Control and Cybernetics

vol. **25** (1996) No. 2

Semi-active dynamic absorber using fuzzy theory for a beam subjected to a moving load

by

J. Hino*, K. Nanki and T. Yoshimura***

* Department of Mechanical Engineering
University of Tokushima, Tokushima, Japan

** Graduate Student
University of Tokushima, Tokushima, Japan

Abstract: This paper presents a semi-active dynamic absorber for controlling the vibration of a beam subjected to a moving load. The characteristics of the beam and moving load system are change with the location of the load. The semi-active dynamic absorber proposed here is of cantilever type and the mass of the absorber can be moved to adjust the eigenvalues. The adjustment of position for the absorber mass is performed by fuzzy reasoning using the displacement of the beam.

1. Introduction

Many researchers have considered controlling vibration of machinery and constructions, see Fujino (1992). Dynamic vibration absorbers are often used to attenuate the responses, see Shadley and Sorem (1992). The passive dynamic absorbers are generally tuned to certain frequency characteristics of systems. The effectiveness of these absorbers deteriorates significantly for the characteristics different from the design values. On the other hand, active vibration control techniques can adapt to the variation of the disturbance and the parameters without losing too much of their intended effectiveness. Hence, active dynamic absorbers may be designed to be significantly more effective than the passive ones. The active control technique is to generate a continuous control force of the same amplitude as the external disturbance and of the opposite direction. However, providing the continuous control force may cause difficulties due to the limitation of energy available for control purposes. To overcome the difficulties semi-active dynamic absorbers have been proposed. The semi-active control means that the characteristic parameters of the dynamic absorber are continuously tuned to the control object.

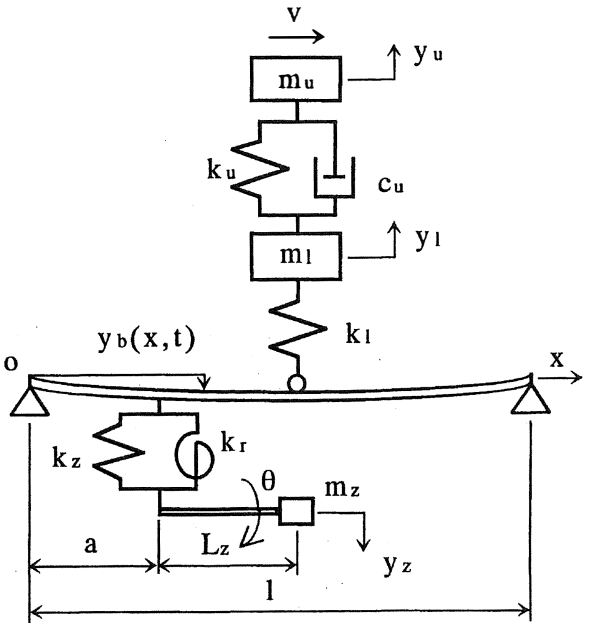


Figure 1. Model of beam, absorber and vehicle.

Eigenvalues of a beam system containing moving loads vary according to the location and number of the loads on the beam. A semi-active dynamic vibration absorber can be considered to attenuate the response of the beam subjected to moving loads. A cantilever type of dynamic vibration absorber is proposed in this paper. The eigenvalue of the rotational mode is adjusted to the first mode of the beam system by moving the absorber mass. The movement of the absorbers mass is determined by fuzzy reasoning see Pedrycz (1993), Ying, Siler, Buckley (1990). Fuzzy control theory has the advantage of robustness for uncertain systems. It is important to decide on control rules in the fuzzy control algorithm. A rule table is produced by the simulation for analytical models. The dynamic responses and the spectra of displacements at the mid-span of the beam are shown in various cases. The effectiveness of the semi-active dynamic absorber is demonstrated experimentally.

2. Equations of motion

Fig.1 shows the configuration of the control system. We considered a simply supported beam, a vehicle and a semi-active dynamic absorber. The equations of motion for this system are derived further in this section.

