag{5}$$

where:

P_L^* - parameter, maximum value of the average productivity of labor $P_L = Q/L$;

U_2 - parameter, value of the capital-to-labor ratio $U = K/L$, for which the average productivity of labor $P_L(U)$ achieves maximum P_L^*, $P_L(U_1) = P_L^*$; corresponding to the gradient of the u_2 line in Fig. 4;

β - parameter, $\beta > 1$.

In the short term analysis, assuming constant capital K_0, the production function attains its maximum when the capital-to-labor ratio is equal to $U_3 = U_2 \frac{\beta - 1}{\beta}$, that corresponds to the slope of the line u_3, Fig. 2. Assuming the capital-to-labor ratio equal to $U_1 = U_2 \left(1 + \frac{\sqrt{\beta}}{\beta}\right)$, which is equal to the slope of the u_1 line, the function (5) achieves the inflection point that corresponds to

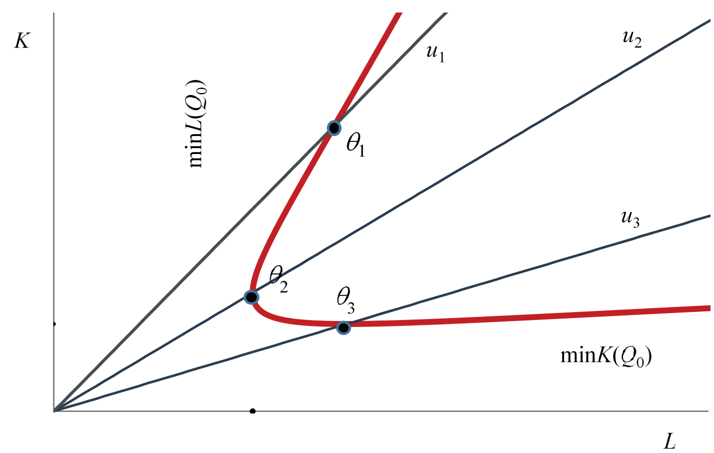

Figure 2: Contour line for the output Q_0 of the Frisch type production function

the maximum of the marginal productivity of labor

$$(\frac{\partial^2 Q}{\partial L^2} = \left(\frac{\beta}{U_2 L}\right)^2 Q \left[U - U_2\left(1 + \frac{\sqrt{\beta}}{\beta}\right)\right]\left[U - U_2\left(1 - \frac{\sqrt{\beta}}{\beta}\right)\right]),$$

see Fig. 2.

Another important property of the production function (5) is that the following variables:

$$\frac{Q}{L},\ \frac{Q}{K},\ \frac{\partial Q}{\partial L},\ \frac{\partial Q}{\partial K}$$

can be expressed as a function of a single variable, the capital-to-labor ratio $U = \frac{K}{L}$, because:

$$\frac{Q}{L} = P_L\left(U\right) = P_L^* \left[\frac{U}{U_2}\exp\left(1 - \frac{U}{U_2}\right)\right]^\beta,$$

$$\frac{Q}{K} = P_K\left(U\right) = \frac{P_L^*}{U} \left[\frac{U}{U_2}\exp\left(1 - \frac{U}{U_2}\right)\right]^\beta,$$

and

$$\frac{\partial Q}{\partial L} = P_L\left(U\right)\frac{\beta}{U_2}\left(U - U_2\frac{\beta - 1}{\beta}\right),\ \frac{\partial Q}{\partial K} = P_K(U)\frac{\beta}{U_2}\left(U_2 - U\right).$$

5. Short-term production costs

Costs reflect the money value of the production inputs used in the process of production. The following costs are distinguished in this analysis: monetary values of the main production factors (capital and labor), and value of the intermediary factors (materials, energy, services, etc.) consumed within production.

Two categories of cost are considered: the fixed cost FC and variable cost VC. The fixed costs FC are the ones that are independent of the amount of the output and consist of the amortization of the capital (fixed assets) $\delta p_K K_0$, where δ stands for the depreciation rate, p_K - price or index price of the capital unit and K_0 denotes the stock of capital at the beginning of the period; as well as other cost such as licenses, leases etc., FC_0. The fixed cost FC is described by the following expression:

$$FC\,(Q) = \delta p_K K_0 + FC_0. \tag{6}$$

Average fixed cost AFC is defined as the average cost of producing unit of the output AFC, $AFC{=}FC/Q$:

$$AFC\,(Q) = \frac{\delta p_K K_0}{Q} + \frac{FC_0}{Q}. \tag{7}$$

Variable cost VC is defined as the sum of the labor cost and the cost of the intermediate factors used in production Q, which are assumed to be linearly dependent on the output:

$$VC\,(Q) = wL + p_m qtQ, \tag{8}$$

where:

w – unit labor cost, wage rate,

p_m - price (index) of the intermediary factor,

q - material intensity of production.

Variables L and Q have been defined above. As to the wage rate w and unit price/index p_m of the intermediary factors it is assumed that they are determined at their respective markets and remain constant. In some cases, as, for example, in the case of the monopoly, it would be sensible to assume certain relation between the demand for the production factors and their prices. However, that aspect of the analysis will be not considered in this paper.

Note that the model of variable cost (8) consists of two parts: the linear $p_m qQ$ and non-linear wL, because the relation between the demand for labor L and the output Q is an inverse function $L = F^{-1}(Q)$, which is non-linear and defined for the range $[0\ ,\ L_3]$, see Fig. 2; note also that the expression $p_m q$ can be interpreted as the marginal cost of the intermediary input.

