

PIDF control for high order MIMO system based on coupling compensation*

by

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Abstract: Complex systems, considered in control engineering, are often modeled in the form of a multi-input-multi-output (MIMO) coupled model. These systems pose definite challenges, due to their coupling behavior and the associated high computational costs. This paper describes a proposal for a technique for designing proportional-integral-derivative plus filter (PIDF) controllers for high order MIMO systems, while compensating the inherited coupling effect. The high-order MIMO system is decoupled into single-input-single-output (SISO) subsystems, which are then investigated by analyzing their time and frequency responses, pole-zero maps, and closed-loop stability. PIDF controllers with phase lead compensators are designed for each subsystem model so as to satisfy the desired closed-loop specifications. Additional controllers are introduced in order to eliminate the coupling effects in the coupled control system, while respecting the physical constraints. The proposed technique is validated through numerical simulations on a coupled control system with a high-order MIMO model, which demonstrate that the proposed approach can achieve the control requirements with acceptable performance for various reference inputs.

Keywords: high order model, flight control, PIDF control, MIMO model, morphing wings, coupling compensation

1. Introduction

Control of high-order models is a challenging problem in many engineering, industrial, and scientific domains. High-order models are mathematical representations of complex systems with a large number of state variables, parameters, and interconnected feedback loops. Such models can capture the system's dynamics more accurately than lower-order models, but they also pose difficulties

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List of abbreviations

DOF	Degree of Freedom
FEM	Finite Element Method
IMC	Internal Model Control
MFC	Macro Fiber Composite
MIMO	Multi Input Multi Output
PIDF	Proportional-Integral-Derivative plus Filter
SISO	Single Input Single Output
SMC	Sliding Mode Control
UAV	Unmanned Aerial Vehicle

for the analysis, design, and implementation of control strategies. The applications for MIMO systems include, in particular, Unmanned Aerial Vehicles (UAVs) and radar systems (Emad, Mohammed and Fanni, 2022; Rojhani and Shaker, 2024), autonomous vehicles (Alfeqy, Abdelmunim et al., 2024; Alfeqy, Abd El Munim et al., 2024; Eissa et al., 2026), robotics (Aessa, Shneen and Oudah, 2025; Bekhiti et al., 2025), as well as energy and smart grids (Elias et al., 2025).

One of the main challenges of controlling high-order models is the trade-off between performance and robustness. Performance refers to how well the controller can achieve the desired objectives, such as tracking a reference signal, rejecting disturbances, or minimizing errors. Robustness refers to how well the controller can cope with uncertainties, such as model errors, parameter variations, non-linear effects, or measurement noise. In general, higher-order controllers can achieve better performance than lower-order controllers, but they may also be more sensitive to uncertainties and less stable.

Another challenge of controlling high-order models is the computational complexity and feasibility. Higher-order controllers may require more computational resources and memory than lower-order controllers, and this requirement can limit their applicability in real-time systems or embedded devices. Moreover, higher-order controllers may not be easy to implement or tune in practice, especially when the model structure or parameters are unknown or time-varying.

PIDF control is a type of industrial PID control that includes a filter in the derivative term to reduce noise sensitivity and improve performance. PIDF control is widely used in various applications, such as integrating processes, robotics, biomedical systems, and power electronics. PID control has some limitations, related to sensitivity to noise, overshoot, oscillations, and poor disturbance rejection. To overcome these drawbacks, PIDF control introduces a derivative filter factor that modifies the derivative term in order to reduce its

noise amplification and improve its stability. The derivative filter factor can be either constant or adaptive, depending on the design of the controller (Borase et al., 2021; Siddiqui, Anwar and Laskar, 2020). The combination of PID control with fuzzy logic is a well-known approach in controlling manipulator robots, as this is illustrated in Amit, Shrey and Lal (2019); Khodadadi and Ghadiri (2018); Mai and Dang (2021); Manjeet and Sathans (2013).

There are also papers, which present studies, concerning the use of PID control for high-order models. And so, for instance, Prokopenko et al. (2019) present a generalized PID interpretation for high-order linear active disturbance rejection control (LADRC) and cascade LADRC. The paper referred to showed that the high-order LADRC can be regarded as a generalized PID controller with a high-order filter in the derivative action and a high-order integral action. In Izci, Ekinci and Hussien (2023), a novel hybrid approach, called hybrid atom search particle swarm optimization (h-ASPSO) was proposed. This work aims to solve the problems associated with two high-order models with PID controllers. The results showed that the hybrid scheme outperforms the conventional scheme in terms of convergence speed and quality of solution.

Han et al. (2022) introduced a state-space pole placement approach for designing high-order PID controllers for second-order plus time delay systems. The study, presented in the paper used a single parameter to determine the locations of closed-loop poles and tuned the high-order PID controller with this parameter. It implemented the proposed method in a temperature control system and showed that it can improve the system response. Another work proposed a novel model reduction scheme and an enhanced internal model control (IMC)-PID controller design method for higher-order processes (Nath, Dey and Mudi, 2019). The approach proposed was applied to a level control loop and it was shown that it can achieve better performance than other methods.

Additionally, Tognetti and de Oliveira (2022) presented a general approach for establishing PID controllers for high-order models, including time delay and uncertainty parameters. Further, a high-dimensional controller was proposed by Yin and Chen (2022) to achieve accurate control of a doubly fed wind power system with a multivariable high-order model imposed by non-linear behaviors.

Concerning recent work on PIDF controllers for high-order MIMO systems, several key studies should be highlighted. A new generation of higher-order proportional-integral-derivative (HO-PID) controllers was introduced in Huba, Vrancic and Bistak (2024) and Huba et al. (2025) to leverage the performance of classic PID control so as to improve system response. The work there reported encourages the use of a high-order controller with a double integrator plus dead time (DIPDT) model. Another work (Konkov and Mitrishkin, 2024) extends the technique of tuning the PID controller using iterated linear matrix inequality (LMI) restrictions. It applies the extended method to discrete-time control sys-

tems, unlike the related previous paper (Boyd, Hast and Åström, 2016), which dealt with continuous-time systems. Further, a quite recent paper (Apparasu et al., 2025) proposed a novel fractional-order filter PID controller design method, applied to second-order plus time delay (SOPTD) processes contained by an Internal Model Control (IMC) based Smith predictor framework.

The limitations and challenges for PID controllers with high-order MIMO models were previously discussed in a number of articles. A PID controller was designed and implemented by Aguiar et al. (2023) to control diesel hydrotreating units with inverted decoupling. This paper investigated the importance of a decoupler and the effect of interactions in the controller on the desired performance. A comparison between sliding mode control (SMC) and PID control was investigated for an actuated orthosis in Ashmi, Anila, and Sivanandan (2021). The study showed that SMC is more efficient in reference tracking. Hosseini, Ranjbar Noei and Rostami (2023) designed a high-order sliding mode controller for the trajectory tracking of six degrees of freedom (DOF) underwater vehicles. They compared the results from the proposed SMC with the conventional PID controller, which showed the superiority of the proposed controller. Also, the work performed by Liu et al. (2023) showed that PID control has significant limitations when applied for the nonlinear coupled models of aircraft. Moreover, another paper (Alla, Rajani and Yadlapalli, 2023) confirmed that designing a PID controller for a higher-order system is challenging. So, it proposed to reduce the model's order with a technique called fractional order PID.

While classic PIDF controllers remain the industry standard, due to their simplicity and ease of implementation, they face significant limitations when applied to morphing-wing UAVs. Specifically, morphing mechanisms involve the high-order MIMO systems that traditional PID structures cannot compensate for without extensive advanced control methods, which increases design complexity and significantly reduces implementation feasibility.

Furthermore, existing decoupling techniques often rely on low-order MIMO systems that break down when applied to the complex model of the wing morphing. Consequently, a gap remains with regard to a control strategy that can handle the high-order coupling of a morphing platform, while retaining the structural simplicity, required for practical, onboard implementation. The study here presented proposes a systematic decoupling technique, intended to address this gap by investigating and understanding the inherent behavior of the plant model and adding compensator controllers to eliminate the inherent coupling effects, ensuring robustness where traditional fixed-structure controllers fail.

This paper proposes a classic PIDF controller design for a high-order coupled system, which exhibits unstable behaviors as well as inherited dynamic coupling effects. The paper presents the design of lead compensators with the aim of stabilizing the system and improving its performance. It also introduces another

controller to eliminate the coupling effects and enable the PIDF controller design. The paper demonstrates the application of this approach to a high-order system with physical constraints. The analysis of the plant in both coupled and uncoupled schemes facilitates the design of a well-established controller that meets the performance requirements of the closed-loop system.

The detailed modeling of the plant used in this paper was previously published by Emad, Mohamed and Fanni (2022). This model describes the dynamics of active flexible morphing wings and their involved interactions with the aerodynamics of the UAV body. The active morphing wings considered were controlled by MFC piezoelectric actuators. A new modeling technique with low computational costs was developed for the MCF actuators and published by Emad, Fanni and Mohamed (2020), and Emad et al. (2021), using the finite element method (FEM).

The paper is organized as follows: after reviewing the previous research works of PIDF control for high-order models in Section 1, Section 2 presents the mathematical modeling of the plant, actuated by MFCs. Section 3 demonstrates the analysis of the plant model in coupled and decoupled schemes, and Section 4 discusses the classic PIDF controller design for the plant models. Then, Section 5 covers the design of lead compensators. After that, Section 6 illustrates a coupled control system for the MIMO plant model, and Section 7 shows the results of the conducted work. Lastly, Section 8 presents the conclusions from the presented work and the suggested future directions of research.

2. Plant design and modeling

2.1. State space modeling of morphing wing

The general form of the state space model is:

Block diagram for the assembly of the models for the right and left morphing wings with the UAV body model

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (1)$$

$$y(t) = Cx(t) + Du(t), \quad (2)$$

where x , u , and y are the state vector, input or control vector, and output vector, respectively. A , B , C , and D are the dynamics matrix, input or control matrix, output or sensor matrix, and feedthrough matrix, respectively.

The complete model of one morphing wing (Emad, Mohammed and Fanni, 2022) can be represented in state space form as:

$$\mathbf{x}(t) = \begin{bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \\ \mathbf{v} \end{bmatrix} \quad (3)$$

$$\mathbf{A} = \begin{bmatrix} 0 & \mathbf{I} & 0 \\ -\tilde{M}^{-1}\tilde{K} & -\tilde{M}^{-1}\tilde{C} & \tilde{M}^{-1}\tilde{D} \\ \tilde{E}_1 & \tilde{E}_2 & \mathbf{F} \end{bmatrix} \quad (4)$$

and

$$\mathbf{B} = \begin{bmatrix} 0 \\ \tilde{M}^{-1}\tilde{B}_{\text{MFC}} \\ 0 \end{bmatrix} \quad (5)$$

where $\mathbf{q}$ and $\mathbf{v}$ are the modal coordinates and introduced variables, respectively. $\tilde{M}$, $\tilde{C}$, and $\tilde{K}$ are aerodynamic effective matrices in modal coordinates for acceleration, velocity, and displacement, respectively. $\tilde{E}_1$, $\tilde{E}_2$, and $\mathbf{F}$ are the aerodynamic effective matrices in modal coordinates for the introduced variables. $\tilde{B}_{\text{MFC}}$ is the effective coefficient matrix of the MFC actuator in modal coordinates. All these parameters were already acquired from the finite element model of the active morphing wing using the modal decomposition method.