2.1. Beam

The beam model is based on the Bernoulli-Euler hypothesis.

$$\rho A\frac{\partial^2 y_b}{\partial t^2} + EI\frac{\partial^2 y_b}{\partial x^4} = \{(m_u + m_l)g$$

$$-m_u\ddot{y}_u - m_l\ddot{y}_l\}\delta(x - vt) - m_z\ddot{y}_z\delta(x - a) \tag{1}$$

where x is the axial co-ordinate from the origin, t is time, y_b is the transverse deflection, ρ is the density, E is Young's modulus, A is the cross-sectional area, I is the moment of inertia of area, m_u and m_l are the sprung and unsprung masses of the vehicle, m_z is the absorber mass, y_u and y_l are the deflections of the vehicle, y_z is the translational deflection of the dynamic absorber, v is the forward velocity of the vehicle, $\delta(\cdot)$ is Dirac's delta function, a is the distance to the dynamic absorber from the origin and the dots denote differentiation with respect to time.

2.2. Vehicle

$$m_u\ddot{y}_u + c_u(\dot{y}_u - \dot{y}_l) + k_u(y_u - y_l) = 0 \tag{2}$$

$$m_l\ddot{y}_l + c_u(\dot{y}_l - \dot{y}_u) + k_u(y_l - y_u) + k_l(y_l - y_b) = 0 \tag{3}$$

where k_u and k_l are the spring constants, c_u is the damping coefficient and y_b is the deflection of beam under the moving load.

2.3. Dynamic absorber

$$m_z\ddot{y}_z + k_z(y_z - y_{bz} - L_z\theta) = 0 \tag{4}$$

$$J\ddot{\theta} + k_r(\theta - \frac{\partial y_{bz}}{\partial x}) - k_zL_z(y_z - y_{bz} - L_z\theta) = 0 \tag{5}$$

where θ is the angular deflection, J is the moment of inertia at the center of gravity, L_z is the distance from the attached position to the center of gravity, k_z and k_r are respectively the translation and angular spring constants and y_{bz} is the deflection of the beam at the attached position of the dynamic absorber.

The differential equation of motion for the beam is formulated by Galerkin's method, see Finlayson (1972), Yoshimura (1986). The obtained discrete form equations are combined with the equations of motion for the vehicle and the dynamic absorber. A vector differential equation for the beam-vehicle-absorber system is then written as

$$[M]\{\ddot{y}\} + [C]\{\dot{y}\} + [K]\{y\} = \{f_u\} \tag{6}$$

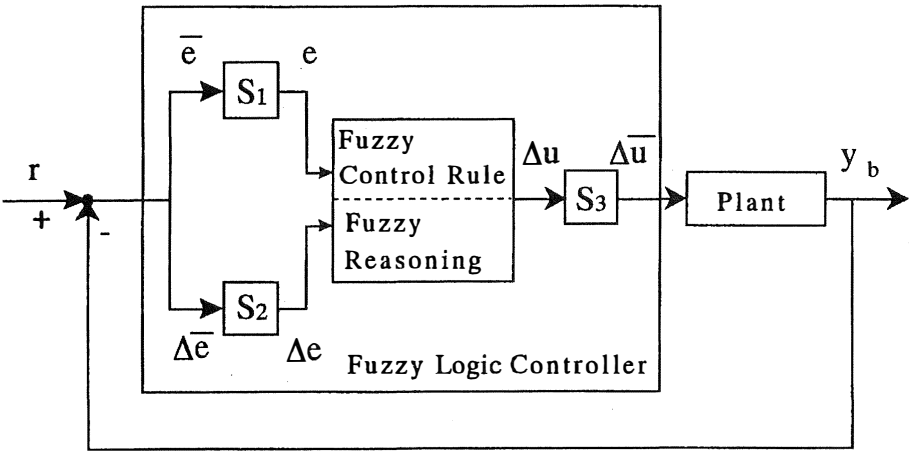


Figure 2. Fuzzy control system.

We then add a trivial equation ($[M]\{\dot{y}\}-[M]\{\dot{y}\} = \{0\}$). These can be combined to form a set of eqautions

$$\begin{Bmatrix} \dot{y} \\ \ddot{y} \end{Bmatrix} = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \begin{Bmatrix} y \\ \dot{y} \end{Bmatrix} + \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix} \{f_u\} \tag{7}$$

where $[I]$ denotes the identity matrix. Eq.(7) is simplified to

$$\{\dot{Y}\} = [A]\{Y\} + \{b\} \tag{8}$$

The deflections $\{Y\}$ can be calculated by numerical integration.

