sciendo
Control and Cybernetics
vol. 50 (2021) No. 1
pages: 69-82
DOI: 10.2478/candc-2021-0005

A fuzzy ranking of negotiation packages for the INSPIRE negotiation support system*

by

Krzysztof Piasecki †

Institute of Economics and Finance, WSB University in Poznań,
ul. Powstańców Wielkopolskich 5, 61-895 Poznań, Poland
krzysztof.piasecki@wsb.poznan.pl

Abstract: Preferential information may be visualized in many different ways, and this constitutes an important issue in the principal-agent decision-making context, e.g., in representative negotiations. In the INSPIRE negotiation support system, the principal's preferences are visualized by circles with different radii. Agents evaluate the principal's preferences in such a manner that they digitize these preferences using numbers directly proportional to the size of the circles, drawn by the principal. The manner, in which an agent understands the concept of the circle size is unknown. The main goal of this paper is to propose such an image of principal's preferences, which is independent of an individual agent's evaluation. Individual negotiators may differ in their understanding of this concept. This means that the notion of "circle size" is a linguistic variable that may be described by a fuzzy set. The empirical studies referred to show that the size of the circle is a value between the radius and the area of this circle. In this paper, the principal's preferences are defined as a fuzzy preorder between fuzzy "circle sizes". We distinguish here two kinds of the INSPIRE method. All considerations are illustrated by means of a short case study based on INSPIRE data.

Keywords: preference visualization, fuzzy ranking, negotiation problem, negotiation offer scoring systems

1. Introduction

An agent's understanding of the principal's preferences is one of the topics of the principal-agent theory (see Orlovsky, 1978; Laffont and Martimort, 2009;

*Submitted: December 2020; Accepted: March 2021.
†See the note in page 7.

Bottom et al., 2006). In particular, the principal's preferences may be visualized in different ways, namely as graphs, bar charts, pie charts, cartograms, bars, circles, etc. (Croxton and Stein, 1932; Korhonen and Wallenius, 2008; Macdonald-Ross, 1977; Miettinen, 2014; Roselli, Frej and de Almeida, 2018; Spence and Lewandowsky, 1991; Liu et al., 2014). However, there is no true consensus regarding the best way of such visualization.

In our paper, we consider the principal's preferences as described in the INSPIRE negotiation support system (Kersten and Noronha, 1999). In this system, the principal's preferences are visualized by circles with different radii. For INSPIRE, agents' understanding of the principal's preferences has already been studied in Roszkowska and Wachowicz (2015), Wachowicz, Kersten and Roszkowska (2019), Weber, Kersten and Hine (2006), as well as Kersten, Roszkowska and Wachowicz (2016, 2017). There, the rankings determined by agents were compared with visualization of the principal's preference. Therefore, the negotiation problem discussed here is stated as the one of a negotiation behind the table.

The main objective of this research is to determine such representation of the principal's preferences, which is independent of an individual agent's rankings. The anticipated result of these studies is to present the principal's preference as fuzzy order relation. The relationship determined in this way can be used as a neutral benchmark for assessing the impact of individual agents on a principal's preference rating. The possibility of determining a neutral benchmark establishes the importance of the research described in this article.

This paper is organized in the following way. In Section 2, the negotiation problem is formalized in the context of the INSPIRE method. Two types of INSPIRE methods are distinguished here. Section 3 presents a discussion on understanding of Principal's preferences. Section 4 summarizes the obtained results and indicates the direction of future research.

2. The negotiation problem

In the formal model of INSPIRE, the negotiation template can be described by means of the ordered pair $T = (F, X)$, where $F = (f_i)_{i=1}^{n}$ is a sequence of negotiation issues f_i and $X = (X_i)_{i=1}^{n}$ is a sequence of option lists X_i related to issues f_i. Each option list $X_i\,(i = 1, ..., n)$ may be considered as the sequence $X_i = (x_{i,j})_{j=1}^{m_i}$ of options. Then, any negotiation package is given as the vector

$$\bar{P} = \left(x_{1,j(p)}, x_{2,j(p)}, \ldots, x_{n,j(p)}\right) \in X_1 \times X_2 \times \ldots \times X_n = P, \tag{1}$$

where $x_{i,j(p)} \in X_i$ denotes an option of issue i used to build the package $\bar{P}$.