The average variable cost AVC is interpreted as the variable cost of producing one unit of product:

$$AVC\,(Q) = \frac{VC(Q)}{Q} = w\frac{L}{Q} + p_m q. \tag{9}$$

Expression (9) indicates that the average variable cost is a linear function of the intensity of labor L/Q and the defined above constant $p_m q$ (marginal cost of the intermediary input).

Total cost C is the sum of the variable and fixed costs:

$$C\,(Q) = wL + p_m qQ + \delta p_k K_0 + FC_0, \tag{10}$$

and the average cost AC is given by the following relationship:

$$AC\,(Q) = \frac{C(Q)}{Q} = \frac{wL}{Q} + p_m q + \frac{\delta p_k K_0}{Q} + \frac{FC_0}{Q}\,. \tag{11}$$

The marginal cost of producing Q units of product is expressed by the following equation:

$$MC\,(Q) = \frac{dC}{dQ} = \frac{w}{dQ/dL} + p_m q, \tag{12}$$

on the basis of differential of the inverse function.

It should be noted that both the average variable cost AVC and the marginal cost MC do not depend on the quantity of Q, but on the proportion of capital and labor that is expressed by the capital-to-labor ratio $U, U = K/L$. One should note that this property is not valid in the case of the average fixed cost AFC. It will be shown further on that it plays an important role in determining the location of the minimum of the average cost and the long-term optimum of the profit maximizing firm.

When the production function is described by the formula (5), the relationships between the unit costs, i.e. AVC, AFC, AC and MC, and the output are represented by the respective curves in Fig. 3. This figure shows that the use of the production function (5) provides for the compatibility of the production theory and the theory of the short-term production costs.

The marginal cost MC attains its minimum MC_{min}, equation (12), at the point $L_1 = \frac{K_0}{U_1}$:

$$L_1 = L_2\,\frac{\beta - \sqrt{\beta}}{\beta - 1},$$

which is identical with the point, at which dQ/dL attains its maximum. The value of the marginal cost MC in that point is equal:

$$MC_{min} = MC\,(P_L(U_1)\,) = \frac{wL_1}{Q(L_1)} + p_m q = \frac{w}{P_L(U_1)} + p_m q. \tag{13}$$

The average variable cost AVC attains its minimum for the same input of labor L_2 at which the average productivity of labor P_L attains its maximum, i. e. for the input of labor equal to $L_2 = \frac{K_0}{U_2}$:

$$AVC_{min} = AVC\,(P_L(U_2)\,) = \frac{wL_{21}}{Q(L_2)} + p_m q = \frac{w}{P_L(U_2)} + p_m q. \tag{14}$$

The average fixed cost AFC is minimal when labor equals L_3, $L_3 = K_0/U_3$, where also production capacity attains its maximum.

$$AFC_{min} = AFC\,(Q(L_3)\,) = \frac{\delta\, p_K\, K_0 + FC_0}{Q(L_3)}\,. \tag{15}$$

The values of L_1, L_2 and L_3 depend on the parameters U_2 and β of the production function (5), while the labor L_{minAC}, for which the average cost AC achieves the minimum depends on the prices of the production factors w – of labor, and p_K – of capital, as well as δ - capital depreciation rate.

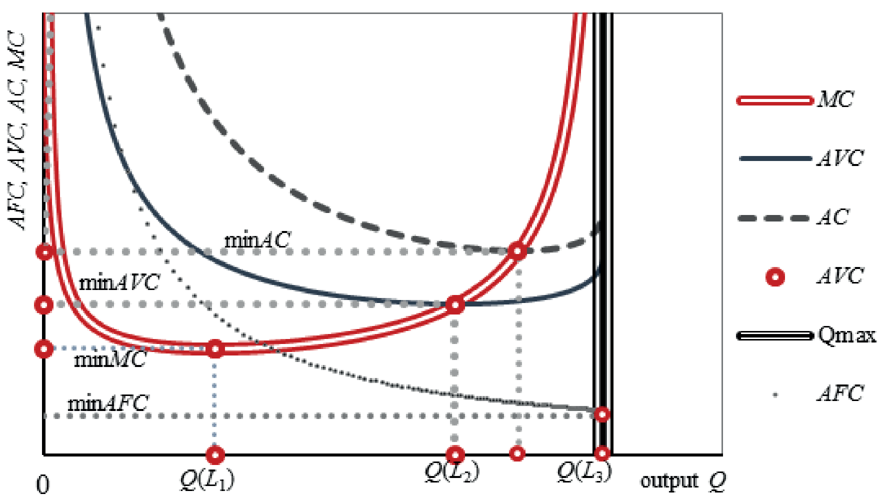

Figure 3: Average costs AC, AFC, AVC and marginal cost. The short-term analysis

Theory of production costs implies that the labor input L_{minAC}, at which the average cost AC attains its minimum belongs to the range $(L_2\,,\, L_3)$. The expression given below has been arrived at using the condition dAC/dL=0, which can be transformed into the binomial with two roots, of which only the positive one is considered (Gadomski, 2020):

$$U_{minAC} = \frac{1}{2}\left[\left(U_3 - \frac{w}{\delta p_K} - \frac{FC_0}{\delta p_K L}\right) + \sqrt{\left(U_3 - \frac{w}{\delta p_K} - \frac{FC_0}{\delta p_K L}\right)^2 + 4\left(\frac{w}{\delta p_K}U_2 - \frac{FC_0}{\delta p_K L}U_3\right)}\,\right]. \tag{16}$$

If one assumes that the component FC_0 of the fixed cost FC equals zero, the above equation reduces to the simpler form:

$$U_{minAC} = \frac{1}{2}\left[\left(U_3 - \frac{w}{\delta p_K}\right) + \sqrt{\left(U_3 - \frac{w}{\delta p_K}\right)^2 + 4\frac{w}{\delta p_K}U_2}\right].$$

The two formulae, provided above, will be discussed in the part devoted to the long-term analysis.