3. Fuzzy control

The fuzzy control rules used herein are presented as an if-then formula:

if e is E_i and Δe is DE_i then Δu is U_i

where e is a input variable, Δe is the change of e, Δu is an increment of the controlled variable, E_i and DE_i are premise rules and U_i denotes the conclusion part. Fig.2 shows the fuzzy control system, where $\bar{e}$ denotes the deflection at mid-span of the beam, $\Delta\bar{e}$ is the increment of $\bar{e}$, $\Delta\bar{u}$ is the length δL_z in the dynamic absorber, r is a reference input, and S_1, S_2 and S_3 are scaling factors. Membership functions of these fuzzy variables are shown in Fig.3. Table 1 is the rule table which is referred to in order to decrease vibrations of the beam system.

		Δe						
		NB	NM	NS	ZO	PS	PM	PB
e	P1	P1	P2	P2	P2	P2	P3	P4
	P2	P2	P2	P3	P3	P3	P4	P5
	P3	P3	P3	P4	P4	P4	P5	P5
	P4	P4	P4	P5	P5	P5	P6	P6
	P5	P5	P5	P6	P6	P6	P7	P7
	P6	P5	P6	P7	P7	P7	P8	P8
	P7	P6	P7	P8	P8	P8	P8	P9

NB: Negative Big NM: Negative Medium NS: Negative Small
PB: Positive Big PM: Positive Medium PS: Positive Small
ZO: Zero

P1: Position 1 P2: Position 2 P3: Position 3 P4: Position 4
P5: Position 5 P6: Position 6 P7: Position 7 P8: Position 8
P9: Position 9 ($P1 < P2 < \cdots < P8 < P9$)

Table 1. Rule Table

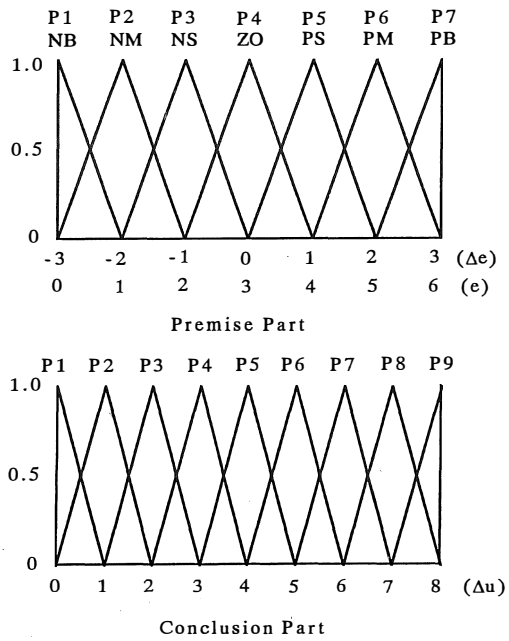


Figure 3. Membership functions

3.1. Fuzzy control rule

The fuzzy control rules are constructed from the empirical knowledge. Here, the dynamic characteristic of the absorber will be changed from the deflections at mid-span of the beam and their increments. The beam and moving load constitute a time-varying system, and the eigenvalues of the whole system depend on the load. The dynamic characteristic of the absorber depend on δLz. Therefore, δL_z can be obtained to tune the absorber to the beam and moving load system. $L_z = L_{z0} + \delta L_z$, where L_z is the length between the attached position and the center of gravity of the dynamic absorber, and L_{z0} is the initial length. It should be noted that the values of u are always positive.

(a) if e is big and Δe is positive, then Δu is very big.
if e is small and Δe is negative, then Δu is very small.
(b) if e is big and Δe is negative, then Δu is big.
if e is small and Δe is positive, then Δu is small.

Using (a) and (b), various control rules are derived.

3.2. Fuzzy reasoning

Fuzzy reasoning can be carried out by using the min-max-center of gravity method proposed by Mamdani. The fuzzy inputs e_n and Δe_n are measured at sampling each interval, where the time step is indicated by the subscript n. The controlled variables are calculated by the rule table based on fuzzy theory.

$$\mu_i(\Delta u_j) = (\mu_{E_i}(e_n) \wedge \mu_{DE_i}(\Delta e_n)) \wedge \mu_{U_i}(\Delta u_j) \tag{9}$$

$$(i = 1, ..., 49 \quad j = 1, 2, ..., 9)$$

where i is the number of control laws in the rule table (7×7 rules) and $\wedge$ is the min composition. μ_{E_i}, μ_{DE_i} and μ_{U_i} denote the grade of E_i, DE_i and U_i, respectively. The results of reasoning can be obtained by

$$\mu_M(\Delta u_j) = \vee_{i=1}^{49}\{\mu_i(\Delta u_j)\} \quad (j = 1, 2, ..., 9) \tag{10}$$

where $\vee$ is the max composition. Since the reasoning results are fuzzy variables, they must be translated into practical controlled values. Therefore, the following formula (weighted average method) is used for fuzzy decision making.