In the next step of the prenegotiation preparation phase, the principal is asked to express their preferences over the elements of the template T. In general, we assume that the preferences are additive. Then, each negotiation package is evaluated with the use of the scoring function $S : P \longrightarrow R_0^+$, determined by the identity

$$S\left(\bar{P}\right) = S\left(x_{1,j(p)}, x_{2,j(p)}, \ldots, x_{n,j(p)}\right) = \sum\nolimits_{i=1}^{n} U\left(x_{i,j(p)}\right), \tag{2}$$

where the symbol $U\left(x_{i,j(p)}\right)$ denotes the utility of option $x_{i,j(p)}$. We have implicitly assumed that the principal's preferences exist. This is a sufficient condition for the existence of option utility. The literature of the subject contains an extensive discussion on the use of utility in describing the negotiators' preferences. Discussants often take opposing positions. For this reason, we ignore the results of this discussion when building the scoring function. In this manner we get a scoring function model that is as general as possible. The INSPIRE method of determining the respective utilities will be discussed later on in this article.

Example 1: (Kersten and Noronha, 1999) We observe a negotiation, in which a musician and a broadcasting company "KAMET-music" talk over the terms of a potential contract. The negotiation template is defined using four issues, each having a predefined list of options that altogether allow for building 240 various offers (see Table 1).

Table 1. Example of negotiation template

Negotiation issues	**Lists of predefined options**
Number of promotional concerts (per year)	5; 6; 7 or 8 concerts
Number of new songs introduced and performed each year	11; 12; 13; 14 or 15 songs
Royalties for CDs (in percent)	1.5; 2; 2.5 or 3 %
Contract signing bonus (in dollars)	\$125 000; \$150 000 or \$200 000

We assume that at least one of the negotiating parties is an Agent representing the Principal. The Principal visualizes its preferences using circles $C(\phi)$ of various radii ϕ, which are unknown to the Agent. The Principal can draw any circle belonging to the family

$$O = \left\{C(\phi) : \phi \in R_0^+\right\}. \tag{3}$$

The symbol $V(C(\phi))$ denotes the size of the circle $C(\phi)$.

This is done separately for issues, where the sequence $(C(R_{i,0}))_{i=1}^{n} \subset O$ visualizes the importance of individual issues. The guiding principle here is:

If the issue f_i is more important than the issue f_k, then
$V(C(R_{i,0})) > V(C(R_{k,0}))$.

Then, for each list X_i of predefined options, the Principal separately visualizes the preferences between options by the sequence $(C(R_{i,j}))_{j=1}^{m_i} \subset O$. The rule is that:

If the option $x_{i,j}$ is better than the option $x_{i,k}$, then
$V\left(C\left(R_{i,j}\right)\right) > V\left(C\left(R_{i,k}\right)\right)$.

Therefore, we can consider each sequence $\left(V\left(C\left(R_{i,j}\right)\right)\right)_{j=1}^{m_i}$ as relative utilities, determined for options assigned to the issue f_i $(i = 1, 2, \ldots, n)$. In practice, the sequence $\left(\left(R_{i,j}\right)_{j=0}^{m_i}\right)_{i=1}^{n}$ of all the applied radii is usually unknown to us. However, for purposes of theoretical discussion only, we assume that the radii used are known to us.

Example 2: (Kersten and Noronha, 1999) In the negotiation described in Example 1, the management board of "KAMET-music" is the Principal. Its preferences are visualized using circles, as shown in Fig. 1.

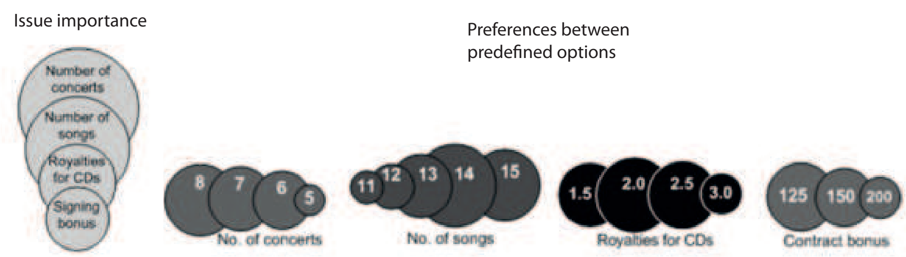

Figure 1. Visualization of Principal's preferences

Concerning the visualization of the Principal's preferences from Fig. 1, Table 2 shows the radii of the circles used for the Principal's visualizations.