6. Conditions at the product market

In this paper we focus on two models of the product markets: the perfect and imperfect competition, and in the case of the latter two submodels are distinguished. The first model describes the market with monopolistic producer and the other describes the oligopolistic competition.

The perfect competition model is the one where there are many producers, of which no one is dominant. At a given market price the market absorbs the entire output of the firm. The elasticity coefficient of demand tends to infinity.

Generally, the revenue R of the firm producing Q units is assumed to be equal to:

$$R(Q) = P(Q)\,Q, \tag{17}$$

and in the case of the perfect competition $P(Q) = P = \textit{const.}$, because the market price P is given and independent of the output.

The marginal revenue MR is defined in the following relationship:

$$MR = \frac{dR}{dQ}, \tag{18}$$

which can be interpreted as the price obtained from the sale of the last unit and in general can be expressed in the following way:

$$MR = \frac{dR}{dQ} = \frac{dP}{dQ}Q + P(Q). \tag{19}$$

Note that in the case of the perfect competition:

$$MR = \frac{dR}{dQ} = P = const, \tag{20}$$

because $\frac{dP}{dQ} = 0$.

All cases of imperfect competition have a common property, namely:

$$\frac{dP}{dQ} < 0, \tag{21}$$

which implies that at a given demand the increase of output causes a decrease of price, while the decrease of output causes an increase of price.

The price function, i.e. the inverse demand function, is described by the following expression:

$$P\left(Q\right)=a_0 exp\left(-a_1 Q\right) \tag{22}$$

where a_0 and a_1 are positive coefficients. The relationship (21) is characterized by the variable elasticity of price relative to output, which is equal $a_1 Q$.

The revenue $R(Q)$, established on the basis of relationship (21), has the following form:

$$R\left(Q\right)=Q\times P\left(Q\right)=Qa_0 exp\left(-a_1 Q\right) \tag{23}$$

which implies that the revenue achieves the maximum when the output is equal:

$$Q_{R_{max}}=1/a_1,$$

because:

$$\frac{\partial P}{\partial Q}=-a_1 P(Q)$$

and

$$\frac{\partial R}{\partial Q}=\frac{\partial Q}{\partial L}\left[\frac{\partial P}{\partial Q}Q+P(Q)\right]=P\left(Q\right)\left[1-a_1 Q\right].$$

The choice of the form of equation (23) is motivated by the fact that the exponential function is more elastic, as it allows for the maximum of the revenue to be located for U smaller than U_3. It is shown, equation (23) and later, that when

$$L_{R_{\max}}=1/\mathrm{a}_1<L_{Qmax}$$

the employment, at which revenue attains maximum can be smaller than the employment, at which production attains its maximum. This property (not fulfilled in the case of power function of the price) is vital for further investigations, not mentioned in this paper.

If demand is represented by the expression (21), then for all positive values of α_1 satisfying following relation:

$$a_1>1/Q_{max},$$

the revenue attains the maximum for output Q belonging to the range $(0, Q_{max})$.

The price function/the inverse demand function in the case of oligopoly is defined by the following relationship:

$$P\left(Q\right)=P\left(D\right)=a_0 \mathrm{exp}\left[-a_1(a_2+Q)\right], \tag{24}$$

where the positive coefficient a_2 represents the supply of other producers, while a_0 and a_1 have the same interpretation as in equation (5). This solution makes it possible that halting the production of the firm would set the price at the level $a_0 \exp[-a_1 a_2]$.

One can note also that the model (22) can be applied in the case of the perfect competition, if it were assumed that $a_1 = 0$.

7. Goals of firms

Goals of enterprises depend on many factors and differ in both the time-horizon and the form of ownership, various internal interests (like those of the owners/shareholders, high-rank managers, and employees) that are related to the motivation system used in a given firm. Market economy is dominated by private enterprises whose assumed formal goal is to maximize their profit. The term formal goal is used here on purpose, because apart from the interest of the owner there are groups of interest, such as those of managers and of workers, which have a real impact on the effectively pursued goal. In some large firms with dispersed ownership the goals of managers dominate, while in the firms with strong trade unions that formal goal is biased in a different way. It is necessary to indicate that owners and firm managers are usually interested also in the increase of the market share.

There are also firms that maximize income and not profit. This is the case of small family firms that do not employ wage-earning labor, or cooperatives and firms with the employee ownership. The respective problems were analyzed in Furubotn (1999, 2001) and Pejović (2001, 2008).

In this paper we focus on the firms, in which internal groups of interests have different goals, as those of the owners/shareholders who pursue profit maximization and the employed labor that pursues production maximization, because in most cases the employee-motivation systems are correlated with the volume of production. As shown in the cited literature, the goals of the production and income maximization are equivalent. However, those of profit maximization and income maximization differ; in the case of the imperfect competition they provide different optimum points, as shown in Fig. 6, but also in the cited literature: Furubotn (2001), Pejović (2001), Gadomski (2020).

7.1. Profit maximization

In the short-term framework it is assumed that labor L is only the variable, while constant are: the capital $K = K_0$, the production technology and the management system.