$$\Delta u = \frac{\sum_{i=1}^{9} \Delta u_i \cdot \mu_M(\Delta u_i)}{\sum_{i=1}^{9} \mu_M(\Delta u_i)} \tag{11}$$

3.3. Scaling factors

When the measured values are first converted to fuzzy variables, they must be scaled by suitable factors for the purpose of fuzzification. The inputs are translated into linguistic fuzzy subsets. Value ranges for e_n and Δe_n are respectively, $(0 \leq e_n \leq 6)$ and $(-3 \leq \Delta e_n \leq 3)$. When the values measured at time n are given by $\bar{e}_n = y_{b_n}$ and $\Delta\bar{e}_n = \bar{e}_n - \bar{e}_{n-1}$, the fuzzification can be written as follows:

$$e = 6(\frac{\bar{e}_n}{\bar{e}_{max}}) \tag{12}$$

$$\Delta e = 3(\frac{\Delta\bar{e}_n}{\Delta\bar{e}_{max}}) \tag{13}$$

where $\bar{e}_{max}$ and $\Delta\bar{e}_{max}$ are the scaling factors. When $|\bar{e}_n| > \bar{e}_{max}$ and $|\Delta\bar{e}_n| > \Delta\bar{e}_{max}$,

$$e = 6(\frac{\bar{e}_n}{|\bar{e}_n|}) \tag{14}$$

$$\Delta e = 3(\frac{\Delta\bar{e}_n}{|\Delta\bar{e}_n|}) \tag{15}$$

The change in controller output can be determined by the fuzzy controller. Then, the results must be converted to actual controlled values by

$$\Delta\bar{u} = \frac{\Delta u \Delta\bar{u}_{max}}{8} + L_{z0} \tag{16}$$

where $\Delta\bar{u}_{max}$ is also a scaling factor.

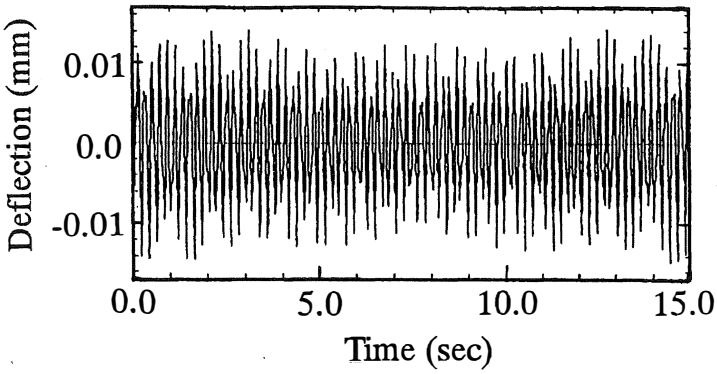
4. Numerical examples

Table 2 shows the values of the parameters used in this paper. The deflections at mid-span of the beam ($\bar{e}_n$) and their change ($\Delta\bar{e}$) are chosen as the inputs. The length between the of attachement position and the center of gravity in the dynamic vibration absorber ($\Delta\bar{u}$) is calculated by the fuzzy controller. The rotation mode in the absorber is initially adjusted to the first mode in the beam without moving loads. The Crank Nicolson method – Yoshimura (1986) – is used to integrate (8). The time step is taken to be 0.0025 sec.

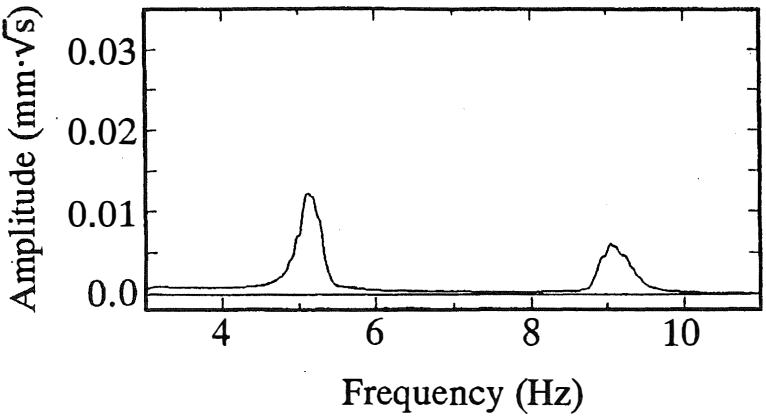
The time history and the power spectrum of the beam at mid-span without control are shown in Figs.4 (a) and (b), where the dynamic absorber is regarded as a passive dynamic absorber. Here, one moving load (moving load 1) runs on the beam, with a velocity of 0.06 m/s. The time history shown in Fig.4 (a) denotes the dynamic deflection in which the static deflection is subtracted from the transient response. The power spectrum for the dynamic deflection is shown in Fig.4 (b). The time history and the power spectra of the beam at