Table 2. Original radii appearing in preference visualization

Issue	**Issue importance**	**Preferences between options**				
Concerts	5.59	4.30	3.85	3.45	1.85	
Songs	4.74	2.00	2.70	3.70	4.90	4.20
Royalties	3.54	3.80	4.50	4.00	2.90	
Bonus	2.89	4.00	3.40	2.50		

Example 2 shows that the importance and preferences of the issues may be visualized using different scales. We can only notice that each circle $C\left(R_{i,j}\right)$, $(i = 1, 2, \ldots, n; j = 0, 1, 2, \ldots, m)$ is uniquely represented by its radius $R_{i,j}$. For the purposes of the INSPIRE system, these circles are standardized separately for visualization of $\left(C\left(R_{i,0}\right)\right)_{i=1}^{n}$, i.e. of issue importance, and for the relative utilities $\left(V\left(C\left(R_{i,j}\right)\right)\right)_{j=1}^{m_i}$ $(i = 1, 2, \ldots, n)$. We standardize the issue importance visualization, and, in this manner, we calculate weights

$$\forall_{i-1,2,\ldots,n}: \quad r_{i,0} = \frac{R_{i,0}}{\sum_{q=1}^{n} R_{q,0}}, \tag{4}$$

$$\forall_{i=1,2,\ldots,n} \forall_{j=1,2,\ldots,m_i}: \\ r_{i,j}^{(1)} = \frac{R_{i,j} - \min\{R_{i,q} : q = 1, 2, \ldots, m_i\}}{\max\{R_{i,q} : q = 1, 2, \ldots, m_i\} - \min\{R_{i,q} : q = 1, 2, \ldots, m_i\}}, \tag{5}$$

which may also be interpreted as standardized relative utility.

Example 3: Table 3 shows the radii of the circles, standardized with the use of INSPIRE 1 for visualization of the Principal's preferences.

Table 3. Preference visualization standardized with the use of INSPIRE 1

Issue	**Issue weights**	**Standardized radii for preference visualization**				
Concerts	0.3335	1	0.8162	0.6531	0	
Songs	0.2828	0	0.2414	0.5862	1	0.7582
Royalties	0.2112	0.5625	1	0.6875	0	
Bonus	0.1724	1	0.6000	0		

The above example implies some more general conclusions. Let us compare the standardized relative utilities assigned to issues: f_3 =" Royalties" and f_4 ="Bonus". In each of these issues, the relative utilities of the options $x_{3,3}$ ="2%" and $x_{4,1}$ ="$125 000" are visualized by the same circles. Moreover, the worst option $x_{3,4}$ ="3%" is visualized by a circle greater than the circle visualizing the worst option $x_{4,3}$ ="$200 000". In INSPIRE 1, the relative utility of option $x_{3,3}$ is lower than the relative utility of $x_{4,1}$. According to common sense, the relative utility of option $x_{3,4}$ should be greater than the relative utility of option $x_{4,3}$. In INSPIRE 1, the relative utility of option $x_{3,4}$ is equal to the relative utility of option $x_{4,3}$. This is a significant drawback of the INSPIRE 1 method.

For this reason, we propose the second variant of the INSPIRE method – INSPIRE 2. This means that for visualization of the preferences between predefined options, we calculate the standardized radii

$$\forall_{i=1,2,\ldots,n} \forall_{j=1,2,\ldots,m_i}: \qquad r_{i,j}^{(2)} = \frac{R_{i,j}}{\max\{R_{i,q} : q = 1, 2, \ldots, m_i\}}, \tag{6}$$

which can also be interpreted as standardized relative utility.

Example 4: Table 4 shows the radii of the circles standardized with the use of INSPIRE 2 for visualization of the Principal's preferences.

Table 4. Preference visualization standardized with the use of INSPIRE 2

Issue	**Issue weights**	**Standardized radii for preference visualization**				
Concerts	0.3335	1	0.8953	0.8023	0.4302	
Songs	0.2828	0.4081	0.5510	0.7551	1	0.8571
Royalties	0.2112	0.8444	1	0.8888	0.6444	
Bonus	0.1724	1	0.8500	0.6250		

Let us note that in INSPIRE 2, the relative utility of options $x_{3,4}$ is greater than the relative utility of options $x_{4,3}$. This is a significant advantage of INSPIRE 2.

$$\forall_{i=1,2,\ldots,n}\,\forall_{j=1,2,\ldots,m_i}: \qquad r_{i,j}^{(2)} = \frac{R_{i,j}}{\max\left\{R_{i,q} : q = 1,2,\ldots,n_i\right\}} \tag{7}$$

3. Understanding the Principal's preferences

In practice, the radii of the circles, drawn by the Principal, are unknown. Understanding of the phrase "circle size" depends on the applied pragmatics of the natural language. Therefore, the linguistic variable "circle size" is imprecise. Agent subjectively interprets the notion of "circle size" by means of definite nonnegative numbers. In this way, the Agent assesses the circle size $V\left(C\left(R_{i,j}\right)\right)$ by a value $V_{i,j} \in R_0^+$ $(i = 1,2,\ldots,n; j = 0,1,2,\ldots,m_i)$. The rules are that:

$$V\left(C\left(R_{i,0}\right)\right) > V\left(C\left(R_{k,0}\right)\right) \Longrightarrow V_{i,0} > V_{k,0}, \tag{8}$$

$$V\left(C\left(R_{i,j}\right)\right) > V\left(C\left(R_{i,k}\right)\right) \Longrightarrow V_{i,j} > V_{i,k}. \tag{9}$$