Profit is defined by the following expression:

$$\pi(Q) = R(Q) - C(Q), \tag{25}$$

where $\pi\,(Q)$ denotes the profit obtained from the sale of Q units of output.

Profit $\pi\,(Q)$ attains an extremum, if the following condition is satisfied:

$$MR{=}\frac{dR}{dQ} = \frac{dC}{dQ} = MC.$$

In the short term, the above condition can be presented in the alternative way:

$$\frac{dR}{dL} = \frac{dC}{dL}.$$

Note that the above condition can be satisfied by the maximum and the minimum of the profit. Obviously, the focus is on maximization.

7.2. Income maximization

Income $INC(Q)$ is defined as the revenue $R(Q)$ minus the costs unrelated to the employment (material cost $p_m qQ$, fixed cost $\delta p_k K_0 + FC_0$, and the capital depreciation cost $\delta p_k K$):

$$INC\,(Q) = P\,(Q)\,Q - p_m qQ - \delta p_k K_0 - FC_0. \tag{26}$$

In the model of the perfect competition the income $INC\,(Q)$ is an increasing function of output Q, because price P is constant and does not depend on the output Q. An enterprise that maximizes income in such a setting realizes its goal by maximizing output, i.e. at the point $L = L_3$.

An enterprise maximizing income $INC\,(Q)$ in the imperfect competition achieves the optimum when the employment L is equal $L = L_{max_{INC}}$, $L_{max_{INC}}$ $<L_2$.

Income $INC\,(Q)$ attains the maximum if the following equality is satisfied:

$$\frac{\partial INC}{\partial L} = \frac{\partial Q}{\partial L}\left[\frac{\partial P}{\partial Q}Q + P\,(Q) - p_m q\right] = 0, \tag{27}$$

this equality having two solutions; the first, for $L = L_3$, i.e. when the short-term output attains the maximum, and the second, when the marginal revenue, MR, $MR = \frac{\partial P}{\partial Q}Q + P\,(Q)$ is equal to the marginal cost of used materials $p_m q$ (the case of $Q = 0$ is not considered). Note that the solution can be located beyond the admissible range of labor, $[0\ ,\ L_3]$, that is, for $L{>}L_3$.

The comparison of the equations (26) and (27) implies the following relationship:

$$\frac{\partial \pi}{\partial L} = \frac{\partial INC}{\partial L} - w, \tag{28}$$

and, moreover,

$$L_{opt\,\pi} < L_{opt\,INC}. \tag{29}$$

The above relations show that in the short-term, *ceteris paribus*, the firm maximizing income has bigger output than the firm maximizing profit.

The goals of firms differ because enterprises differ as to the form of ownership; some of them maximize profit (private enterprises with hired labor) and some pursue maximum income (such as cooperatives, family businesses without hired workforce).

Within larger firms, there coexist different interests of distinct groups of actors involved: owners, managers and employees. Autonomous interests of the firm managers may emerge in corporations with dispersed ownership, but such a case is not discussed in this paper. Only two groups are considered here: owners, who are interested in the maximal profit, and employees, who are interested in the maximal income; the latter being equivalent to the output maximization wherever the employee motivation systems are correlated with the output.

The diversity of goals makes it impossible to simultaneously achieve the optimum points for each distinct criterion. Hence, the conflict of interests may be an immanent feature and can evolve into a more acute form.

Mitigation of internal conflicts can be accomplished by negotiations, aimed at reconciliation of conflicting interests.

8. Bi-criteria analysis

Let us consider a firm, in which decisions made take into account two groups of interests. The groups of interest have different goals: one pursues income maximization, $INC(Q)$, and the other pursues profit maximization, $\pi(Q)$. The decision-making problem can be described as the bi-criteria optimization problem (with respect to the inputs of capital K and labor L), shown below:

$$V_{max}[INC(Q), \pi(Q)], \tag{30}$$

where income $INC(Q)$ is defined by equation (26), profit $\pi(Q)$ – by (25). The production quantity $Q = F(K, L)$ is described by the assumed production function (2) and (5) as dependent on the capital and labor.

The problem is being solved with respect to the factors of capital and labor treated as variables. The requirements of positive values of the variables have to be satisfied:

$K, L \geq 0$, and $F(K, 0) = 0$, $F(0, L) = 0$.

The conditions are described in detail by relation (3).

Let Ω denote the set of all admissible values of variables K, L, satisfying the above conditions. The operator V_{max} means that we are looking for pairs of variables, which are Pareto-optimal in the set Ω. The pair (K, L) is Pareto-optimal if there exist no other pairs (K', L'), that would improve one of the criteria (INC or π) and would not be worse for the other criterion in the set Ω.

In the general case there exist many Pareto-optimal solutions of the problem (30), including solutions maximizing only the income, or only the profit, but also the intermediate ones, which can be negotiated while striving for a consensus.

The above problem is analyzed in the cases of two models of product markets: perfect and imperfect competition of firms. The above formulation relates to the case of long-term analysis in which both inputs, K and L, are treated as decision variables. We consider also the case of short-term analysis. In this case the capital is assumed to be constant and the labor L is treated as the decision variable. Pareto-optimal solutions of the problem are derived and discussed for the assumed parameters of the model, describing the enterprise activity. The computational results are illustrated in a graphical form.

8.1. Model of perfect competition

Figures 4 and 5 present the values of income and profit, respectively, as dependent on the variables of capital and labor. One can observe that in both these cases there exists a narrow ridge of the maximal values of income and profit with respect to the variables. Both income and profit increase almost linearly with the increase in capital and labor. It should be pointed out that small irregularities at the bottom of the figure are an effect of numerical inaccuracy.