	Beam	
l	Length	: $0.9(m)$
EI	Flexural stiffness	: $69.525(N \cdot m^2)$
ρ	Density	: $7.84 \times 10^3(kg/m^3)$
A	Sectional area	: $4.5 \times 10^{-4}(m^2)$
	Moving load 1	
m_{u1}	Sprung mass	: $0.87(kg)$
m_{l1}	Unsprung mass	: $0.48(kg)$
k_{u1}	Secondary spring	: $549.5(N/m)$
k_{l1}	Primary spring	: $2.0 \times 10^6(N/m)$
c_{u1}	Damping coefficient	: $0.01(N \cdot s/m)$
	Moving load 2	
m_{u2}	Sprung mass	: $0.625(kg)$
m_{l2}	Unsprung mass	: $0.50(kg)$
k_{u2}	Secondary spring	: $370.5(N/m)$
k_{l2}	Primary spring	: $2.0 \times 10^6(N/m)$
c_{u2}	Damping coefficient	: $0.01(N \cdot s/m)$
	Dynamic absorber	
m_z	Absorber mass	: $1.39(kg)$
k_z	Translation spring	: $9.4 \times 10^5(N/m)$
k_r	Angular spring	: $74.5(N \cdot m)$
a	Mounted location	: 0.6 (m)

Table 2. Parameters of the system



(a) Time history



(b) Power spectrum

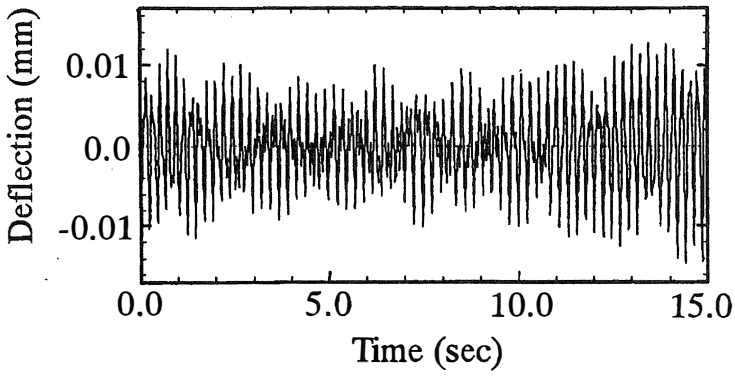
Figure 4. Deflections without control

mid-span with control are shown in Figs.5 (a) and (b), respectively. The fuzzy reasoning is carried out at intervals of 1.0 sec. The scaling factors in Table 3 may be decided heuristically. It is evident from Figs.4 and 5 that the vibrations are attenuated by the semi-active dynamic absorber. The effectiveness of the absorber has been verified in the calculations.

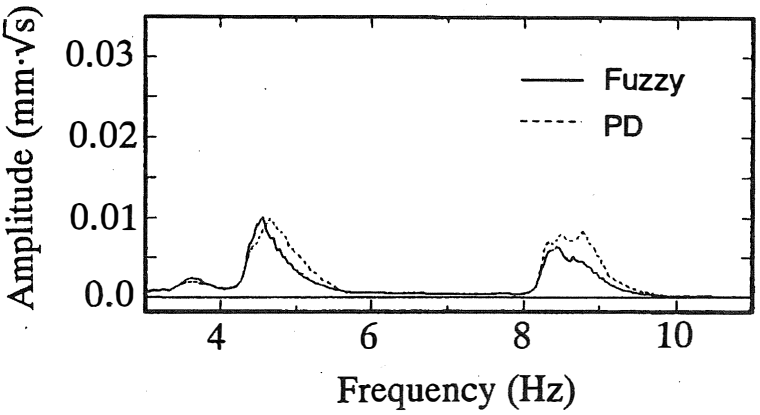
The result of power spectrum for PD control, Douglas (1972), is also represented in Fig.5 (b). The results show that fuzzy control is superior to PD control. The controlled variable in the PD control is defined by follows:

$$L_z = L_{z0} + \delta L_z \tag{17}$$

where $\delta L_z = K_c(1 + T_d \frac{d}{dt})y_b$, $K_c = 20$ and $T_d = 5$. The K_c and T_d denote gain and D-action time constant, respectively.