Three outputs are generated from this model, y_{lift} , y_{pitch} , and y_{roll} which are defined as the total lift force, total pitching moment, and total rolling moment of the morphing wing, respectively. The aerodynamic outputs are calculated as follows:

$$y_{\text{lift}} = \int_L^{-L} Q_L \, dz \quad (6)$$

$$y_{\text{pitch}} = \int_L^{-L} (Q_L a + Q_M) \, dz \quad (7)$$

$$y_{\text{roll}} = \int_L^{-L} Q_L z \, dz, \quad (8)$$

where L and z are the wing semi-span and distance from the center of gravity of each strip to the fixation at the fuselage, and the width of each strip along the spanwise direction, respectively. It is to be noted that the total pitching moment is equal to the unsteady aerodynamic moment applied at the center of gravity of the airfoil Q_M , as the non-dimensional distance a is equal to zero in

the current model. Moreover, the aerodynamic outputs are calculated from the FE model in modal coordinates as:

$$Q_L = M_l \ddot{q} + C_l \dot{q} + K_l q + D_l v \quad (9)$$

$$Q_P = M_p \ddot{q} + C_p \dot{q} + K_p q + D_p v \quad (10)$$

$$Q_R = M_r \ddot{q} + C_r \dot{q} + K_r q + D_r v \quad (11)$$

where Q_L , Q_P , and Q_R are the total lift force, total pitching moment, and total rolling moment of the morphing wing, respectively. All other parameters in the above equations are matrices relating the aerodynamic outputs to the modal coordinate q with its derivatives and the introduced aerodynamic variables v . Hence, the three outputs can be represented in state space form as

$$y = \begin{bmatrix} y_{\text{lift}} \\ y_{\text{pitch}} \\ y_{\text{roll}} \end{bmatrix} \quad (12)$$

$$C = \begin{bmatrix} K_l - M_l \tilde{M}^{-1} \tilde{K} & C_l - M_l \tilde{M}^{-1} \tilde{C} & D_l + M_l \tilde{M}^{-1} \tilde{D} \\ K_p - M_p \tilde{M}^{-1} \tilde{K} & C_p - M_p \tilde{M}^{-1} \tilde{C} & D_p + M_p \tilde{M}^{-1} \tilde{D} \\ K_r - M_r \tilde{M}^{-1} \tilde{K} & C_r - M_r \tilde{M}^{-1} \tilde{C} & D_r + M_r \tilde{M}^{-1} \tilde{D} \end{bmatrix} \quad (13)$$

$$D = \begin{bmatrix} M_l \tilde{M}^{-1} \tilde{B}_{\text{MFC}} \\ M_p \tilde{M}^{-1} \tilde{B}_{\text{MFC}} \\ M_r \tilde{M}^{-1} \tilde{B}_{\text{MFC}} \end{bmatrix}. \quad (14)$$

2.2. Modeling of small UAV with morphing wings

After developing the state space model of one morphing wing influenced by aerodynamic loads in the previous subsection, the complete model of a small-light UAV will be developed. This complete model consists of the UAV body model, including the fuselage and other aircraft components, combined with the preceding model of the morphing wing. The UAV is assumed to be in equilibrium, where it moves horizontally with a constant velocity of 20 m/s, and the equivalent weight is applied at the center of gravity of the UAV body, which is also equal to the static lift force due to the initial angle of attack. Moreover, the aircraft is assumed to have an initial height of 5 m, an initial rolling angle of 0° , and zero drag force in the vertical direction, as the velocity of the UAV in ascending or descending is small compared to the horizontal velocity. Hence,

the fuselage and the rest of the parts of the aircraft are modeled as a rigid body with mass (m) and moments of inertia (I_{xx}, I_{yy}, I_{zz}).

The net mass of the UAV is estimated as 0.3 Kg, while its moments of inertia are estimated based on the statistical method (Pegram and Anemaat, 2000). This method supports the assumption of similarity for the non-dimensional radii of gyration between aircraft with nearly similar configurations and orientations. The moments of inertia for the current UAV can be calculated from equations (15), (16), and (17), below, where b and l are the wing total span and overall length of the UAV, respectively. $\bar{R}_x$, $\bar{R}_y$, and $\bar{R}_z$ are the non-dimensional radii of gyration of aircraft influencing rolling moment, pitch moment, and yaw moment, respectively.

$$I_{xx} = \left(\frac{b\bar{R}_x}{2} \right)^2 m \quad (15)$$

$$I_{yy} = \left(\frac{l\bar{R}_y}{2} \right)^2 m \quad (16)$$

$$I_{zz} = \left(\frac{e\bar{R}_z}{2} \right)^2 m; \quad e = \frac{b+l}{2}. \quad (17)$$

The UAV is assumed to be a small homebuilt propeller-driven aircraft with a span of 400 mm and a total length of 300 mm. The radii of gyration for the current design can be approximately referred to the airplane type "Cessna 177A". So, the values of the radii of gyration are: $\bar{R}_x = 0.212$, $\bar{R}_y = 0.362$, and $\bar{R}_z = 0.394$.

The control objectives in the current work are to achieve the desired altitude, pitch angle, and rolling angle of the aircraft. Therefore, three outputs are generated from the morphing wing model, which are the total lift force, total pitching moment, and total rolling moment. These three outputs are the actuating inputs to the UAV, where the translational and rotational equations of motion of the UAV rigid body are:

$$m\ddot{p} = F_L; \quad I_{xx}\ddot{\theta} = M_R; \quad I_{yy}\ddot{\alpha} = M_P \quad (18)$$

where p , α , and θ are the altitude, pitch angle, and roll angle of the UAV body, respectively. F_L , M_P , and M_R are the total lift force, total pitching moment, and total rolling moment generated from each morphing wing, respectively.

Based on equation (1), the state space model of the UAV body is:

$$\begin{bmatrix} \dot{p} \\ \ddot{p} \\ \dot{\alpha} \\ \ddot{\alpha} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p \\ \dot{p} \\ \alpha \\ \dot{\alpha} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 1/m & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1/I_{yy} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1/I_{xx} \end{bmatrix} \begin{bmatrix} F_L \\ M_P \\ M_R \end{bmatrix} \quad (19)$$

$$\begin{bmatrix} p \\ \alpha \\ \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} p \\ \dot{p} \\ \alpha \\ \dot{\alpha} \\ \theta \\ \dot{\theta} \end{bmatrix} + [0] \begin{bmatrix} F_L \\ M_P \\ M_R \end{bmatrix}. \quad (20)$$

The complete model of the UAV is achieved by assembling its body model with the left and right morphing wings models. The right and left morphing wings models are connected in parallel, then the resulting model is connected in series with the body model, as demonstrated in the block diagram in Fig. 1. Each morphing wing is controlled by two MFC actuators. Consequently, the complete model of the UAV is a multi-input, multi-output model (MIMO) with four inputs and three outputs. This assembled model will be called the plant model (G_p) in the control system design

3. Analysis of the plant model

Some analyses of the characteristics and behaviors of the complete UAV model are presented here. The analysis of the UAV model is covered by studying the step time response, frequency response (Bode diagram), pole-zero map, and closed-loop stability. The behaviors of the morphing wing model are evaluated first, then the influence of the UAV body model on the wing model is evaluated so that a full analysis of the complete UAV model is accomplished. After this analysis, the control design can be implemented efficiently.

3.1. Analysis of the decoupled morphing wing model

The assembled MIMO morphing wing model (G_w), represented in Fig. 1, has 12 transfer functions for its four inputs and three outputs. These transfer functions are nominated as G_w (Output number, Input number). Since the models of the

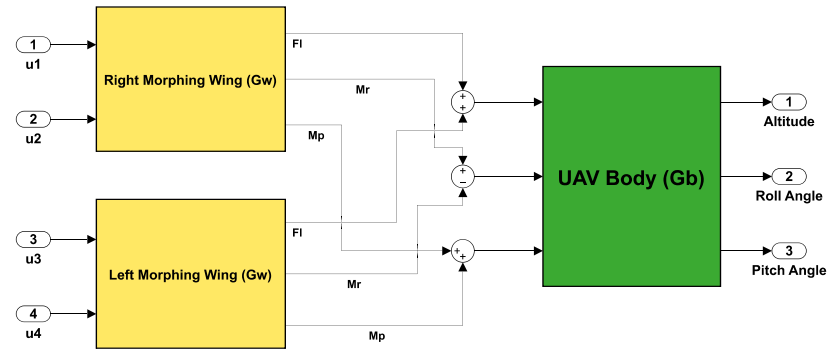


Figure 1: Block diagram for the assembly of the models for the right and left morphing wings with the UAV body model

left and the right morphing wings are identical, the transfer functions due to the inputs u_1 and u_2 are the same as the transfer functions due to the inputs u_3 and u_4 . Hence, the analysis will focus on six transfer functions only, which are $G_w(1,1)$, $G_w(1,2)$, $G_w(2,1)$, $G_w(2,2)$, $G_w(3,1)$, and $G_w(3,2)$.

Also, the transfer functions $G_w(1,1)$ with $G_w(1,2)$, the transfer functions $G_w(2,1)$ with $G_w(2,2)$, and the transfer functions $G_w(3,1)$ with $G_w(3,2)$ are expected to have great similarity, as the upper MFC actuator with u_1 voltage and the lower MFC actuator with u_2 voltage is of the same type and located at the same position on the upper and lower surfaces of the wing substrate. This similarity is declared in the Bode diagram of Fig. 2 for the open wings models (G_w) in the frequency domain.

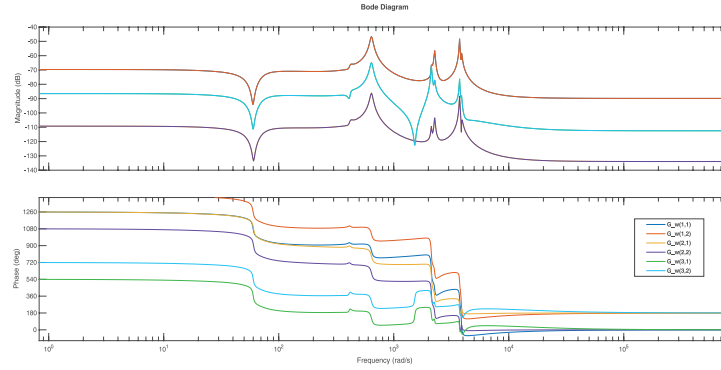


Figure 2: Bode diagram for the open-loop system (G_w)

The magnitude plot of the Bode diagram shows that $G_w(1,1)$ is almost equal to $G_w(1,2)$ with a percentage difference of about 0.0033%, and also $G_w(2,1)$ is almost equal to $G_w(2,2)$ with the same percentage difference. The phase plot of the Bode diagram shows that a phase shift of 180° exists between $G_w(1,1)$ and $G_w(1,2)$ as well as between $G_w(2,1)$ and $G_w(2,2)$. These results explain the need of applying inputs for each wing with equal magnitudes and opposite signs so that each output from the wing can be doubled.