The results of studies by Roszkowska and Wachowicz (2015) and Wachowicz, Kersten and Roszkowska (2019) demonstrated that the agents also use different scales for assessing the size of the circles. Moreover, in determining the relative utility, each unknown radius $R_{i,j}$ may be replaced by any value $V_{i,j}$. For these reasons, we standardize Agent's assessments in the same way as that used for the radii of the circles, drawn by the Principal. Therefore, we standardize the issue importance visualization is such a way that we calculate the weights

$$\forall_{i=1,2,\ldots,n} \quad : v_{i,0} = \frac{V_{i,0}}{\sum_{q=1}^{n} V_{q,0}} \quad . \tag{10}$$

Then, the standardized description of preferences between predefined options is determined in the following way for the INSPIRE 1 method:

$$\forall_{i=1,2,\ldots,n}\,\forall_{j=1,2,\ldots,m_i} \quad :$$

$$v_{i,j}^{(2)} = \frac{V_{i,j} - \min\left\{V_{i,q} : q = 1,2,\ldots,m_i\right\}}{\max\left\{V_{i,q} : q = 1,2,\ldots,m_i\right\} - \min\left\{V_{i,q} : q = 1,2,\ldots,m_i\right\}}, \tag{11}$$

while for the INSPIRE 2 method:

$$\forall_{i=1,2,\ldots,n}\,\forall_{j=1,2,\ldots,m_i}\ :\qquad v_{i,j}^{(1)}=\frac{V_{i,j}}{\max\left\{V_{i,q}\,:\ q=1,2,\ldots,m_i\right\}}\tag{12}$$

Brinton (1914) recognized some problems with using circles as a tool for information presentation. We describe his considerations in modern language. The guiding principle of the method considered by him was that the greater utility of a characterized object implies the larger size of the representing circle. He showed that circle sizes evaluated by circle radius or by circle area make the reader misperceive the relative utility of the objects described by these circles. Brinton noticed that:

- comparison between radii causes overestimation of the relative utility of the worse object;
- comparison between areas causes underestimation of the relative utility of the worse object.

Many authors confirm these observations. They conclude, accordingly, that the relative sizes of the circles are misperceived, and these mistakes are systematic (see Macdonald-Ross, 1977). Therefore, they propose such a formula of the function of "circle relative size", which allows for the "psychologically correct" circle sizes to be calculated. Their proposition implies that the "circle size" function $V\left(\bullet\,|\,\gamma\right):O\rightarrow R_0^+$ is given by the identity

$$V\left(C\left(r\right)|\,\gamma\right)=\alpha\bullet r^{\gamma},\tag{13}$$

where $\alpha\in R^+$ is the size of a benchmark circle $C\left(1\right)$. The exponent γ characterizes an agent's understanding of the Principal's preferences. Brinton's (1914) observations demonstrate that $\gamma\in[1,\,2]$. For example, the exponent γ, derived in the empirical studies of Macdonald-Ross (1977), varies from 1.6 to 1.82. It means that the circle size is a value between the circle radius and the circle area.

Due to (2) and (3), we can pre-specify the form of the scoring function $S\left(\bullet\,|\,\gamma\right):P\rightarrow[0,1]$ as follows

$$\begin{aligned}&S\left(\bar{P}\,|\,\gamma\right)=S\left(x_{1,j(p)},x_{2,j(p)},\ldots,x_{n,j(p)}\,|\,\gamma\right)\\&=\sum\nolimits_{i=1}^{n}r_{i,\,0}^{\gamma}\bullet r_{i,\,j(p)}^{\gamma}=\sum\nolimits_{i=1}^{n}\left(r_{i,0}\bullet r_{i,j(p)}\right)^{\gamma}.\end{aligned}\tag{14}$$