The maximal values of income and profit as dependent on capital are shown in Fig. 6. The maximum is reached for capital $K = 175$ and labor $L = 320$ in the first case, and for the same capital and labor $L = 220$ in the second case. It means that there exists a set of Pareto-optimal solutions.

As it was mentioned already before, different internal interests in the firm may lead to different goals, represented by criteria like income and profit, considered jointly. The Pareto set represents a set of solutions, which can be the subject of possible negotiations. It is the set of solutions, for which the income and profit cannot be improved jointly. For the capital $K = 175$ the maximal profit is obtained at the labor $L = 220$. Increase of the labor variable value above this value leads to a decrease of the profit, but entails an increase of the income till $L = 320$, when the income is maximal. Both of the criteria decrease below $L = 220$ and beyond $L = 320$. A similar situation can be observed for a given lower value of the capital, $K = 70$. There also exists the Pareto set with the maximal profit at $L = 90$, and the maximal income at $L = 125$. In general, the simulation results indicate that the range of the Pareto set decreases at lower values of the capital

Figure 7 shows the values of income and profit as dependent on capital for different values of labor in the case of perfect competition. For a given value of labor, the maximal values of income and profit are reached at the same value of capital. The set of Pareto-optimal solutions reduces to just a single point.

For the given value of labor, $L = 120$, these maximal values of both criteria are reached at the capital $K = 120$, and for labor $L = 50$, at $K = 50$. Increasing

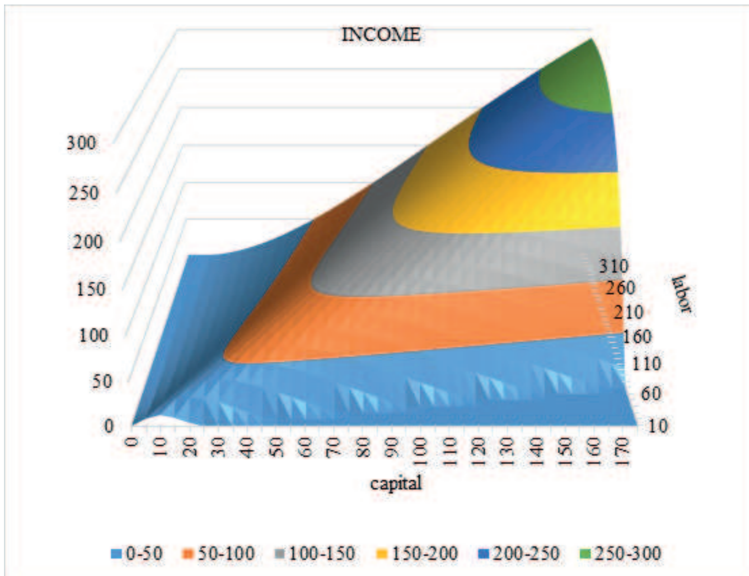

Figure 4: The dependence of income on capital and labor; the model of perfect competition; long-term analysis

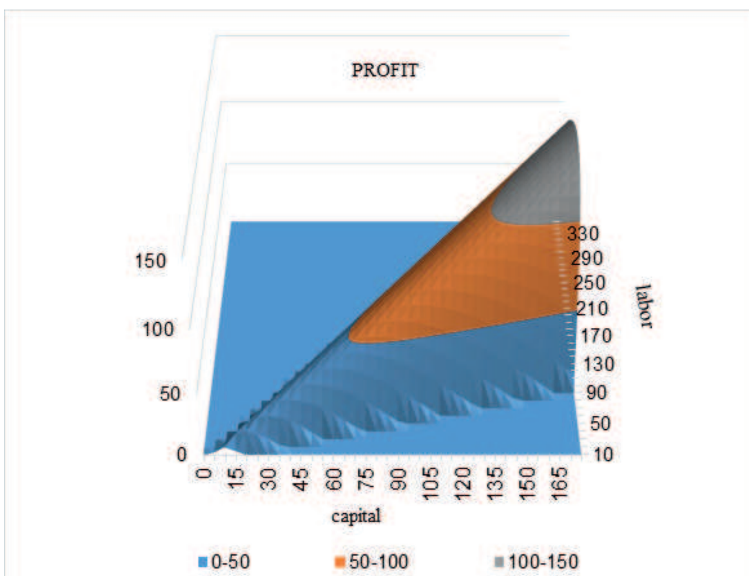

Figure 5: The dependence of profit on capital and labor; the model of perfect competition; long-term analysis

the input of the capital entails the necessity of increasing the amount of labor required to reach the maximal values of the criteria.

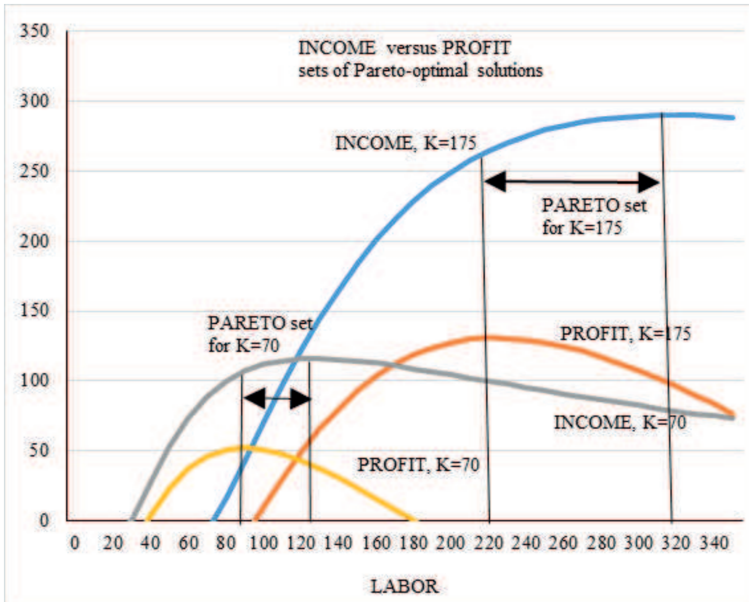

Figure 6: Income versus profit; dependence on labor for different values of capital; the model of perfect competition; long-term analysis