Investigating the behavior of the open-loop system is essential; however, the most important analysis for the system stability is achieved for the closed-loop system with feedback. Consequently, the closed-loop system is created by connecting the open-loop model (G_w) in series with the controller model (G_c) and feedback with the sensor model (H) as this is shown in the block diagram of Fig. 3. In this figure, r , y , e , and u are the reference trajectory, the controlled output, the error signal, and the controller signal, respectively. It is assumed in this study that a unit negative feedback is applied in the closed-loop system. It is convenient in control system design to start the study for the closed-loop

system by using the controller model of unit static gain ($K_c = 1$). Then, the influence of the controller gain on the closed loop system can be investigated by varying the controller gain (K_c) from 0 to ∞ .

The analysis for the closed-loop stability is performed on the basis of the characteristic equations of the denominators of the closed-loop system models:

$$1 + G_c G_w H = 0.$$

This stability analysis is carried out for the closed-loop systems of the wing models (G_w) with unity controller gain and unity feedback gain. The stability margins are shown in Table 1 and represented as gain margin (GM), phase margin (PM), gain crossover frequency (w_{cg}), and phase crossover frequency (w_{cp}).

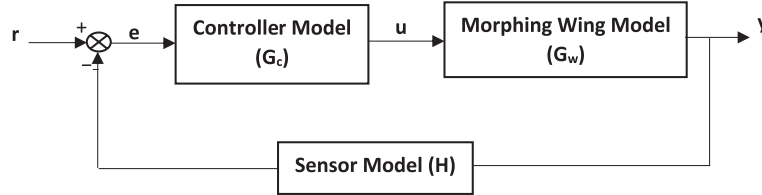


Figure 3: Block diagram for the closed-loop system

Here, GM and PM indicate the relative stability of the closed-loop system compared to that of an open-loop gain of 1 and an open-loop phase of -180° , respectively, while w_{cg} and w_{cp} are the frequencies at which the magnitude and phase of the open loop system are 1 and -180° , respectively. The results show that the closed-loop system for each morphing wing model is stable as GM and PM are both positive.

It is important to investigate the open-loop poles and zeros of the morphing wing models on the pole-zero map, as shown in Fig. 4, which affect the stability of the closed-loop system. In the figure, “x” and “o” denote poles and zeros on the pole-zero map, respectively. The number of the open-loop poles is equal to 94, this being the same number as for the open-loop zeros for each wing model. The open loop poles are the same for the models $G_w(1,1)$ and $G_w(1,2)$, as well as for the models $G_w(2,1)$ and $G_w(2,2)$. On the other hand, the open loop zeros differ between the models $G_w(1,1)$ and $G_w(1,2)$, as well as between the models $G_w(2,1)$ and $G_w(2,2)$. Also, all open-loop poles for each wing model are located within the stable region of the left-hand side (LHS) of the real axis of the pole-zero map. At the same time, open-loop zeros for each wing model are located on both LHS and the right-hand side (RHS) of the real axis of the

Table 1: Stability margins for the closed-loop systems of the morphing wing models

Morphing wing models	Gain margin (GM)	Phase margin (PM) ($^{\circ}$)	Gain crossover frequency (w_{cg}) (rad/s)	Phase crossover frequency (w_{cp}) (rad/s)
$G_w(1,1)$	397.06	∞	612.3	Not Determined
$G_w(1,2)$	282.2	∞	3725.3	Not Determined
$G_w(2,1)$	61526	∞	3780	Not Determined
$G_w(2,2)$	53341	∞	680.8	Not Determined
$G_w(3,1)$	2465.7	∞	2117.9	Not Determined
$G_w(3,2)$	7740.5	∞	3748.8	Not Determined

pole-zero map. This means that at certain controller gains (K_c), also called critical gains, the closed-loop poles for each wing model will cross over to the unstable region on the RHS of the real axis of the pole-zero map. These critical controller gains are the gain margins, which are demonstrated in Table 1.

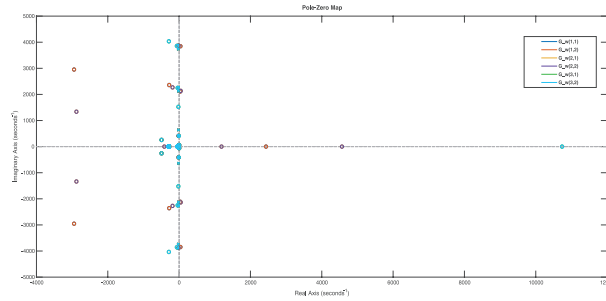


Figure 4: Pole-zero map for the morphing wing model

Moreover, the pole-zero map is plotted again for the open-loop wing models $G_w(1, [1 \ 2])$ and $G_w(2, [1 \ 2])$, considering both of the two inputs (u_1 and u_2) for every single output, as this is shown in Fig. 5. This study demonstrates the influence of linking the two models of the two inputs for each output. Some remarkable changes resulted from this linking of models. All open-loop poles and zeros are now located on the LHS of the real axis of the pole-zero map. Concerning the open loop poles for the wing models $G_w(1, [1 \ 2])$ and $G_w(2, [1 \ 2])$ there is the same total number of 94 poles and the same locations on the pole-zero map. Then, with regard to the open loop zeros for the wing model

$G_w(1, [1 \ 2])$ there is a total number of 65 zeros, where 33 zeros are located at approximately -43.33 , while 32 zeros are located at approximately -285.7 on the LHS of the real axis of the pole-zero map. On the other hand, concerning the open loop zeros for the wing model $G_w(2, [1 \ 2])$ there is the total number of 63 zeros, where 32 zeros are located at approximately -43.33 and 31 zeros, which are located at approximately -285.7 on the LHS of the real axis of the pole-zero map. Therefore, the influence of linking these models is revealed as it increases the stability of the system by pushing the open-loop zeros and poles to the stable region on the RHS of the real axis of the pole-zero map.

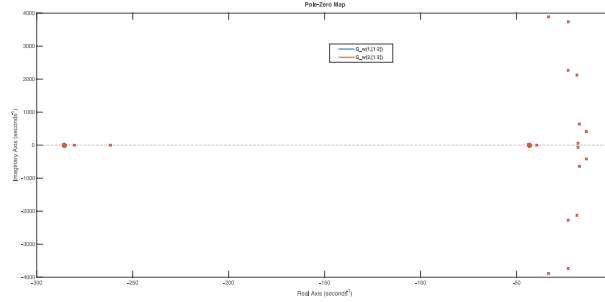


Figure 5: Pole-zero map after linking the wing models of the two inputs for each output

3.2. Analysis of the coupled plant model

The step response for the open loop plant model (G_p) with input's gain taken equal $u = [-500 \ 500 \ -500 \ 500]^T$ is illustrated in Fig. 6a, this affecting the altitude and pitch angle of the UAV, while the step response, having gain equal $u = [-500 \ 500 \ 500 \ -500]^T$ is illustrated in Fig. 6b, and it affects the roll angle of the UAV. The step responses of the morphing wing model have bounded outputs for the four bounded inputs based on the bounded-input-bounded-output (BIBO) theory (Emad, Mohamed and Fanni, 2022). However, the step responses of the UAV model (Fig. 6) after adding the body's model have unbounded outputs for the bounded inputs. This behavior for the UAV model is expected as the UAV body's model (G_b) has double integrators for each output generated from the undamped model, expressed in equation (18).

In order to get insight into the behavior of the open-loop UAV model, the zero-pole map is plotted and investigated. The zero-pole map in Fig. 7 reveals the four integrators of the UAV body's model at the origin of the map, while the other zeros and poles of the morphing wings models are located within the stable (LHS) part of the real axis. Also, all zeros and poles are extracted from

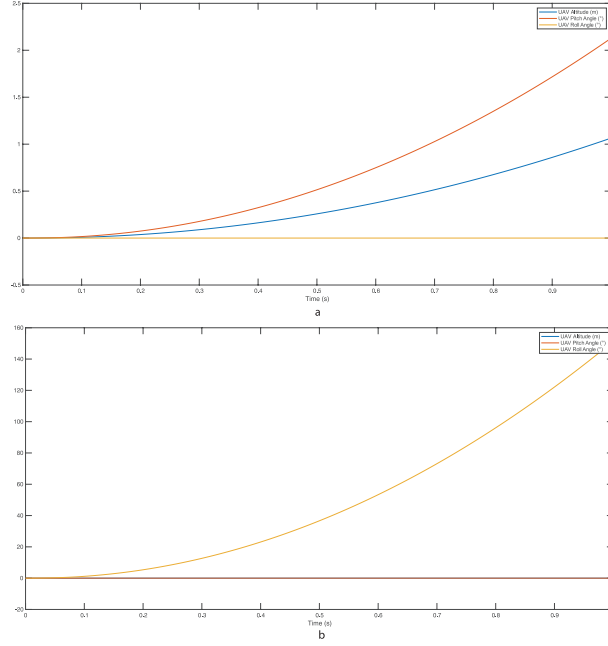


Figure 6: Step responses of the complete open-loop UAV model (G_p) (a) response due to step input with gain $u = [-500 \ 500 \ -500 \ 500]^T$. (b) response due to step input with gain $u = [-500 \ 500 \ 500 \ -500]^T$

the MIMO model (G_p), showing 215 zeros and 380 poles. All zeros have only negative real parts, four poles have zero values for both real and imaginary parts, some poles have only negative real parts, and the rest of the poles have pairs of negative real parts with conjugate imaginary parts.

Similarly to the Bode diagram of the morphing wing model of Fig. 2, the Bode diagram is plotted for the open loop UAV model (G_p) as displayed in Fig. 8, which shows that $G_p(1,1)$ equals $G_p(1,2)$, $G_p(2,1)$ equals $G_p(2,2)$, and $G_p(3,1)$ equals $G_p(3,2)$ in magnitude, whereas they display a phase shift of 180° in the phase plot. Due to the double integrators' influence, the phase plots for the four models of the UAV model (G_p) are shifted by -180° , while the magnitude plots are inclined with a slope of -40 dB/dec (decibel per decade).