It is evident that the rating of the negotiation packages $\left\{\bar{P}_1,\bar{P}_2,\ldots,\bar{P}_m\right\}\subset P$, obtained with the use of the values $S\left(\bar{P}_k\,|\,\gamma\right)$ is the same as the rating obtained with the use of the values $\sqrt[\gamma]{S\left(\bar{P}_k\,|\,\gamma\right)}$. Moreover, then the values $\sqrt[\gamma]{S\left(\bar{P}_k\,|\,\gamma\right)}$ and $\sqrt[\delta]{S\left(\bar{P}_k\,|\,\delta\right)}$ can be compared for $\gamma\neq\delta$, because they are expressed in the same measurement unit. For these reasons, we propose the following final form of the scoring function $\hat{S}\left(\bullet\,|\,\gamma\right):P\rightarrow[0,1]$, given by the identity

$$\hat{S}\left(\bar{P}\,|\,\gamma\right) =$$

$$\hat{S}\left(x_{1,j(p)}, x_{2,j(p)}, \ldots, x_{n,j(p)}\,|\,\gamma\right) = \sqrt[\gamma]{S\left(\bar{P}\,|\,\gamma\right)} = \sqrt[\gamma]{\sum\nolimits_{i=1}^{n}\left(r_{i,0}\bullet r_{i,j(p)}\right)^{\gamma}}. \tag{15}$$

For any negotiation package $\bar{P}$, the function $\hat{S}\left(\bar{P}\,|\,\bullet\right) : [1,2] \to [0,1]$ is decreasing, convex and continuous. This implies that

$$\forall_{\bar{P}\in P}\,\forall_{\gamma\in[1,2]}\ : \qquad A\left(\bar{P}\right) = \hat{S}\left(\bar{P}\,|\,2\right) \leq \hat{S}\left(\bar{P}\,|\,\gamma\right) \leq \hat{S}\left(\bar{P}\,|\,1\right) = B\left(\bar{P}\right). \tag{16}$$

We conclude that for any negotiation package $\bar{P}$, all of its possible scoring ratings form the interval

$$I\left(\bar{P}\right) = \left[A\left(\bar{P}\right), B\left(\bar{P}\right)\right] \tag{17}$$

which is called the scoring interval. These intervals will be the basis for comparisons between negotiation packages.

We consider a pair $(\bar{P}\bar{Q}) \in P^2$ of negotiation packages, evaluated by the scoring function (15). On the space P, we can consider the relation $\bar{P}NW\bar{Q}$, which reads:

$$\textit{Package } \bar{P} \textit{ is not worse than package } \bar{Q}. \tag{18}$$

For fixed $\gamma \in [1,2]$, the relation is equivalent to the inequality

$$\hat{S}\left(\bar{P}\,|\,\gamma\right) \geq \hat{S}\left(\bar{Q}\,|\,\gamma\right). \tag{19}$$

It is evident that the fulfilment of condition (19) depends on the value of $\gamma \in [1,2]$. The monotonicity, together with convexity of the function $\hat{S}\left(\bar{P}\,|\,\bullet\right) : [1,2] \to [0,1]$ causes that the inequality (19) has exactly one solution $\alpha \in [1,2]$. The length of the interval $[1,\alpha] \subset [1,2]$ is determined by the integral

$$I\left(\bar{P}, \bar{Q}\right) = \int_{\left\{\gamma\in[1,2]:\hat{S}(\bar{P}|\gamma)\geq\hat{S}(\bar{Q}|\gamma)\right\}} dx. \tag{20}$$

Because the interval $[1,2]$ is of unit length, the value $I\left(\bar{P}, \bar{Q}\right)$ may be interpreted as a degree, in which the inequality (19) is fulfilled. Therefore, we define the relation (18) as a fuzzy one, determined by its membership function $\mu_{NW} : P \times P \to [0,1]$, given as follows

$$\mu_{NW}\left(\bar{P}, \bar{Q}\right) = I\left(\bar{P}, \bar{Q}\right). \tag{21}$$

For the cases $B\left(\bar{Q}\right) \leq A\left(\bar{P}\right)$ or $B\left(\bar{P}\right) < A\left(\bar{Q}\right)$, the value $\mu_{NW}\left(\bar{P}, \bar{Q}\right)$ is obvious. When considering the other cases, we will apply the following linear approximation of the scoring function

$$\hat{S}\left(\bar{P}\,|\,\gamma\right) \approx (2-\gamma)\bullet A\left(\bar{P}\right) + (\gamma-1)\bullet B\left(\bar{P}\right). \tag{22}$$

Then, the inequality (19) is replaced by the inequality

$$(2-\gamma)\bullet A\left(\bar{P}\right)+(\gamma-1)\bullet B\left(\bar{P}\right)\geq(2-\gamma)\bullet A\left(\bar{Q}\right)+(\gamma-1)\bullet B\left(\bar{Q}\right). \quad (23)$$

