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Book review:
Minimum Entropy Control for Time Varying Systems

by

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The minimum entropy control is a kind of optimal control problem in which the performance index takes the form of a functional interpreted as entropy. The problem received some, but not very significant, attention in the literature, however, the respective publications concern the linear time invariant systems. Using concepts from operator theory a time-domain interpretation of entropy is presented in the book reviewed.

The book is composed of seven chapters. In Chapter 1, first the two related optimal control problems, H2 and H_{00} are considered. It is stressed that these two optimal control theories were proposed to achieve two conflicting goals in the system: performance and robustness, respectively. It is shown that minimum entropy control allows to trade off these two goals.

In Chapter 2 the class of the considered systems, as well as the notation used, are determined. The linear time varying systems are here represented as infinite-dimensional operators. Some general properties of these input-output operators are discussed. Next the special class of systems, namely those which may be represented by a state-space realization is considered. Finally, the concept of duality for time-varying systems and the so-called time inverse operator is described.

In Chapter 3 a characterization for the induced norm of both discrete-time and hybrid linear time varying system in terms of operator algebraic equations is provided. The sample operator, as well as the hold operator is considered and used in system description. Using these operators the hybrid feedback systems are also described.

In Chapter 4 it is shown that for discrete-time time-varying systems, a natural extension of entropy exists. The definition of entropy for this class of systems is based on the spectral factorization problem considered in Chapter 3. The rest of the Chapter 4 is devoted to deriving properties of this entropy.

Chapter 5 is devoted to connection between the entropy and some related optimal control problems. It is shown that entropy is an upper bound for H2 norm, while maintaining a bound on the $\boldsymbol{H_{00}}$ norm. It follows that the minimum entropy controller allows the designer to trade off between the robystness of the system and its performance. The relationship between the average entropy of a

causal system and the average entropy of its anti-causal adjoint is shown. The connection with risk-sensitive control is also investigated.

The results for the minimum entropy control problem for linear discrete-time time-varying systems are considered in Chapter 6.

Finally, in Chapter 7 the extension of the definition of entropy for linear continuous-time time-varying systems is considered. The definition of entropy given in this chapter has been chosen to preserve most of the properties of discrete-time system entropy, but some inevitable differences also appear.

The minimum entropy control problem for linear discrete-time time-varying systems, described in the book, is based on the use of the infinitely dimensional operators. The compact notation used in the book does not, however, remove the calculation difficulties. In reality some of the problems considered, have not been solved yet at all. For instance, the Riccati operator equations frequently exploited in the book, may be only approximately solved when the system is relatively slowly time-varying. Generally, in the case of linear time-varying systems computing the solution to the Riccati operator equations remains an open problem.

To sum up, the problem of minimum entropy control for linear time-varying systems is both interesting and still open; it is presented in the book from the theoretical point of view. It may be hoped that a further treatment of a more applied nature will follow.

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