Like the stability margins, investigated for the closed-loop systems of the morphing wing models in Table 1, the stability margins for the closed-loop systems of the UAV models are studied and demonstrated in Table 2. The stability margins show that the closed-loop systems of the UAV models become

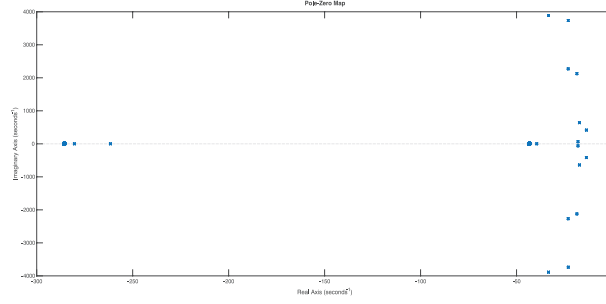


Figure 7: Zero-pole map for the open-loop system of the UAV model (G_p)

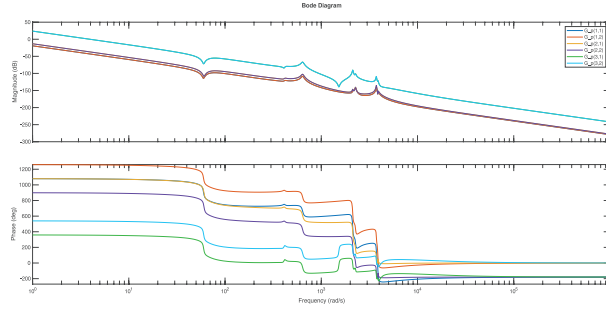


Figure 8: Bode plot for the open-loop system (G_p)

unstable. This unstable behavior is also revealed by the closed-loop UAV models plotted in the pole-zero map in Fig. 9, which shows the existence of some poles in the unstable region on the RHS of the imaginary axis but with very small values.

4. Classic PIDF controller design for the plant models

A classic PIDF controller will be designed with the aim of achieving the aerodynamic requirements for the UAV through active control shaping of the morphing wing. The PIDF controller can be designed for a single-input single-output (SISO) model only, this being difficult to use in the current MIMO plant model.

As a result of the analysis, presented in the previous subsection, the UAV MIMO model was declared to demonstrate great similarities in its internal eight transfer functions. Therefore, the control system is decoupled, and the PIDF controller will be designed for only one transfer function for each output in a single independent feedback control loop, as demonstrated in the block dia-

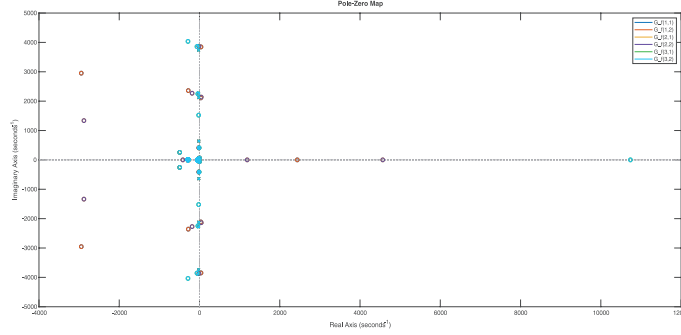


Figure 9: Pole-zero map for the closed-loop feedback system of the UAV models

Table 2: Stability margins for the closed-loop systems of the UAV models

UAV models	Gain margin (GM)	Phase margin (PM) ($^{\circ}$)	Gain crossover frequency (ω_{cg}) (rad/s)	Phase crossover frequency (ω_{cp}) (rad/s)
$G_p(1,1)$	5.4×10^5	-180.3°	59.84	0.3311
$G_p(1,2)$	4.5×10^5	-0.3°	612.33	0.3311
$G_p(2,1)$	2.7×10^5	-180.4°	60.4	0.473
$G_p(2,2)$	1.1×10^5	-0.4°	144.4	0.473
$G_p(3,1)$	1.2×10^4	-182°	60.02	2.243
$G_p(3,2)$	1.1×10^4	-2°	613.3	2.243

gram, shown in Fig. 10. Afterward, the MIMO overall control system will be coupled and investigated regarding the interaction between the single-feedback loop control systems.

The aerodynamic requirements for the control design are assumed to increase the UAV's altitude with a range of 40 cm, increase the pitch angle with a range of 4° , and increase the roll angle with a range of 40° . These requirements should be achieved with a fast and robust performance, described as a time response overshoot of $\leq 15\%$, settling time of ≤ 2 seconds, gain margin of ≥ 15 dB, and phase margin of $\geq 60^{\circ}$. The PIDF controller will be designed first to achieve the altitude requirement, then the pitch and roll angles requirements will be investigated.

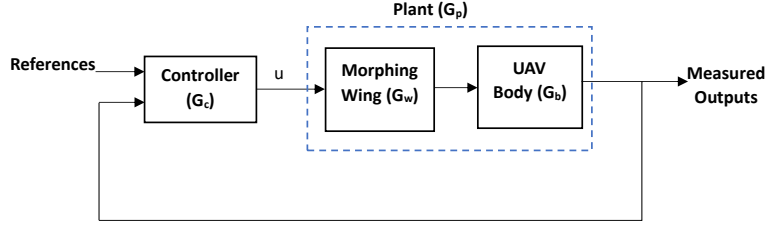


Figure 10: Generalized block diagram of the single closed-loop control system for each transfer function of the UAV models

The transfer function of the PIDF controller is:

$$G_c = K_p + K_i \frac{1}{s} + K_d \frac{s}{T_f s + 1} \quad (21)$$

where K_p , K_i , K_d , T_f are the proportional gain, integral gain, derivative gain, and filter time constant for the derivative action, respectively. The controller parameters are tuned using MATLAB software to achieve the control objectives.

The controller is initially tuned to achieve a fast and robust response for the $G_p(1,1)$ model, as shown in Fig. 11a with a desired altitude of 10 cm. Also, the equivalent error and control signals for this controller design are demonstrated in Figs. 11b and 11c, respectively. The tuning parameters for the designed PID controller are $K_p = -119$, $K_i = -49$, $K_d = -71.4$, and $T_f = 0.00112$. This controller design successfully achieves the requirements with an overshoot of 15%, a settling time of 1.6 s, a gain margin of 16.9 dB, and a phase margin of 70.2°. However, this PIDF controller cannot be practically applied for the morphing wings, as the MFCs have a limited excitation voltage of -500 V, whereas the designed control signal largely exceeds this limit, as shown in Fig. 11c.

This PIDF controller is tuned again to realize the voltage limitation of the MFCs. However, the respective controller design yields a slower and more aggressive controller compared to the previous controller design, as shown in the time response of Fig. 12a, and its equivalent error signal of Fig. 12b. The tuning parameters for the redesigned PIDF controller are $K_p = -0.138$, $K_i = -0.0024$, $K_d = -1.95$, and $T_f = 0.0397$.

This controller design failed to achieve the performance criteria, demonstrating the overshoot of 19.4 %, the settling time of 43.8 s, the gain margin of -21.9 dB, and the phase margin of 71.1°. This design of the PIDF controller is, of course, applicable to the control of the morphing wings, since the control signal is within the voltage limitation, as this is shown in Fig. 12c. These results show that the PIDF controller can control the UAV's altitude by actuating its morphing wings, but with very weak performance.

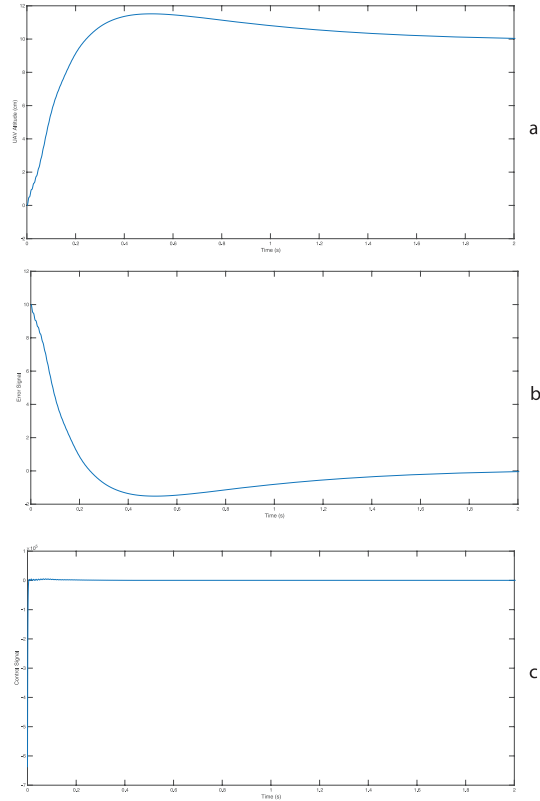


Figure 11: Results for the fast and robust controller design: (a) time response; (b) error signal; (c) control signal

The current performance of the control system can be explained as the PIDF controller is exerting a large effort to compensate for the unstable behavior of the UAV model, due to the inherited double integrator, as well as the closed-loop poles of the wing model, located near to the imaginary axis of the pole-zero map. This large effort can be seen in the high gains of the PIDF parameters as well as in the resulting control signal.

Hence, the unstable behavior of the UAV model needs to be enhanced in such a way that the PIDF controller can be efficiently designed to achieve the performance criteria. Some efficient solutions can overcome the problem of the unstable model of the UAV and achieve the desired response with reasonable control effort, such as adding a phase lead compensator, applying pole placement technique, optimal control technique, or using one of the robust control

techniques.

The pole placement technique cannot be used directly in this control system because the UAV model is neither completely controllable nor completely observable, these being the essential conditions in designing the state gains for the pole placement method. It is known that the phase lead compensator is widely used with PID controllers to enhance overall performance. According to Table 2, the gain margin for the UAV model is stable, whereas the phase margin is unstable, so only the phase margin for the UAV model needs to be enhanced. Hence, a lead compensator will be designed and investigated for its effect on the performance of PIDF control for achieving the performance requirements.