(a) Time history



(b) Power spectra

Figure 5. Deflections with control

Scaling factors	
$\bar{e}_{max}$	: $3.8 \times 10^{-3}(m)$
$\Delta\bar{e}_{max}$	: $1.20 \times 10^{-3}(m)$
$\Delta\bar{u}_{max}$	: 0.072
Initial length	
L_{z0}	: $0.12[m]$
Intervals for control	
$\Delta\bar{t}$	: $1.0[sec.]$

Table 3. Parameters of the controller

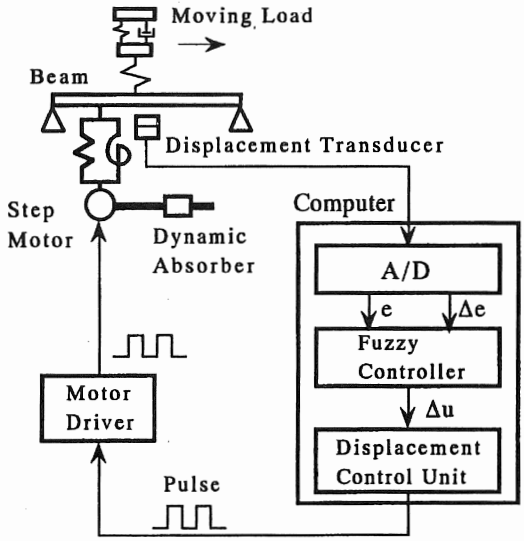


Figure 6. Experimental scheme

5. Experiment

5.1. Experimental procedure

The experimental scheme is shown in Fig.6. The values of parameters used in this experiment have already been presented in Table 2. The transient responses at mid-span of a beam subjected to the passage of vehicles are measured by using a non-contact displacement transducer. The controlled values are determined by the fuzzy logic controller. A stepping motor drives the dynamic absorber mass to the optimal position according to the results of fuzzy reasoning. The dynamic vibration absorber is assumed to be of a cantilever type, in which the natural frequency of the rotational mode can be adjusted to the first mode of the beam system. Fig.7 shows the natural frequency of the rotational mode depends on the position of the absorber mass. The initial position of the mass was decided so as to decrease the free vibration of the beam's fundamental mode (8.61Hz). Two vehicle models were made and moved along on beam in this experiment. The moving load 1 and 2 have the first natural frequencies of 3.99Hz and 3.87Hz respectively. The scaling factors from Table 3 are also employed for the experiments.

5.2. Results and discussion

Time histories and spectra of dynamic deflections of the beam at mid-span are shown in Figs.8 (a)-(d). These results were obtained for one load moving with velocity of 0.06 m/s. The dynamic deflections without and with control are