8.2. Model of imperfect competition

Values of income as dependent on the variables of capital and labor are presented in Fig. 8. In this case there exists a ridge of maximal values of the income for increasing values of capital and labor; however, it reaches its global maximal value and splits into two almost flat but gently sloping ridges. It means that the same locally maximal values of income can be obtained for two different and separate pairs of the capital and labor values.

The dependence of profit on both variables is shown in Fig. 9. In this case also a rising ridge can be seen along with the increasing values of capital and labor till the maximal value of the profit is reached. The ridge splits into two branches for further increases of capital and labor. In fact, these results are counterintuitive. The calculation process was, therefore, very carefully checked. The result may be an effect of the Frisch postulates and of the form of the applied production function, satisfying these postulates.

Let us look again at Fig. 1. The production quantity initially increases with increasing labor, reaches its maximum and then decreases. Moreover, in the case

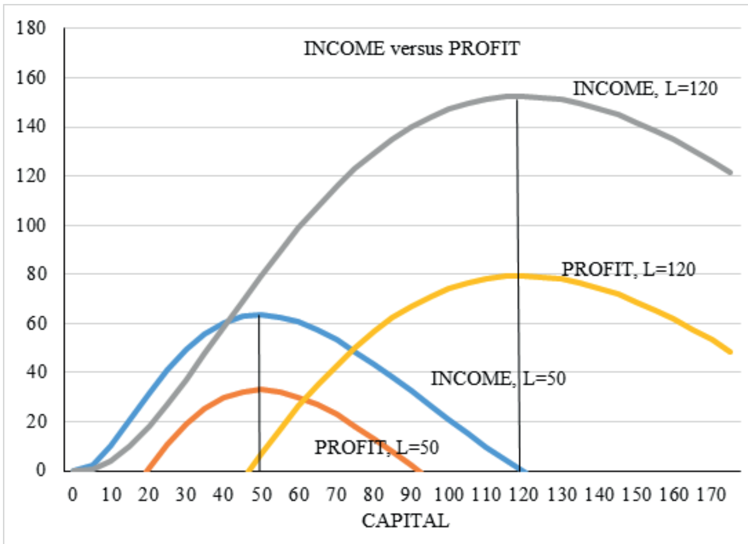

Figure 7: Income versus profit; dependence on capital for different values of labor; the model of perfect competition

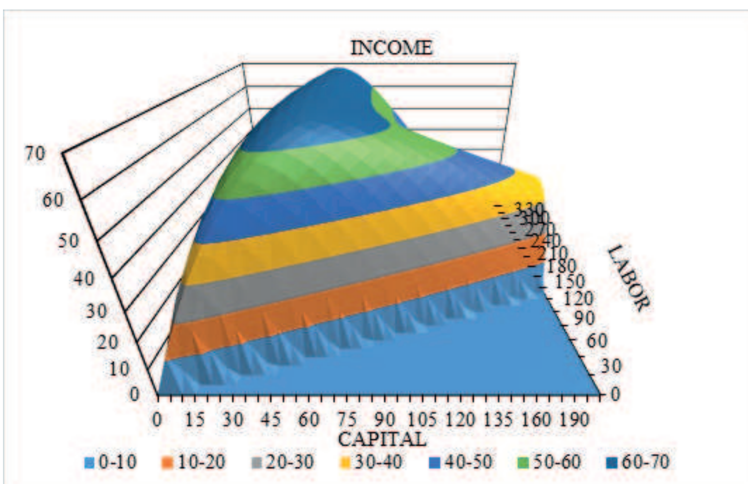

Figure 8: Dependence of income on capital and labor; the model of imperfect competition; long-term analysis

of imperfect competition, the product price depends on the volume of production and decreases when the production volume increases. In this case decisions of the firm concerning the output volume significantly affect the resulting price. All these reasons may result in that the ridge reaches its maximum and splits into two branches for further increase in capital and labor.

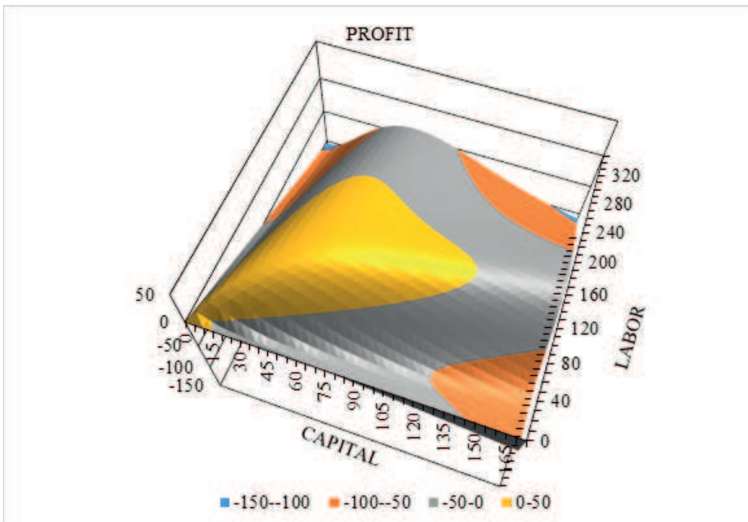

Figure 9: Dependence of profit on capital and labor; the model of imperfect competition; long-term analysis

The following part of the paper includes the results of the short term analysis. In this case the capital K is constant and only the labor L is the variable, which can be changed.