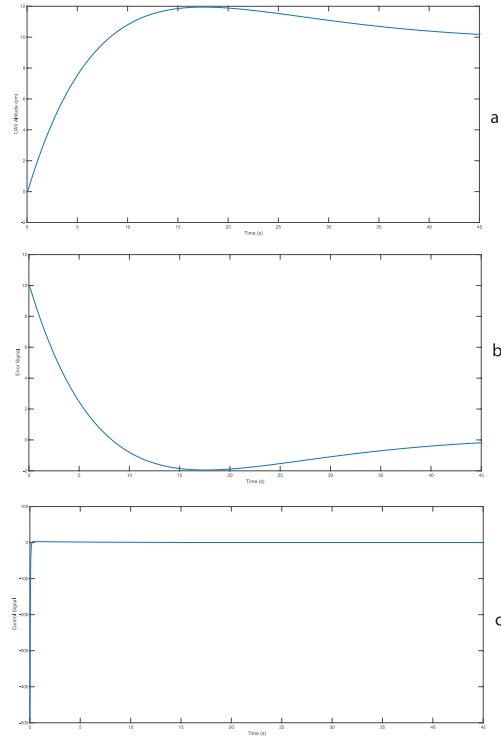


Figure 12: Results for the slow PIDF controller: (a) time response; (b) error signal; (c) control signal

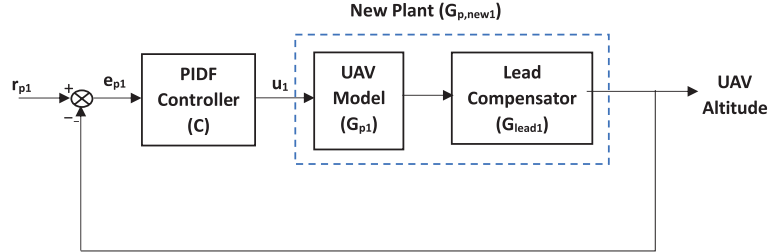


Figure 13: Block diagram for the control system of the UAV model with a lead compensator

5. Lead compensator design

A lead compensator is widely used in control systems to enhance the system's stability and increase the speed of response. The transfer function of a first-order phase lead compensator is:

$$G_{\text{lead}} = \frac{1 + aTs}{1 + Ts} \quad (22)$$

where s is a complex variable for the Laplace domain, while a and T are design parameters. These design parameters are determined from:

$$\omega_{\max} = \frac{1}{T\sqrt{a}} \quad ; \quad \sin \varphi = \frac{a - 1}{a + 1} \quad (23)$$

where $\omega_{\max}$ and $\sin \varphi$ are the frequency at which the maximum added phase occurred and the maximum added phase, respectively. The maximum added phase from the lead compensator is equal up to 90° , and it depends on the design parameter a . The frequency $\omega_{\max}$ is chosen on the basis of the value of the phase crossover frequency for each plant model, also the maximum added phase (φ) is set to 70° . However, the effective added phase on the phase margin of the UAV model after adding the lead compensator is about 50° . Increasing the maximum phase of the lead compensator by more than 70° has a nearly negligible influence on the effective added phase on the plant model. The stability margins for the UAV models after designing and adding a lead compensator for each model are shown in Table 3. The results show that the phase margins are increased, but at the cost of decreasing their gain margins. Also, the UAV models $G_p(1,2)$, $G_p(2,2)$, and $G_p(3,2)$ are now stable after adding the lead compensators.

The PIDF controller is designed again for the UAV altitude models $G_p(1,1)$ and $G_p(1,2)$ after adding the lead compensator according to the block diagram of Fig. 13. It was found that a 2^{nd} order lead compensator was required. The

Table 3: Stability margins for the closed-loop systems of the UAV models with lead compensators

Plant models with lead compensators	Gain margin (GM)	Phase margin (PM)	Gain crossover frequency (ω_{cg})	Phase crossover frequency (ω_{cp}) (rad/s)
$G_p(1,1)$	1.7×10^4	-131.8°	59.9	1.5
$G_p(1,2)$	33.3	48.2°	10.6	1.5
$G_p(2,1)$	8.4×10^3	-132.5°	60.45	2.1
$G_p(2,2)$	22.1	47.5°	12.3	2.1
$G_p(3,1)$	383.7	-139.1°	60.2	9.9
$G_p(3,2)$	5.6	40.9°	26.7	9.9

1^{st} order design largely reduced the control effort but still exceeded the voltage limitations. It achieves an overshoot of 9.8%, a settling time of 0.8 seconds, a gain margin of 22 dB, and a phase margin of 65.7° .

For controlling the UAV's pitch angle, the control systems for the UAV models $G_p(2,1)$ and $G_p(2,2)$ are designed similarly as in the previous design of the PIDF controller for the models of the UAV's altitude. A 2^{nd} order lead compensator was required. While a 1^{st} order compensator met the required performance for a 1° pitch angle, it produced a maximum control effort of approximately -400 V, which is too high for the subsequent coupling phase. It achieves an overshoot of 5.5%, a settling time of 0.84 seconds, a gain margin of 20.9 dB, and a phase margin of 69.3° .

For controlling the UAV's roll angle, the control systems for the UAV models $G_p(3,1)$ and $G_p(3,2)$ are designed similarly to the aforementioned design. A 1^{st} order lead compensator was sufficient. This configuration successfully achieves the required 10° set point while keeping control efforts below the acceptable 200 V threshold. It achieves an overshoot of 8%, a settling time of 1.2 seconds, a gain margin of 16.5 dB, and a phase margin of 67.5° . The equivalent time responses, error signals, and control signals for all closed-loop control systems are consolidated in Fig. 14. Table 4 summarizes the results of controller design with respect to the tuned controller gains.

To quantitatively validate the robustness of the chosen lead compensator design, a sensitivity analysis was conducted with respect to the design parameters a and T . These parameters dictate the maximum added phase φ_{max} and the frequency at which it occurs, ω_{max} . The altitude control model $G_p(1,1)$ was

Table 4: Summary for reference tracking performance with respect to tuned controller gains

		G_p (1,1)	G_p (2,1)	G_p (3,1)
PIDF tuned controller gains for each plant model				
K_p		-0.47	-0.27	-1.24
K_i		-0.576	-0.41	-1.68
K_d		-0.082	-0.041	-0.11
T_f		0.0095	0.0095	0.011
	Required performance	Closed loop performance for each plant model		
Overshoot %	$\leq 15\%$	9.8%	5.5 %	8%
Settling time	≤ 2 sec	0.8 sec	0.84 sec	1.2 sec
Gain margin	≥ 15 dB	22 dB	20.9 dB	16.5 dB
Phase margin	$\geq 60^\circ$	65.7°	69.3°	67.5°

used as the baseline for this analysis. The nominal parameters were varied by $\pm 20\%$, and the effects on the system's stability margins, transient overshoot, and maximum control effort (voltage) were recorded.

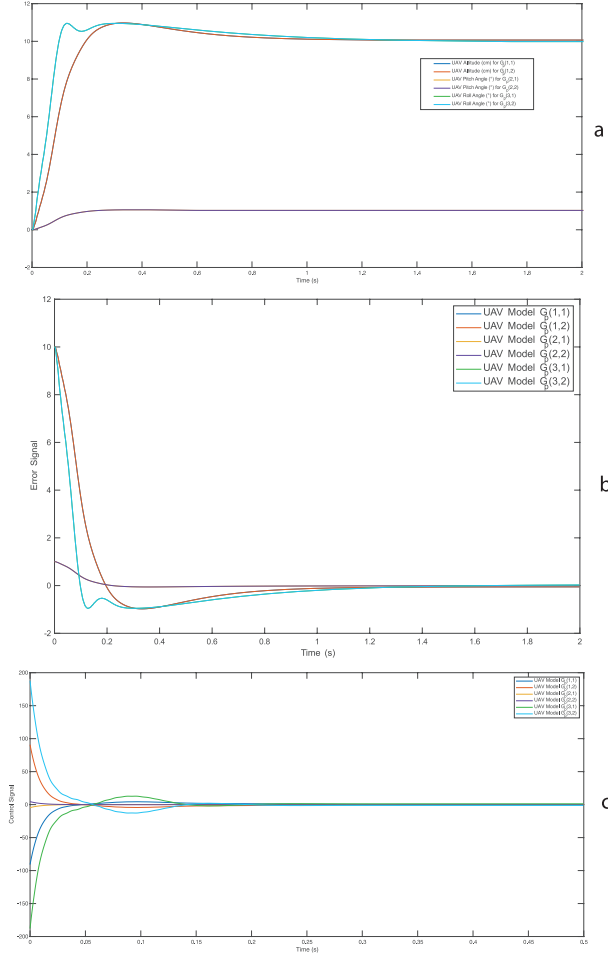


Figure 14: Results for the PIDF controllers with lead compensator control systems of $G_p(1,1)$ and $G_p(1,2)$; $G_p(2,1)$ and $G_p(2,2)$; $G_p(3,1)$ and $G_p(3,2)$: (a) time response. (b) error signal. (c) control signal

As shown in Table 5, varying the lead compensator parameters by $\pm 20\%$, leads the phase margin to vary between 62.3° and 70.9° , the gain margin to range from 18.9 dB to 25.6 dB, and the maximum overshoot to fluctuate between 4.2% and 13.7%. Notably, the maximum control effort remains strictly unchanged at

Table 5: Sensitivity analysis of lead compensator parameters for G_p (1,1)

Condition	a	T	PM	GM	$M_p\%$	$ u_{\max} $
Nominal design	32.2	0.53	65.7	22	9.8	90.8V
+20% variation in a	38.6	0.47	64.2	18.9	8.13	90.8V
−20% variation in a	25.7	0.6	63.6	25.6	13.7	90.8V
+20% variation in T	32.2	0.64	62.3	21.8	13.7	90.8V
−20% variation in T	32.2	0.43	70.9	22.3	4.2	90.8V

90.8 V across all tested variations. These results confirm that the required performance control objectives and controller effort limitations are consistently achieved despite considerable variations in the lead compensator parameters, successfully demonstrating the robustness of the proposed design.

6. Coupled control system for the MIMO plant model

6.1. Design of closed-loop system with disturbance rejection controller

The control systems for the decoupled independent closed loops of the UAV models are successfully designed as this is described in the previous section and can be represented by the block diagrams shown in Fig. 15a, Fig. 15b and Fig. 15c for the altitude (p), pitch angle (α), and roll angle (θ) outputs, respectively.

Regarding the notation, used in these figures, r_p is the total required altitude, with

$$r_{p1}=r_{p2}=r_{p3}=r_{p4}=\frac{r_p}{4},$$

r_α is the total required roll angle, with

$$r_{\alpha1}=r_{\alpha2}=r_{\alpha3}=r_{\alpha4}=\frac{r_\alpha}{4},$$

and r_θ is the total required roll angle, with

$$r_{\theta1}=r_{\theta2}=r_{\theta3}=r_{\theta4}=\frac{r_\theta}{4}.$$

The error signals for each output control system are e_{p1} , e_{p2} , e_{p3} , e_{p4} ; $e_{\alpha1}$, $e_{\alpha2}$, $e_{\alpha3}$, $e_{\alpha4}$; and $e_{\theta1}$, $e_{\theta2}$, $e_{\theta3}$, $e_{\theta4}$, concerning the altitude, pitch angle, and roll angle, respectively. P is the total actual altitude of the UAV, α is the total actual pitch angle of the UAV, and θ is the total actual roll angle of the UAV. Then, u_1 , u_2 , u_3 , and u_4 are the control signals generated from each controller.