The solution of inequality (23) shows that the approximation of the membership function μ_{NW} is given by means of the identity

$$\mu_{NW}\left(\bar{P},\,\bar{Q}\right)=\begin{cases}1 & A\left(\bar{P}\right)\geq\ A\left(\bar{Q}\right)\ \&\ B\left(\bar{P}\right)\geq\ B\left(\bar{Q}\right)\\ \left(\frac{A\left(\bar{P}\right)-A\left(\bar{Q}\right)}{B\left(\bar{Q}\right)-B\left(\bar{P}\right)}+1\right)^{-1} & A\left(\bar{P}\right)\leq\ A\left(\bar{Q}\right)<B\left(\bar{Q}\right)<B\left(\bar{P}\right)\\ \left(\frac{B\left(\bar{Q}\right)-B\left(\bar{P}\right)}{A\left(\bar{P}\right)-A\left(\bar{Q}\right)}+1\right)^{-1} & A\left(Q\right)\leq\ A\left(\bar{P}\right)<B\left(\bar{P}\right)<B\left(\bar{Q}\right)\\ 0 & A\left(\bar{P}\right)<\ A\left(\bar{Q}\right)\ \&\ B\left(\bar{P}\right)\leq B\left(\bar{Q}\right)\end{cases}. \quad (24)$$

From the viewpoint of multivalued logic, the value $\mu_{NW}\left(\bar{P},\,\bar{Q}\right)$ is interpreted as the truth-value of the sentence (18). The relation NW is a fuzzy preorder in the sense given by Orlovsky (1978). This preorder is linear, because we have

$$\forall_{\left(\bar{P},\,\bar{Q}\right)\in P^2}:\qquad \max\left\{\mu_{NW}\left(\bar{P},\,\bar{Q}\right),\mu_{NW}\left(\bar{Q},\,\bar{P}\right)\right\}\geq\frac{1}{2}. \quad (25)$$

Therefore, for any subsets $P^*\subset P$, the subset $\max P^*$ of its maximal elements is distinguished in the following way

$$\max P^*=\left\{\bar{P}_i\in P^*:\forall_{\bar{P}_j\in P^*}:\ \bar{P}_i.NW.\bar{P}_j\right\}. \quad (26)$$

The set $\max P^*$ is a fuzzy one. Due to the Zadeh's Extension Principle, this fuzzy subset is determined by its membership function $\mu_{\max P^*}:P\to[0,1]$, given as follows

$$\mu_{\max P^*}\left(\bar{P}_i\right)=\left\{\mu_{NW}\left(\bar{P}_i,\bar{P}_j\right)\right\}. \quad (27)$$

From the viewpoint of multivalued logic, the value $\mu_{\max P^*}\left(\bar{P}\right)$ is interpreted as the truth-value of the sentence:

Package $\bar{P}$ is the best.

The set of maximal elements is a very useful tool for evaluating any sets of non-dominated negotiation packages. We often meet the corresponding situations in the subsequent negotiation phase when the parties submit alternative offers.

4. Case study

We consider the space P_e of negotiation packages, listed in Table 5. These negotiation packages will be compared separately for the INSPIRE 1 method and the INSPIRE 2 method.

Table 5. Considered negotiation packages and their scoring

Symbol	**Package content**	**Negotiation package**
$\bar{P}_1$	$(x_{1,1},\, x_{2,1}, x_{3,1}, x_{4,2})$	(5, 11, 1.5, 150 000)
$\bar{P}_2$	$(x_{1,1},\, x_{2,1}, x_{3,1}, x_{4,3})$	(5, 11, 1.5, 200 000)
$\bar{P}_3$	$(x_{1,1},\, x_{2,1}, x_{3,3}, x_{4,3})$	(5, 11, 2.5, 200 000)
$\bar{P}_4$	$(x_{1,1},\, x_{2,2}, x_{3,3}, x_{4,3})$	(5, 12, 2.5, 200 000)
$\bar{P}_5$	$(x_{1,3},\, x_{2,3}, x_{3,4}, x_{4,1})$	(7, 13, 3.0, 125 000)

4.1. The INSPIRE 1 case

Using the identities (10), (11), (3), and (16) for each considered negotiation package, we calculate the scoring intervals, associated with the INSPIRE 1 method. The determined scoring intervals are presented in Table 6. These intervals are applied for determining fuzzy preorder on the space P_e. The membership function of this relation is calculated with the use of (23). Table 6 contains the respective membership functions values.