The maximal values of income and profit in the function of labor for selected values of capital are presented in Fig. 10. The maximal value of the income is reached for considerably greater value of labor than the maximal value of the profit. For the capital $K = 55$, the maximal value of income is obtained at labor $L = 100$, while the maximal value of profit at $L = 50$. In this range of labor quantities we deal with the set of Pareto-optimal solutions. Increasing the values of labor beyond $L = 100$ and decreasing them below $L = 50$ results in decrease of both criteria. This range of labor values opens a field for possible negotiations - looking for a compromise among the possibly maximal values of the income and profit.

Figure 11 presents the outcomes obtained in the space of the two considered criteria, income and profit, for the capital $K = 55$. The set of Pareto-optimal outcomes is indicated by bolded curves.

Let the representatives of two groups of interests negotiate the decisions on labor for given capital investment. The first group tries to obtain the maximum

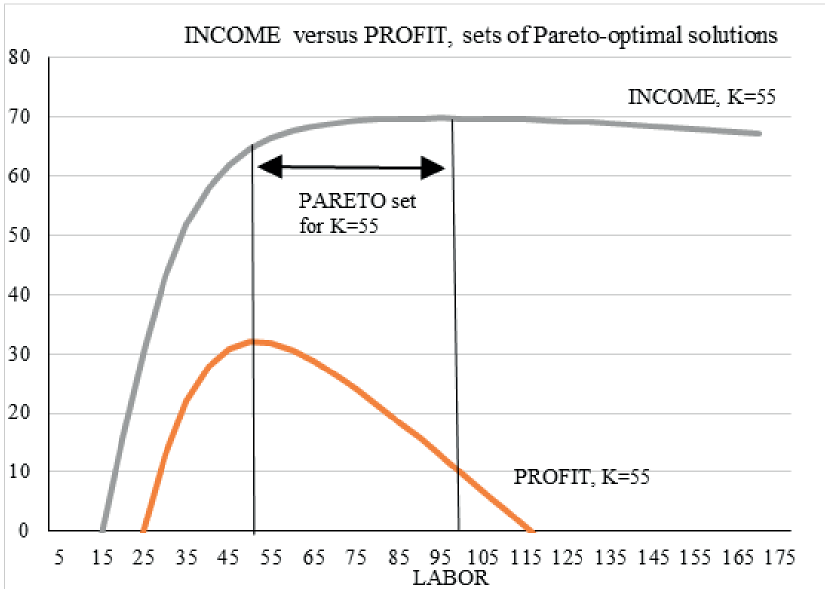

Figure 10: Income versus profit; dependence on capital for different values of labor; the model of imperfect competition; short-term analysis

income possible, while the other is interested in maximization of profit. The obtained results show that the criteria are contradictory. The groups negotiate the decision on the amount of labor for a given value of capital assets. The Pareto sets can be the basis for possible negotiations. They limit the range of the negotiations to nondominated pairs of the criteria: income and profit. The negotiations should concern labor values as decisions, which are defined by the Pareto set. It can be noticed, see Figs. 10 and 11, that each decision out of the Pareto set can be improved for both criteria by moving to a decision belonging to the set. Such decision would be irrational for both groups of interests. We observe it for the labor lower than 50 and greater than 100. It means that the Pareto set narrows down sufficiently the domain of choice in the negotiations.

Negotiation should be preceded by an analysis of the optimum points of each group of interests as well as a bi-criteria optimization in order to determine a Pareto set. The results should be passed on to a mediator/moderator. However, the question arises: should the parties be informed on the results of the bi-criteria analysis, or should they be accessible to the mediator only?

A separate problem is the role of the mediator. No doubt she/he should be impartial and her/his main task should be the determination of some neutral starting point of the negotiation.

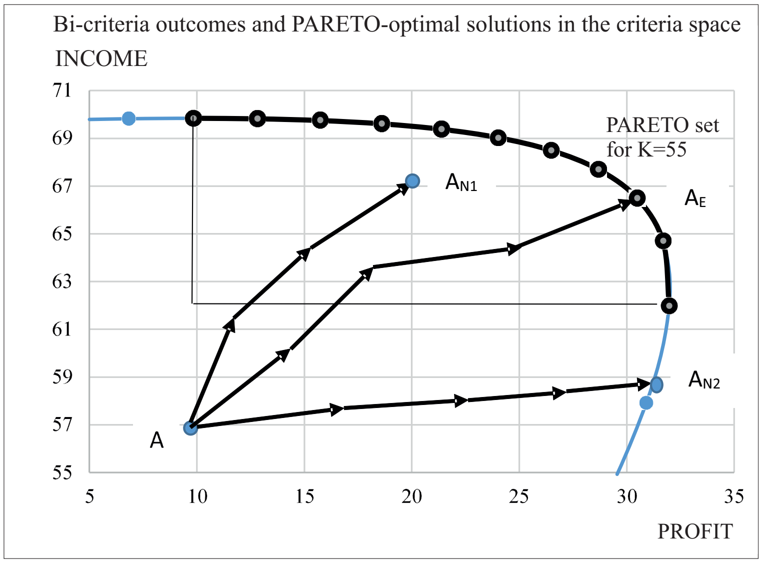

Figure 11: The Pareto-optimal solutions in the criteria space: income and profit; the model of imperfect competition; short term analysis

Benefits for the participants in the negotiations are as follows:

Owners:

•social peace in the firm, avoidance of the industrial actions/strikes and the associated costs

•providing higher pay for employees that enables recruitment of better-skilled labor

•higher market share than that achieved in the profit maximization.

