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Swing–up control of double pendulum using genetic algorithms

by

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Abstract: Genetic Algorithm (GA) is a random search algorithm which is modeled after the natural evolution and selection. In the algorithm, the search process progresses with evaluating only the attainment of the objective of multiple candidate solutions, so a priori knowledge about the objective is not needed, and it is expected that the process is prevented from terminating by falling into the local optimum points. From these advantages, GA is powerful and usable for the nonlinear optimization problems.

In this paper, in order to confirm the feasibility of GA for solving nonlinear dynamic control problems, we use a swing-up control problem of double-pendulum system. In the system, the cart can move along a rail symmetrically with respect to its initial position and the pendulum can rotate clockwise or counterclockwise. So, the problem is an ill-conditioned problem and it is hard to solve analytically. Because of those features, it is a good example for checking the feasibility of GA for solving the nonlinear dynamic control problems. Finally, some simulation results are given. From the results, it is found that GA has a good ability of searching for the optimum solution of the nonlinear dynamic control problem.

Keywords: genetic algorithms, swing-up control, double pendulum, nonlinear control

1. Introduction

Many mathematical techniques for solving the dynamic control problems of linear system have been proposed. But the practical systems have strong nonlinearity and uncertainty, and so those techniques have less ability for solving the practical control problems. In order to deal with the nonlinearity and the uncertainty of the system, new techniques have been discussed. For example, in order to deal with the nonlinearity of the system without approximation, the nonlinear control techniques based on the differential geometry were proposed, see Ishijima (1993), and in order to deal with the uncertainty of the system, the robust control techniques, e.g. H_∞ control, μ-synthesis were proposed, see Dorato and Yedavalli (1991). Whereas, intelligent control such as provided by fuzzy, neural networks, is aimed to introduce the human's intelligence into the design of control system, and is widely applicable for the practical control problems. In this case, it is the key issue how to express the human knowledge as accurately as possible. In the design of the fuzzy controller Kawaji (1991), the control strategy is mainly represented by fuzzy rules and the membership functions and it is necessary to understand the properties of the plant a priori and deeply. Neural networks have the similar difficulties in the design of the teaching signals in the networks where the derivative of the performance index is further needed, Kawaji (1992).

Recently, genetic algorithms (GAs) have acquired major interest as an alternative optimization algorithm. GA is one of the random search algorithms which are modeled after the process of evolution and the natural selection, see Holland (1975), Goldberg (1989). In GA, the search process is executed at multiple points in the search space concurrently and stochastically, and it only uses the fitness value of current search point which represents the degree of attainment of the control objective. From these features, GA is a powerful and easily usable algorithm for nonlinear optimization problems. Some studies on applications of GA to the control problems have been developed, see e.g. Ichikawa (1992), Unemi (1993). The control problems in general are regarded as optimization problems for the operation sequence of the system to satisfy the control objective, and GA may be used for solving the problem.

This paper aims to study the feasibility of GA for solving nonlinear dynamic control problems. As an application example, the swing-up control problem of double inverted pendulum is discussed. The purpose of the swing-up control is to drive double pendulum from the pendant position to the upright position by moving a cart appropriately, Furuta (1991), Xia (1992). But, in the system, the cart can move symmetrically with respect to its initial position, and the pendulum can rotate clockwise or counterclockwise, and so the problem is ill-conditioned and very difficult to be solved analytically. But some methods have been proposed for this problem, Furuta (1991;1993), Xia (1992), Kawaji (1994).

The paper is organized as follows. In section 2, the swing-up control problem of the double pendulum is formulated. In section 3, GAs are briefly described, and the controller for the swing-up motion is designed by using GA. In section 4, the simulation results are shown, and conclusions are presented in section 5.

2. Problem statement

The double-pendulum system is shown in Fig.1. In the system, the first pendulum is joined to the cart and the second pendulum is joined to the first pendulum, respectively. The cart is driven by DC motor and it moves along a

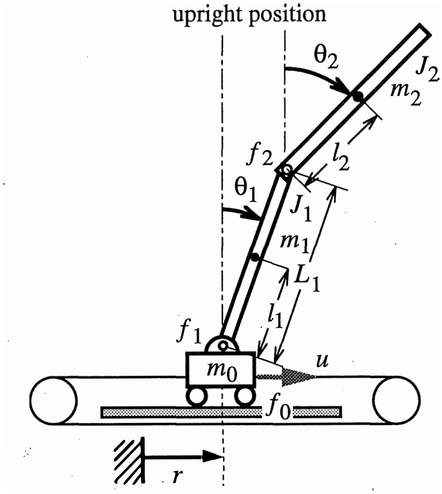

Fig.1. Double pendulum system

rail so as to make the pendulum swing.