Table 6. The membership function of the fuzzy preorder NW on P_e, case of INSPIRE 1

	Scoring interval		**Relation NW on the space $\mathbf{P_e}$**				
	$\mathbf{A}\left(\bar{P}\right)$	$\mathbf{B}\left(\bar{P}\right)$	$\bar{P}_1$	$\bar{P}_2$	$\bar{P}_3$	$\bar{P}_4$	$\bar{P}_5$
$\bar{P}_1$	0.368868	0.555803	1	1	1	0.88	0.99
$\bar{P}_2$	0.354061	0.452342	0	1	0	0	0.23
$\bar{P}_3$	0.363772	0.478744	0	1	1	0	0.34
$\bar{P}_4$	0.370122	0.54701	0.12	1	1	1	0.84
$\bar{P}_5$	0.323518	0.55604	0.01	0.77	0.66	0.16	1

In the next step, we restrict our considerations to the set $P_e^* = \left\{\bar{P}_1, \bar{P}_4, \bar{P}_5\right\}$ of non-dominated negotiation packages. The relation NW on the space P_e^* and the set $\max P_e^*$ are presented in Table 7.

Table 7. The membership function of the fuzzy preorder NW on P_e^* and the set of maximal elements, generated by it, case of INSPIRE 1

	Relation NW on the space $\mathbf{P_e^*}$			$\max \mathbf{P_e^*}$
	$\bar{P}_1$	$\bar{P}_4$	$\bar{P}_5$	
$\bar{P}_1$	1	0.88	0.99	0.88
$\bar{P}_4$	0.12	1	0.84	0.12
$\bar{P}_5$	0.01	0.16	1	0.01

4.2. The INSPIRE 2 case

Using the identities (10), (12), (3), and (16) for each considered negotiation package, we calculate the scoring intervals, associated with the INSPIRE 2 method. The determined scoring intervals are presented in Table 8.

Table 8. The membership function of the fuzzy preorder NW on P_e, case of INSPIRE 2

	Scoring interval		**Relation NW on the space $\mathbf{P_e}$**				
	$\mathbf{A}\left(\bar{P}\right)$	$\mathbf{B}\left(\bar{P}\right)$	$\bar{P}_1$	$\bar{P}_2$	$\bar{P}_3$	$\bar{P}_4$	$\bar{P}_5$
$\bar{P}_1$	0.421740	0.773898	1	1	1	0	0.43
$\bar{P}_2$	0.409874	0.735100	0	1	0	0	0.95
$\bar{P}_3$	0.414045	0.744488	0	1	1	0	0.14
$\bar{P}_4$	0.427075	0.784890	1	1	1	1	0.81
$\bar{P}_5$	0.406790	0.789709	0.57	0.05	0.86	0.19	1

These intervals are applied for determining the fuzzy preorder on the space P_e. The membership function of this relation is calculated with the use of (23). Table 9 contains the respective membership function values.

In the next step, we restrict our considerations to the set $P_e^* = \left\{\bar{P}_4, \bar{P}_5\right\}$ of non-dominated negotiation packages. The relation NW on the space P_e^* and the set $\max P_e^*$ are presented in Table 9.

Table 9. The membership function of fuzzy preorder NW on P_e^* and the set of maximal elements, generated by it, case of INSPIRE 2

	Relation NW on the space $\mathbf{P_e^*}$		$\max \mathbf{P_e^*}$
	$\bar{P}_4$	$\bar{P}_5$	
$\bar{P}_4$	1	0.81	0.81
$\bar{P}_5$	0.19	1	0.19

4.3. The case study summary

In the two subsections above, the INSPIRE 1 and INSPIRE 2 methods have been used to indicate the non-dominated negotiation packages. The indications obtained by INSPIRE 1 are not contrary to the indications obtained by INSPIRE 2.

On the other hand, the indications obtained from INSPIRE 1 are more ambiguous than the indications obtained from INSPIRE 2. This suggests the higher quality of the indications obtained from INSPIRE 2. The reason for this may be the drawback of the INSPIRE 1 method, as discussed in the context of Example 3.

These are the conclusions that relate only to the example discussed in the article. In this situation, it is advisable to repeat this test many times over for different cases. It is possible that in this way we might be able to confirm the universal character of the conclusions formulated above.

5. Final remarks

In this paper, we reconsidered the problem of understanding of the Principal's preferences as visualized in the INSPIRE negotiation support system. We have distinguished two kinds of the INSPIRE methods. In the approach proposed by us, the premise of "an agent's understanding" is replaced by a more general premise, corresponding to "any agent's understanding". In this way, we obtained a more reliable rating method with the use of the scoring intervals. The price for raising the rating credibility was the reduction of order precision. A fuzzy relation discloses an imprecision of this order.

The results obtained in Section 4 show that the disclosed imprecision may have limited consequences. This means that an agent's understanding of the Principal preferences may not exert impact on the comparisons of most packages. On the other hand, the results obtained in Section 4 show that the choice of the kind of INSPIRE approach has an impact on the rating of negotiation packages. In many papers, the negotiation case considered in our examples is a reference point for the discussions on INSPIRE. Therefore, the conclusions, presented here, are very important for future scientific discussion.