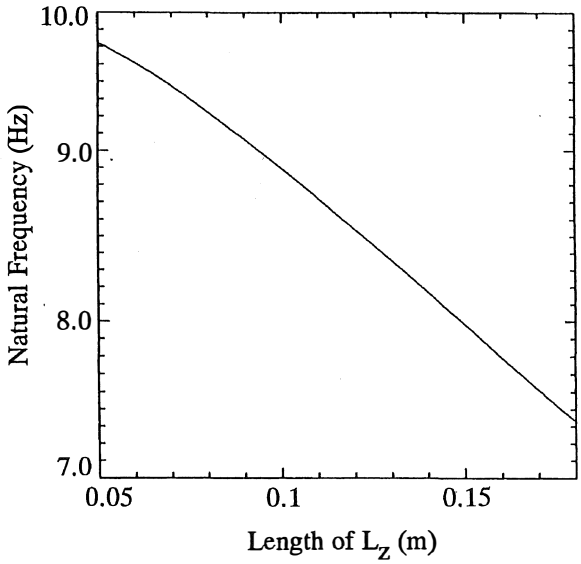


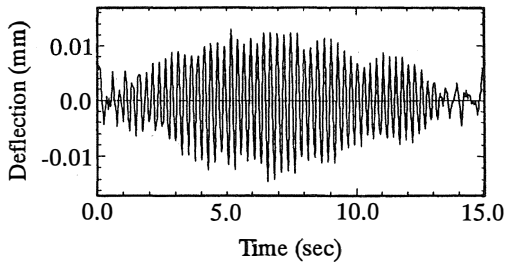
Figure 7. Natural frequency of absorber

shown in Figs.8 (a) and (b), respectively. Their power spectra are shown in Figs.8 (c) and (d). Since the change of the characteristics of the beam system depended on the moving load, updates of the position of the absorber mass were performed at intervals of 1.0 sec. The moving load approached the beam from a runway in this experiment. Thus, it had vertical motion at the entrance to the beam. In Figs.8 (c) and (d), the peak value about 3.9Hz denotes significant effects of the load, and the vibrations of about 7-9Hz were derived from the beam and the absorber. It can be seen that the amplitudes of dynamic deflections with control are smaller than those without.

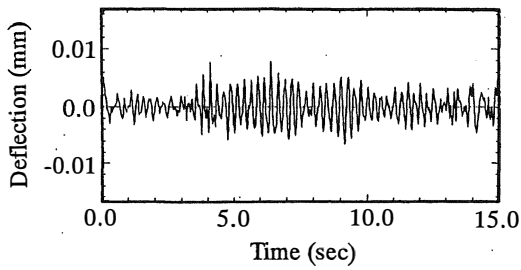
Figs.9 (a)-(d) show the results for two moving loads. The moving load 1 ran to the beam first, and then the moving load 2 followed at a distance of 0.32m. It is obvious on inspection that the dynamic deflections were decreased by using the semi-active dynamic absorber. It can be found that the beam vibrations with fuzzy control were reduced in these figures. Apparently, the effect of the semi-active dynamic absorber with fuzzy control is recognized in the nonstationary vibrations as compared to the passive dynamic absorber. This is because the inertial term was tuned to characteristics of the beam system.

6. Conclusion

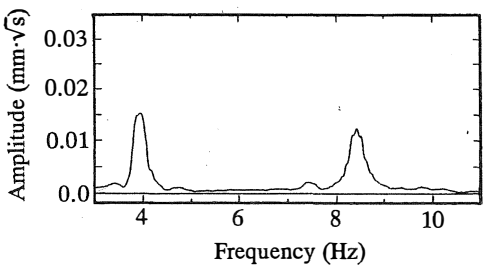
The cantilever type semi-active dynamic absorber was considered to reduce responses of a beam subjected to moving loads. Eigenvalues of the beam system containing moving loads vary according to location of the loads on the beam and the number. The characteristics of the absorber were tuned to the first mode