The controller (C) and the lead compensator (G_{lead}) models are named in the the same way as the UAV plant models (G_p). From the similarities of the control systems,

$$-C(1, 1) = C(1, 2) = -C(1, 3) = C(1, 4) = C_p$$

and

$$G_{lead}(1, 1) = G_{lead}(1, 2) = G_{lead}(1, 3) = G_{lead}(1, 4) = G_{lead,p}.$$

Similarly,

$$-C(2, 1) = C(2, 2) = -C(2, 3) = C(2, 4) = C_\alpha$$

and

$$G_{lead}(2, 1) = G_{lead}(2, 2) = G_{lead}(2, 3) = G_{lead}(2, 4) = G_{lead,\alpha}.$$

Moreover,

$$-C(3, 1) = C(3, 2) = C(3, 3) = -C(3, 4) = C_\theta$$

and

$$G_{lead}(3, 1) = G_{lead}(3, 2) = G_{lead}(3, 3) = G_{lead}(3, 4) = G_{lead,\theta}.$$

From the similarities of the UAV models,

$$G_p(1, 1) = G_p(1, 3), \quad G_p(1, 2) = G_p(1, 4),$$

and

$$G_p(1, 1) \approx -G_p(1, 2).$$

Similarly, $G_p(2, 1) = G_p(2, 3)$, $G_p(2, 2) = G_p(2, 4)$, and

$$G_p(2, 1) \approx -G_p(2, 2).$$

Also, $G_p(3, 1) = -G_p(3, 3)$, $G_p(3, 2) = -G_p(3, 4)$, and

$$G_p(3, 1) \approx -G_p(3, 2).$$

It is to be noted that the control signals are generated as follows:

$$u_1 = e_{p1}C(1, 1) + e_{\alpha 1}C(2, 1) + e_{\theta 1}C(3, 1) \quad (24)$$

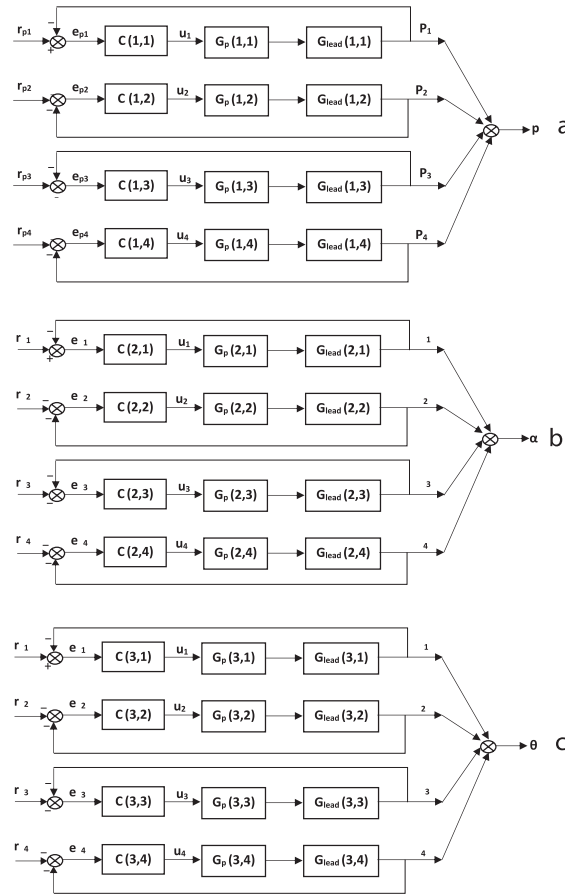


Figure 15: Block diagram for the decoupled control systems of the MIMO UAV model; (a) control system for the altitude (p) output; (b) control system for the pitch angle (α) output; (c) control system for the roll angle (θ) output

$$u_2 = e_{p2} C(1, 2) + e_{\alpha 2} C(2, 2) + e_{\theta 2} C(3, 2) \quad (25)$$

$$u_3 = e_{p3} C(1, 3) + e_{\alpha 3} C(2, 3) + e_{\theta 3} C(3, 3) \quad (26)$$

$$u_4 = e_{p4} C(1, 4) + e_{\alpha 4} C(2, 4) + e_{\theta 4} C(3, 4). \quad (27)$$

The equations above, establishing the control signals, show clearly that each control signal is subjected to the control efforts generated from all outputs. However, the controllers of each independent closed-loop system are designed based on the UAV plant model of every single output. Therefore, the controller of each closed-loop system is affected by other closed-loop systems, as demonstrated in the block diagram for the control signal u_1 in Fig. 16.

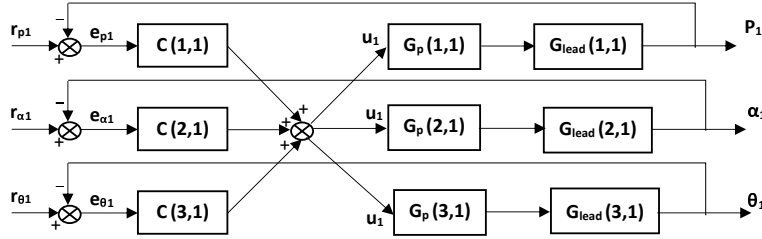


Figure 16: Block diagram for the interaction between closed-loop control systems to generate the equivalent control signal u_1

This interaction between the independent closed-loop systems has caused disturbances in each controller, which poses great challenges during the reference tracking objective. To solve this problem, another controller is added for each closed-loop system. The function of the extra controllers is to eliminate the disturbances coming from other control systems. These added controllers are referred to as disturbance rejection controllers (C_d) while the main PIDF controllers are referred to as reference tracking controllers (C_r).

The models of the disturbance rejection controllers can be determined from the mathematical equations of the outputs as follows

$$p_1 = G_p(1, 1) G_{lead}(1, 1) e_{p1} C_r(1, 1) + G_p(1, 1) G_{lead}(1, 1) [e_{\alpha 1} C_r(2, 1) + e_{\theta 1} C_r(3, 1)] \quad (28)$$

$$e_{p1} = r_{p1} - p_1 = r_{p1} - [C_r(1, 1) G_p(1, 1) G_{lead}(1, 1)] e_{p1} \quad (29)$$

$$e_{p1} = \frac{1}{1 + C_r(1, 1) G_p(1, 1) G_{lead}(1, 1)} r_{p1}. \quad (30)$$

Upon substituting Equation (30) into Equation (28) we get:

$$p_1 = \frac{C_r(1,1) G_p(1,1) G_{lead}(1,1)}{1 + C_r(1,1) G_p(1,1) G_{lead}(1,1)} r_{p1} + G_p(1,1) G_{lead}(1,1) [e_{\alpha 1} C_r(2,1) + e_{\theta 1} C_r(3,1)]. \quad (31)$$

Similarly,

$$\alpha_1 = \frac{C_r(2,1) G_p(2,1) G_{lead}(2,1)}{1 + C_r(2,1) G_p(2,1) G_{lead}(2,1)} r_{\alpha 1} + G_p(2,1) G_{lead}(2,1) [e_{p1} C_r(1,1) + e_{\theta 1} C_r(3,1)] \quad (32)$$

$$\theta_1 = \frac{C_r(3,1) G_p(3,1) G_{lead}(3,1)}{1 + C_r(3,1) G_p(3,1) G_{lead}(3,1)} r_{\theta 1} + G_p(3,1) G_{lead}(3,1) [e_{p1} C_r(1,1) + e_{\alpha 1} C_r(2,1)]. \quad (33)$$

The first part of each equation is the transfer function of the closed-loop system for every single output, while the rest of each equation represents the disturbance model at each output. Hence, the disturbance rejection controller can be added to the control signal of the reference tracking controller, which eliminates the disturbances of other closed-loop systems, as this is illustrated in the example of Fig. 17.

The decoupled control systems here described can be simply converted to a single coupled control system, in the way, which is shown further in Fig. 18, owing to the great similarities between the decoupled effective models. The total altitude, total pitch angle, and total roll angle of the UAV are the only measurable signals from the UAV that can be measured using gyroscope or accelerometer sensors.

From a physical point of view, the internal outputs, like p_1 , p_2 , p_3 , and p_4 , cannot be directly measured, and hence, the lead compensators for each control loop system cannot be physically added to these internal outputs. However, the internal outputs can be estimated by designing a state observer for the internal states of the MIMO UAV model. For simplicity, this issue is solved by moving the lead compensator inside the reference tracking controller, as shown in the coupled control system in Fig. 18. Similarly, the disturbance rejection controller cannot be physically added to the control signals subjected to each output. Therefore, the disturbance rejection controller is moved to the feedback path as demonstrated in Fig. 18. By this configuration, the overall control system depends only on the measured total outputs from the UAV, which are practically feasible.

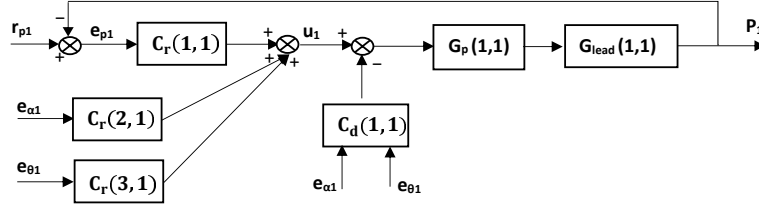


Figure 17: Block diagram showing the influence of the disturbance rejection controller on the closed-loop control system of output p_1

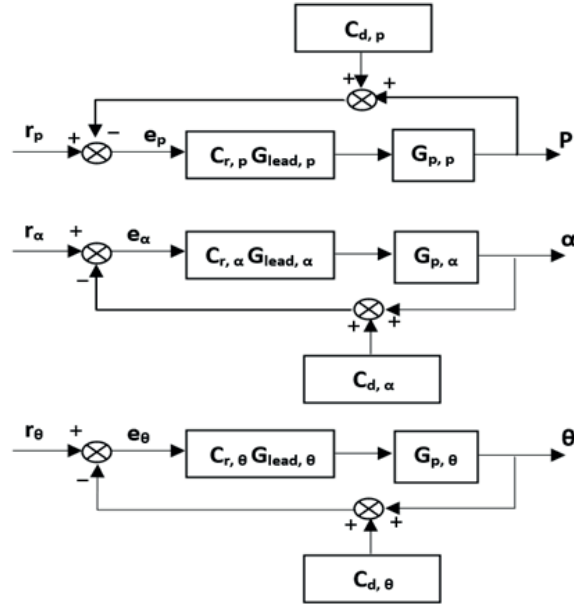


Figure 18: Block diagram for the coupled control systems of the MIMO UAV model

The overall matrix for the reference tracking controller combined with the lead compensator and disturbance rejection controller can be written as:

$$C_r = \begin{bmatrix} -C_p G_{lead,p}^2 & -C_\alpha G_{lead,\alpha}^2 & -C_\theta G_{lead,\theta} \\ C_p G_{lead,p}^2 & C_\alpha G_{lead,\alpha}^2 & C_\theta G_{lead,\theta} \\ -C_p G_{lead,p}^2 & -C_\alpha G_{lead,\alpha}^2 & C_\theta G_{lead,\theta} \\ C_p G_{lead,p}^2 & C_\alpha G_{lead,\alpha}^2 & -C_\theta G_{lead,\theta} \end{bmatrix} \quad (34)$$

$$C_d = \begin{bmatrix} 0 & \frac{C_\alpha G_{lead,\alpha}^2}{C_p G_{lead,p}^2} & \frac{C_\theta G_{lead,\theta}}{C_p G_{lead,p}^2} \\ \frac{C_p G_{lead,p}^2}{C_\alpha G_{lead,\alpha}^2} & 0 & \frac{C_\theta G_{lead,\theta}}{C_\alpha G_{lead,\alpha}^2} \\ \frac{C_p G_{lead,p}^2}{C_\theta G_{lead,\theta}} & \frac{C_\alpha G_{lead,\alpha}^2}{C_\theta G_{lead,\theta}} & 0 \end{bmatrix} \quad (35)$$

where the inputs to the disturbance rejection controller are $[e_p \ e_\alpha \ e_\theta]$, and its outputs are $[d_p \ d_\alpha \ d_\theta]$.