Econometric verification of the model (13) is an important direction for future research on INSPIRE or on other negotiation support systems that use the principal's preference visualizations. Moreover, we suggest using the relation (18) to determine the multiple criteria comparison, describing the preferences of both negotiating parties. An interesting direction of future research may also be constituted by a discussion on various forms of the membership function (20).

The here presented model is a normative one. In the future, a discussion should be started about the use of the membership function (24) in negotiation practice.

Today, a principal can draw circles on the interactive tablet screen. Then, using the proposed algorithm, the principal's preferences can be determined directly. This fact implies the application potential of the proposed algorithm.

Acknowledgments:

The author is very grateful to the anonymous reviewers for their insightful

and constructive comments and suggestions. The use of these comments allowed me to improve this article.

References

Bottom W.P., Holloway J., Miller G.J., Mislin A. and Whitford A. (2006) Building a pathway to cooperation: negotiation and social exchange between principal and agent. *Adm Sci Q* 51:29–58.

Brinton, W. C. (1914) *Graphic methods for presenting facts*. The Engineering Magazine Company, New York.

Croxton, F.E. and Stein, H. (1932) Graphic comparisons by bars, squares, circles, and cubes. *Journal of the American Statistical Association* 27, 54–60.

Kersten, G.E. and Noronha, S.J. (1999) WWW-based negotiation support: design, implementation, and use. *Decision Support Systems* 25, 135-154.

Kersten G.E., Roszkowska E. and Wachowicz T. (2016) An Impact of Negotiation Profiles on the Accuracy of Negotiation Offer Scoring System – Experimental Study. *Multiple Criteria Decision Making*, 11, 77-103.

Kersten G.E., Roszkowska E. and Wachowicz T. (2017) The Heuristics and Biases in Using the Negotiation Support Systems. In: M. Schoop and M. Kilgur, eds., *Group Decision and Negotiation. A Socio-Technical Perspective*, *Lecture Notes in Business Information Processing*, **293**, Springer, 215-228.

Korhonen P. and Wallenius J. (2008) Visualization in the multiple objective decision-making framework. In: J. Branke, K. Deb, K. Miettinen and R. Słowiński, eds., *Multiobjective Optimization*. Springer, Berlin, 195–212.

Laffont J.-J. and Martimort D. (2009) *The Theory of Incentives: The Principal-Agent Model*. Princeton University Press, Princeton.

Liu S., Cui W., Wu Y. and Liu M. (2014) A survey on information visualization: recent advances and challenges. *Vis Comput* 30:1373–1393.

Macdonald-Ross, M. (1977) How numbers are shown. *Audio-Visual Communication Review* 25, 359–409.

Miettinen K. (2014) Survey of methods to visualize alternatives in multiple criteria decision-making problems. *OR Spectrum* 36, 3–37.

Orlovsky, S.A. (1978) Decision making with a fuzzy preference relation. *Fuzzy Sets Systems* 1, 155–167.

Pratt, J. W. and Zeckhauser, R. J. (1985) Principals and Agents: An Overview. In: J. W. Pratt and R. J. Zeckhauser, eds., *Principals and Agents: The Structure of Business*. Cambridge MA, Harvard Business School Press, 1-35.

Roselli L.R.P., Frej E.A. and de Almeida A.T. (2018) Neuroscience experiment for graphical visualization in the FITradeoff decision support system. In: Y. Chen, G.E. Kersten, R. Vetschera and H. Xu, eds., *Group decision and negotiation in an Uncertain World. GDN 2018. Lecture Notes in Business Information Processing*, **315**, Springer, Cham 56–69.

Roszkowska E. and Wachowicz T. (2015) Inaccuracy in Defining Preferences by the Electronic Negotiation System Users. In: B. Kaminski, G.E. Kersten and T. Szapiro, *Outlooks and Insights on Group Decision and Negotiation GND 2015, Lecture Notes in Business Information Processing*, **218**, Springer, Heidelberg, 131-143.

Spence, I. and Lewandowsky, S. (1991) Displaying proportions and percentages. *Application Cognitive Psychology 5*, 61–77.

Wachowicz, T., Kersten, G. E. and Roszkowska, E. (2019) How do I tell you what I want? Agent's interpretation of principal's preferences and its impact on understanding the negotiation process and outcomes. *Operational Research*, **19**(4), 993–1032.

Weber M., Kersten G. and Hine M. (2006) Visualization in e-negotiations: an inspire ENS graph is worth 334 words, on average. *Electronic Markets* 16:186–200.