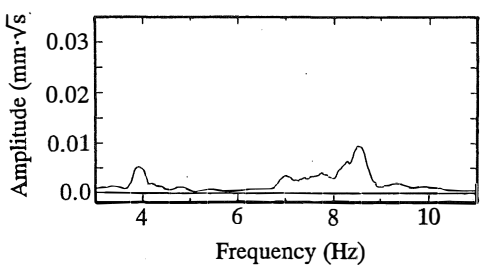
(a) Dynamic deflection without control



(b) Dynamic deflection with control

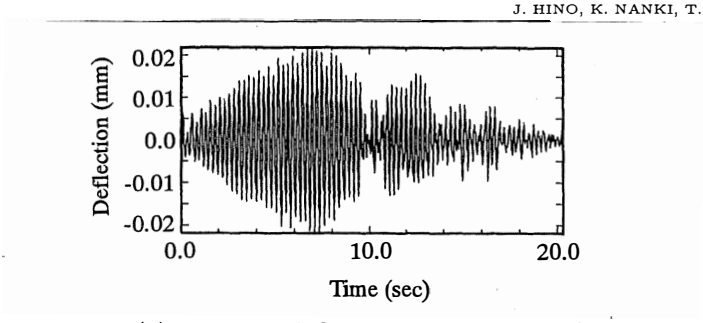


(c) Power spectrum without control

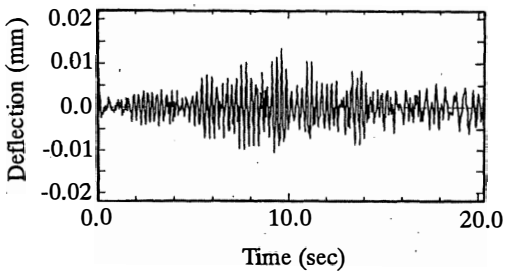


(d) Power spectrum with control

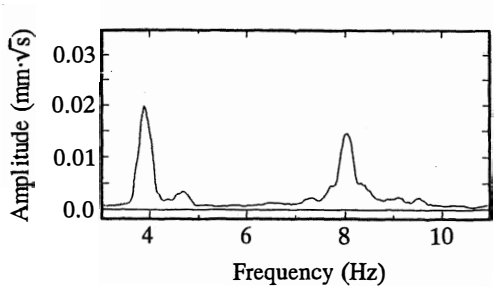
Figure 8. Dynamic deflections (one moving load, $v = 0.06$m/s)



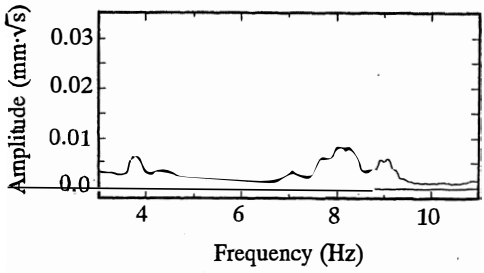
(a) Dynamic deflection without control



(b) Dynamic deflection with control



(c) Power spectrum without control



(d) Power spectrum with control

Figure 9. Dynamic deflections (two moving loads, $v = 0.06$m/s)

of the beam system by fuzzy reasoning. The rule table and the scaling factors in the fuzzy control were decided by numerical calculations of the analytical model. Various cases of dynamic deflections of the beam were shown and, the effectiveness of the semi-active dynamic absorber with fuzzy control was verified experimentally.

References

ABDELNOUR, G.M., et al. (1991) Design of a Fuzzy Controller Using Input and Output Mapping Factors. *IEEE Trans. on Systems, Man and Cybernetics*, **21**, 5, 952.

DOUGLAS, J.M. (1972) *Process Dynamics and Control, Vol.2, Control System Synthesis.* Prentice-Hall.

FINLAYSON, B.A. (1972) *The Method of Weighted Residuals and Variational Principles.* Academic Press.

FUJINO, Y. (1992) On Active Control of Public Structures. *J. Society of Instrument and Control Egnineers* (in Japanese), **31**, 4, 473.

IWAI, Z., et al. (1992) Experimental Considerations of an Active Dynamic Vibration Absorbing Control System Using Fuzzy Control. *Trans. of JSME* (in Japanese), **58**, 546, C, 401.

MANIKANAHALLY, D.N., CROCKER, M.J. (1991) Vibration Absorbers for Hysteretically Damped Mass-Loaded Beams. *ASME J. Vibration and Acoustics*, 113, p.116.

MATSUHISA, H., et al. (1988) Cantilever-Type Dynamic Vibration Absorbers Considering the Effects of Force and Moment. *Trans. of JSME* (in Japanese), **54**, 508, C, 2935.

PEDRYCZ, W. (1993) *Fuzzy Control and Fuzzy Systems.* John Wiley & Sons.

SETO, K., et al. (1991) A Study on Active Dynamic Absorbers. *Trans. of JSME* (in Japanese), **57**, 538, C, 1834.

SHADLEY, J.R., SOREM, J.R, JR. (1992) Damped Absorber Optimization and Stability for Semi-Definite Systems Exhibiting Unstable Self-Excitation During Start-Up. *ASME J. Vibration and Acoustics*, 114, 47.

SHINGU, K., FUNAMOTO, D. (1992) Fuzzy Control of Multi-Degree of Freedom System Structures. *J. Japan Society for Fuzzy Theory and Systems* (in Japanese), **4**, 4, 742.

WANG, B.P., et al. (1985) Synthesis of Dynamic Vibration Absorbers. *ASME J. Vibration Acoustics, Stress, and Reliability in Design*, 107, 161.

YING, H., SILER, W. BUCKLEY, J.J. (1990) Fuzzy Control Theory: A Nonlinear Case. *Automatica*, **26**, 3, 513.

YOSHIMURA, T., et al. (1986) Vibration Analysis of a Non-linear Beam Subjected to Moving Loads by Using the Galerkin Method. *J. Sound and Vibration*, **104**, 2, 1986, 179.