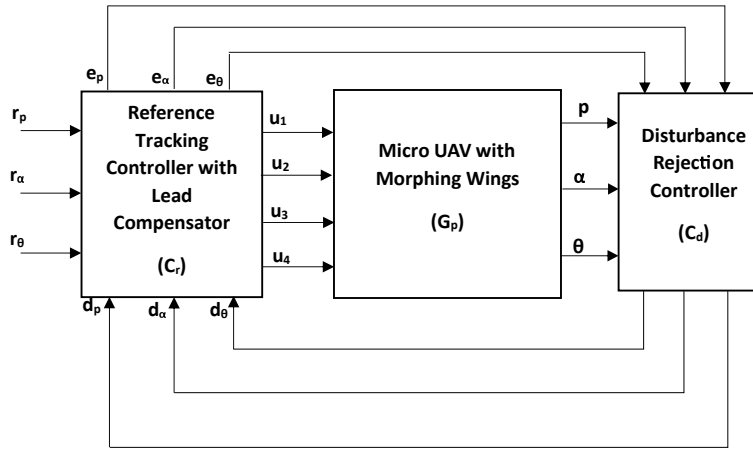


Figure 19: Generalized block diagram for the assembled control system for the UAV with morphing wings

Another generalized block diagram is shown in Fig. 19 for the practically assembled control system of the UAV actuated by morphing wings. The summing points (comparators), the PIDF controllers, lead compensators, and disturbance rejection controllers are programmed inside the control unit, which can be an embedded processor inside an Arduino board or any other programmable microcontroller system. The control signals, which are generated from the control unit are sent to the morphing wings through an amplifier in order to provide a sufficient voltage needed by the MFC actuators. The amplifier can be the commercial-lightweight microHVA-2, which is a high-voltage power amplifier with dual MFC channels. This amplifier can supply a PWM control voltage to two different MFC actuators simultaneously with a range of -500 V to 1500 V. Also, it can be operated with a DC battery from 12 V to 16 V.

6.2. Stability analysis for the closed-loop control system

While the simulation results demonstrate bounded tracking errors, the theoretical stability of the coupled MIMO system is verified by analyzing the closed-loop

poles from characteristic equation of the closed-loop system. Calculation of the system eigenvalues confirms that all closed-loop poles are strictly located in the left-half s-plane. Specifically, the dominant poles are located at $s = -a \pm bj$, ensuring that the controller structure stabilizes the cross-coupled dynamics and satisfies the theoretical stability requirement. The closed-loop system has 533 poles; all with negative real parts; and 482 zeros, which are illustrated in the pole-zero map in Fig. 20.

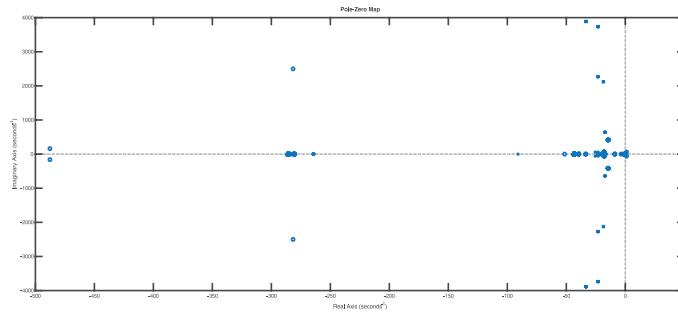


Figure 20: Pole-zero map for the coupled closed loop system with disturbance rejection controller

6.3. Sequence of steps of the proposed method

The sequence of steps for the conducted method is given in the form of the flowchart in Fig. 21. First, the active morphing wing is developed as MIMO state space model. Then the morphing wing model is coupled with the UAV rigid body model. Next, various analyses have been carried out in order to investigate the inherent coupling effects between every morphing wing model as well as the effect of the coupled UAV body model. These analyses facilitate the design of PIDF Controller for each wing model, followed by adding lead compensators to account for the UAV body effect. After that, the closed loop system is investigated and formulated in such a manner that the decoupled closed loop system for each controlled variable is affected by the disturbances coming from the other variables. Finally, disturbance rejection controllers are established and developed to compensate for the influence of the controlled variables exerted on each other.

7. Results and discussion

The designed control system is first investigated for control of only one variable of the step signal, such as the required height of 40 cm with zero rolling angle,

as shown in Fig. 22a, the first one on the series of illustrations in Fig. 22, and the required rolling angle of 40° with zero height as shown in Fig. 22c. Then, the control system is investigated for controlling both height and rolling angle simultaneously with a required step height of 31 cm and step rolling angle of 40° as this is shown in Fig. 22e. If the required height is above 31 cm with a rolling angle of 40° , the control efforts will exceed the MFC voltage limitation. The corresponding control signals for each case are shown, respectively, in Fig. 22b, Fig. 22d, and Fig. 22f. The results show that the control system successfully achieves the aerodynamic requirements of step inputs for the small UAV with fast and robust performance.

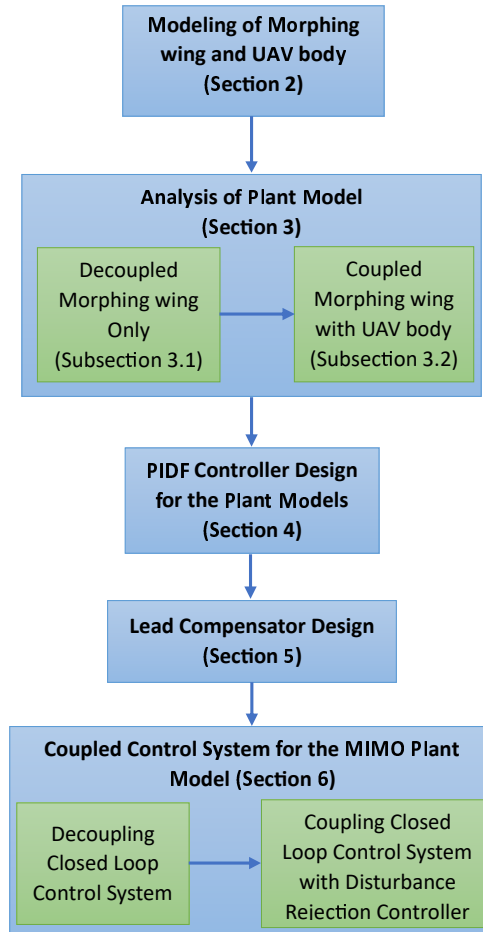


Figure 21: Flowchart describing the sequence of steps of the proposed method

Table 6: Comparison between the classical PIDF control and the proposed coupled controller with disturbance rejection for the performance of UAV altitude tracking

	Overshoot (%)	Settling time	Gain margin	Phase margin
Classical PIDF control	19.4 %	43.8 s	-21.9 dB	71.1°
Proposed coupled control	7.5%	1.9 s	22 dB	68°

Also, the control system for the UAV is investigated for controlling ramp input, as this is shown in Fig. 23a. The corresponding error signals and control signals for the ramp input are shown in Fig. 23b and in Fig. 23c, respectively.

It is worth noting what is the position of the proposed PIDF approach within the broader spectrum of control strategies for morphing UAVs. In a previous work (Emad, Mohamed and Fanni, 2022), Model Predictive Control (MPC) was shown to provide excellent tracking performance; however, this came at the cost of high computational demand, potentially limiting its use regarding the resource-constrained flight controllers.

Moreover, the control system for the UAV is investigated for controlling a sinusoidal input with a frequency of $\frac{1}{2*\pi}$ as this is shown in Fig. 24a. The corresponding error signals and control signals for the sinusoidal input are shown, respectively, in Fig. 24b and Fig. 24c.

Conversely, while a classical PID structure offers simplicity, it fails to provide adequate reference tracking for this high-order model due to inherent coupling behaviors. The proposed PIDF controller, therefore, bridges this gap. It provides a feasible solution that achieves the necessary reference tracking, which is unattainable with standard PID, while retaining computational efficiency superior to high-level schemes like MPC or SMC. The performance improvement, which is claimed here, is thus defined by the balance of successful tracking capabilities combined with practical implementability. Table 6 provides a quantitative comparison between the proposed PIDF approach and a classical PID controller. The comparison highlights key performance metrics, including overshoot, settling time, gain margin and phase margin, clearly demonstrating the improved stability and tracking performance of the proposed method in the presence of inherent coupling behaviors where the standard PID struggles.

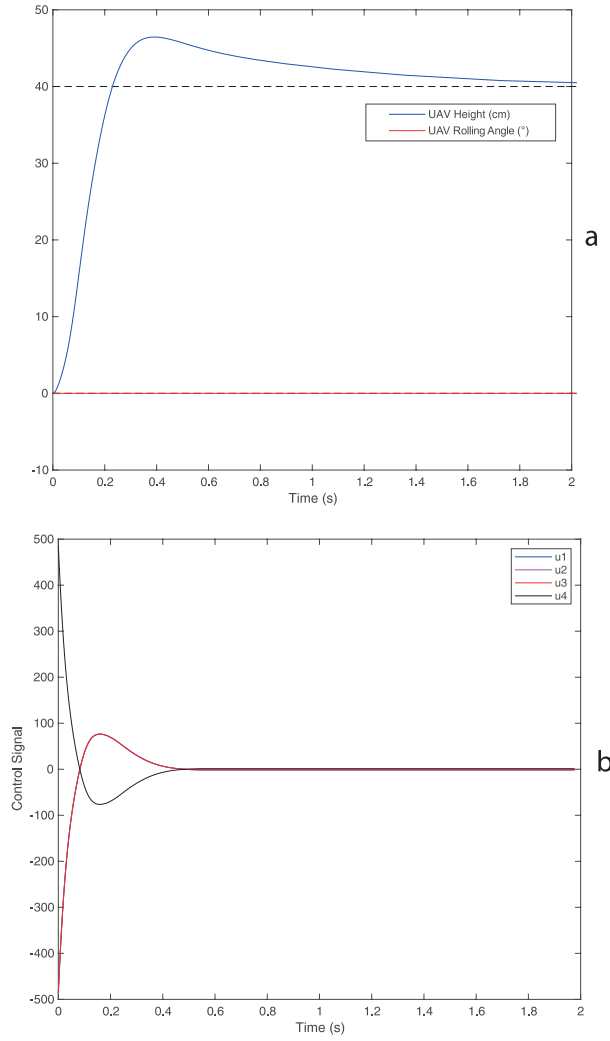


Figure 22: parts "a" and "b", to be continued

8. Conclusion

This paper presents a systematic approach meant for designing and analyzing PIDF controllers for complex high-order MIMO systems with compensatory controllers to leverage the coupling effect of inherited dynamics. First, the modeling of morphing wings actuated by MFC actuators is elaborated. Then, these models are investigated before and after adding the model of the UAV