Employees:

•industrial action/strike is individually risky and costly

•industrial action introduces uncertainty for both owners and employees.

The following example illustrates how the information about the Pareto set can be used in the negotiations in comparison to the case when such information is absent. We assume that the negotiations are conducted by a mediator who has a deep knowledge about the firm, is privy to expectations and wishes of the negotiating parties. He/she also has information about the Pareto set including outcomes and decision variables leading to the outcomes as presented in Figs. 10 and 11.

Let the current state of the firm be defined by the point A in Fig. 11. We can see that the point is far from the Pareto frontier. The mediator proposes a sequence of variants according to the single negotiation text procedure (see

Raiffa, 1982), applied in the Camp David negotiations. The proposals are progressive and are treated as single texts to be considered by the opponents, who analyze them and submit their suggestions. On this basis the mediator proposes a new variant, formulated according to expectations of the opponents. In the discussion, the additional benefits of the parties, as mentioned above, can be taken into account, like for example the increase of the market share, important for the owners together with the profit maximization. The mediator, knowing the Pareto set, can continue the procedure till the Pareto frontier is reached. This case is illustrated by a sequence of arrows leading to the effective point A_E in Fig. 11. Note that the proposed production function defines the maximal production, which can be achieved for given inputs of labor and capital when the firm operates at the maximum production capacity. However, in reality, it happens that the firm's production capacity is not fully utilized. Therefore, the values of the criteria for the same pair of variables L, K can be lower than those calculated with the use of the production function and represented by the Pareto frontier. The point A_{N1} in Fig. 11 relates to such a case, when the production capacity of the firm is not fully utilized. Also the status quo point A relates to this kind of case.

Let the mediator and the negotiating parties have no information about the Pareto set and the single text procedure be also applied. In such a case the procedure can wind down without reaching the Pareto frontier, as in point A_{N1} or can reach the frontier of the possible outcomes of the firm, but in the part, which is not Pareto-optimal, as in point A_{N2}. Both points, A_{N1}, and A_{N2}, are non-effective. Let us see that in this case the procedure is also progressive. Each arrow represents the improvement of both criteria. However, if the information about the Pareto set is absent, the procedure can be finished without reaching the Pareto set, if the opponents are satisfied and decide to agree on a consensus. But, in fact, they could have obtained better outcomes in the Pareto set.

Therefore, the knowledge about the Pareto sets is important and can provide grounds for possible negotiations between groups of interests. The negotiation problems are discussed in a wide bibliography. The exemplary papers (see Kersten, 1988; Kersten and Lai, 2007; de Almeida and Wachowicz, 2017; Wachowicz, Kersten and Roszkowska, 2019) present different aspects of the negotiations including analysis, procedures, decision support and e-negotiation systems.

9. Summary

The paper presents a model that describes economic effects of an enterprise activity. It is constructed using the originally formulated production function, which satisfies the postulates proposed by Ragnar Frisch. The properties of the function are shown and analyzed for the assumed model parameters. The theoretical considerations are illustrated by the results of numerical experiments.

Figures included in the paper present contour lines of the long-term production function, the average costs and marginal cost as functions of the output.

An exemplary bi-criteria optimization problem is formulated. The problem describes joint maximization of two criteria: income and profit, measuring objectives of the firm's activity with respect to capital and labor. The problem is analyzed in two market cases of the perfect and imperfect competition. Exemplary solutions of the problem are derived and shown on three-dimensional figures. The dependence of income and profit on capital and labor is shown. The sets of Pareto-optimal solutions of the problem are derived. Next, the income versus profit dependence on capital for different values of labor and on labor for different values of capital is presented and analyzed. In the case of perfect equilibria, there exists a narrow ridge of the maximal values of income and profit with respect to capital and labor. The value of income as well as that of the profit increase almost linearly on the ridge when the capital and labor are increased. For a given value of the capital the maximum value of the income is reached for a considerably greater value of the labor than the maximum value of the profit. It means that there exists a set of the Pareto-optimal solutions. In general, we can see from the simulation results that the range of the Pareto set decreases at lower values of the capital.

In the case of imperfect competition there also exists a ridge of the maximal values of the income for increasing values of capital and labor, however, it reaches its global maximal value. For a further increase of capital and labor, the ridge splits into two almost flat but gently sloping ridges. It means that the same locally maximal values of the income can be obtained for two different and separate pairs of the capital and labor values. The similar qualitative results are observed in the case of profit. It is also shown how the image of the Pareto set changes with respect to capital and labor in the criteria space.

Conflicting internal interests in the firm may lead to different goals represented by such criteria as income and profit, considered jointly. The Pareto set represents the set of alternatives, which can be analyzed and discussed during possible negotiations. The properties of these solutions are such that both criteria cannot be improved jointly. Therefore, in this set one can look for a possible consensus. It has been shown that the knowledge of the form of the Pareto set can be crucial in the negotiation process. It makes it possible to reach the efficient consensus.

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