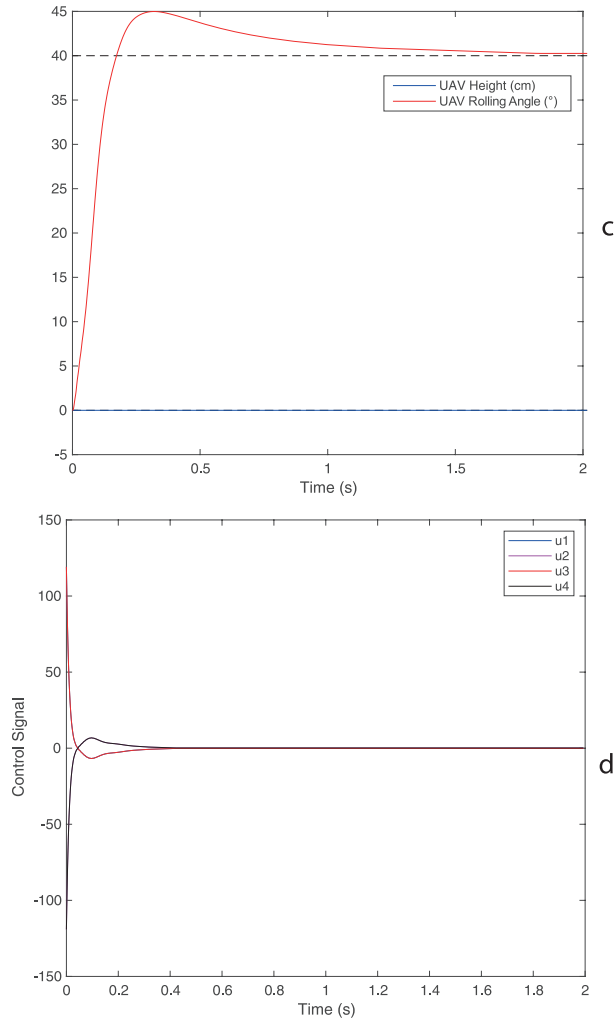


Figure 22: parts "c" and "d", to be continued

body, constructed as a rigid body with multi-DOF. The approach reduces the complexity by decoupling the MIMO system model into equivalent SISO models, which are then tuned separately based on the desired closed-loop specifications. In addition, lead compensators are added to the plant model so as to enhance the PIDF control effect and reduce the controller efforts. The coupled and decoupled models are analyzed in different domains to facilitate the controller design phase. Within the approach we also introduce additional controllers to mitigate the coupling effects after the assembly of the control system, while respecting the

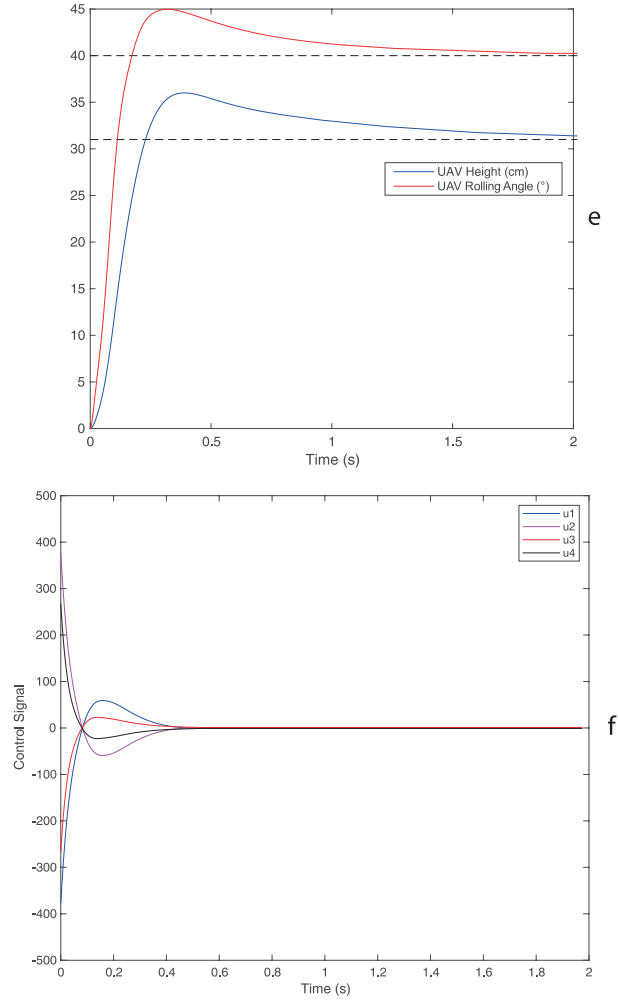


Figure 22: Results for step responses for the designed control system: (a) step response for the required height of 40 cm and rolling angle of 0° ; (b) corresponding control signals for the required height of 40 cm and rolling angle of 0° ; (c) step response for a required rolling angle of 40° and height of 0 cm; (d) corresponding control signals for the required rolling angle of 40° and height of 0 cm; (e) step response for required height of 31 cm and rolling angle of 40° ; (f) corresponding control signals for the required height of 31 cm and rolling angle of 40°

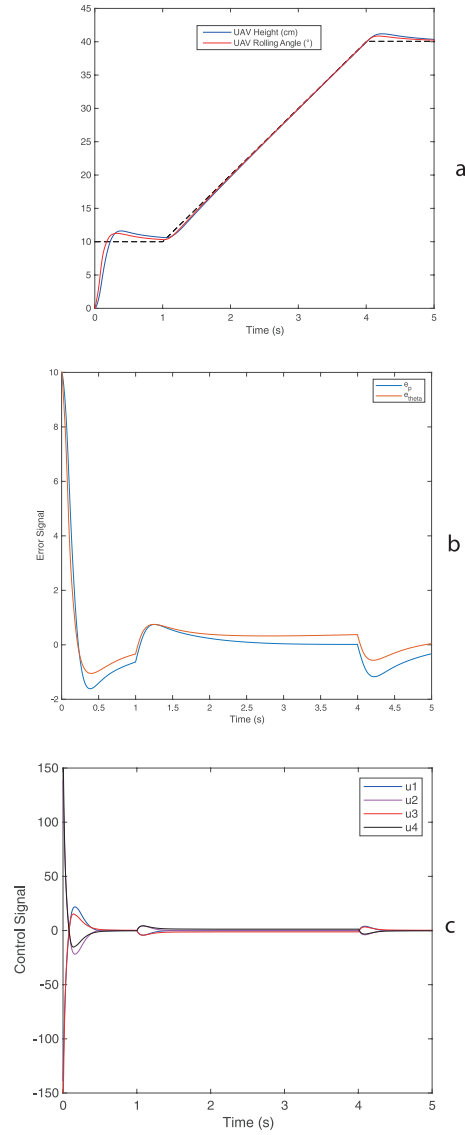


Figure 23: Results for the ramp signal response for the designed control system: (a) time response for the ramp input; (b) corresponding error signals for ramp response; (c) corresponding control signals for ramp input response

physical constraints. The approach is validated through numerical simulations on a high-order MIMO system with inherited coupling behavior, showing that it can achieve good reference tracking with various types of inputs.

It is important to note that the reference trajectory amplitudes in this study were selected to maintain the control effort within the physical saturation limits of the MFC actuators (± 500 V). For the maneuvers presented, the control voltage did not exceed these bounds, ensuring the validity of the linear control assumption.

Some envisaged directions for future work can be described as follows:

- While the current study demonstrates effective tracking within the linear actuation range, the present author acknowledges that during highly aggressive maneuvers or in the presence of large, sustained disturbances, unchecked actuator saturation could lead to integral windup. This would likely result in temporary degradation of the decoupling performance, larger transient overshoots, and delayed settling times. Therefore, future work will focus on integrating anti-windup mechanisms and constrained control allocation to handle actuator non-linearities during aggressive maneuvers.
- Extending the proposed method in order to handle nonlinear MIMO systems and time-varying MIMO systems.
- Developing adaptive and optimal PIDF controllers and compensators for MIMO systems with uncertain or varying parameters.
- Applying the proposed method to other practical MIMO systems, such as robotic manipulators, conventional aircraft, and power grids.

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Data and material availability The data presented in this study are available on request from the author.

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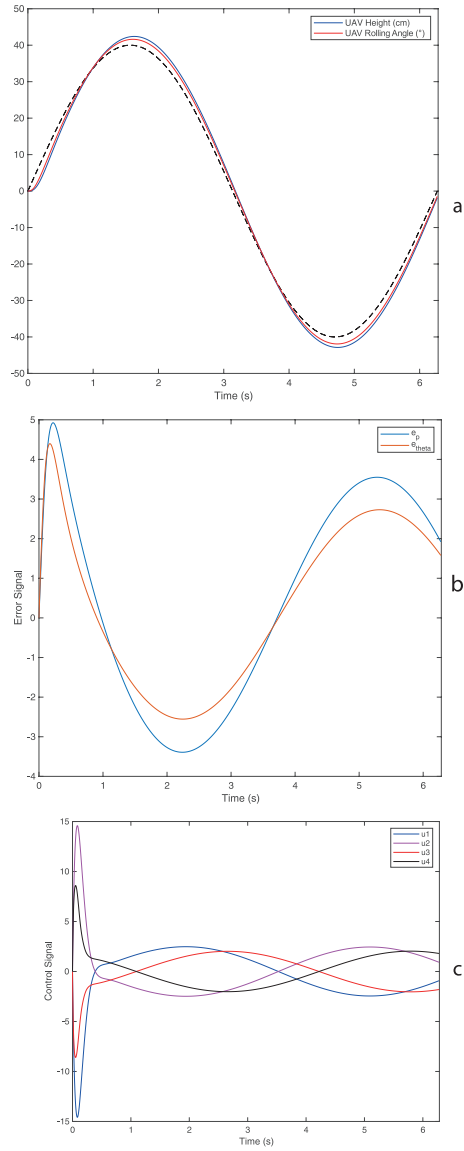


Figure 24: Results for a sinusoidal signal response for the designed control system: (a) time response for the sinusoidal input; (b) corresponding error signals for a sinusoidal response; (c) corresponding control signals for a sinusoidal response