The motion equations of the double pendulum system are derived by the Lagrange method, and are given as

$$\begin{aligned}
&a_{11}\ddot{r} + a_{12}\ddot{\theta}_1 \cos\theta_1 + a_{13}\ddot{\theta}_2 \cos\theta_2 \\
&\quad -a_{14}\dot{\theta}_1^2 \sin\theta_1 - a_{15}\dot{\theta}_2^2 \sin\theta_2 + a_{16}\dot{r} = u \\
&a_{21}\ddot{r}\cos\theta_1 + a_{22}\ddot{\theta}_1 + a_{23}\ddot{\theta}_2\cos\theta_2 + a_{24}\dot{\theta}_1 \\
&\quad +a_{25}\dot{\theta}_2^2 \sin\theta_{12} - a_{26}\dot{\theta}_2 - a_{27} g \sin\theta_1 = 0 \\
&a_{31}\ddot{r}\cos\theta_2 + a_{32}\ddot{\theta}_1\cos\theta_{12} + a_{33}\ddot{\theta}_2 \\
&\quad -a_{34}\dot{\theta}_{12} - a_{35}\dot{\theta}_1^2\sin\theta_{12} - a_{36} g \sin\theta_2 = 0
\end{aligned} \tag{1}$$

where

$$\begin{array}{lll}
a_{11} = m_0 + m_1 + m_2, & a_{12} = m_1 l_1 + m_2 L_1, & a_{13} = m_2 l_2, \\
a_{14} = m_1 l_1 + m_2 L_1, & a_{15} = a_{13}, & a_{16} = f_0, \\
a_{21} = a_{12}, & a_{22} = m_1 {l_1}^2 + m_2 {L_1}^2 + J_1, & a_{23} = m_2 l_2 L_1, \\
a_{24} = f_1 + f_2, & a_{25} = a_{23}, & a_{26} = f_2, \\
a_{27} = a_{14}, & a_{31} = a_{13}, & a_{32} = a_{23}, \\
a_{33} = m_2 {l_2}^2 + J_2, & a_{34} = a_{26}, & a_{35} = a_{23}, \\
a_{36} = a_{23}, & &
\end{array}$$

where r is the position of the cart, θ_i $(i = 1, 2)$ is the angle of the pendulum, which is measured from the upright position clockwise. Then, θ_{12} is $\theta_1 - \theta_2$, u is

Cart	m_0	equivalent mass	5.92	[kg]
	f_0	equivalent viscous friction coefficient	21.5	[kg/s]
First Pendulum	m_1	mass	0.17	[kg]
	f_1	viscous friction coefficient	2.26×10^{-3}	[kgm^2/s]
	l_1	length between the rotation axis and the center of gravity	0.235	[m]
	L_1	total length	0.323	[m]
	J_1	moment of inertia about the center of gravity	2.59×10^{-3}	[kgm^2]
Second Pendulum	m_2	mass	0.041	[kg]
	f_2	viscous friction coefficient	1.0×10^{-3}	[kgm^2/s]
	l_2	length between the axis and the center of gravity	0.118	[m]
	J_2	moment of inertia about the center of gravity	4.45×10^{-3}	[kgm^2]

Table 1. Parameters of the double pendulum system

the control input and g is the gravitational acceleration. The parameters used in (1) are shown in Table 1.

The purpose of the swing-up control is to drive the pendulum from the pendant position to the upright position by moving the cart appropriately. The swing-up control is a typical nonlinear control problem because the system dynamics is represented by highly nonlinear differential equations (1). Furthermore, the pendulum can rotate clockwise and counterclockwise, and the cart can move symmetrically with respect to its initial position, so there are many kinds of swing-up motions. For these reasons, the swing-up control problem is a very difficult problem to solve analytically.

By the way, although the motion equations are strongly nonlinear, the stabilization problem that the pendulum be controlled so as to keep its position in the neighborhood of the upright position can be solved by the linear control method, Furuta (1994). So, we do not consider the stabilization control of the pendulum in this paper.

3. Design of the swing-up controller using GA

3.1. Genetic Algorithms (GAs)

In this subsection, genetic algorithms are briefly discussed. An excellent reference on GA's and their implementation is the book by Goldberg (1989). GA is a search technique that emulates natural genetic evolution process. The search process of GA is executed in a population of some search points, which are called individuals, so that the members of the group are renewed stochastically. For the stochastic renewal of individuals, genetic operators such as selection, crossover and mutation are used. Those operators are explained in subsection 3.4. The probability with which an individual contributes to generate a new individual is determined according to the fitness value. The fitness value is assigned to each individual by the fitness function which evaluates the degree of attainment of the control objective. The search process of GA is summarized as follows:

1. The individuals are generated randomly, and the initial population is composed of those individuals.
2. Each individual in the population is evaluated by the fitness function, and assigned the fitness value.
3. By applying the selection to the population, some recessive individuals are eliminated from the population based on its fitness value, and temporal population called mating pool is formed from the survived individuals.
4. By applying the crossover and mutation to the mating pool, new individuals are created, and new population is formed from the new individuals.

A cycle which consists of step 2. through 4. is called generation. The population evolves stochastically from generation to generation by applied the genetic operators of selection, crossover and mutation. Fig.2 shows the flowchart of the search process of GA after Koza (1994).

Since the search process proceeds concurrently and stochastically at multiple points in the search space, it is expected that the search process is prevented from terminating in falling into local optimum points. Furthermore, in the search process of GA, it is only needed to evaluate to what extent the current search point attains the objective. Therefore, the deep priori knowledge about the plant is not needed.

3.2. Formulation of the optimization problem

In order to apply GA to the swing-up control problem, the latter should be transformed into an optimization problem. In this paper, we shall use a feedforward controller whose output is of bang-bang with zero type as shown in Fig.3 to control the cart so that the swing-up motion may be realized, Furuta (1993). Then, the swing-up problem can be transformed into an optimization problem for a sequence of bang-bang with zero signals to realize the swing-up motion. In this framework, the feedforward controller just reproduces the optimum sequence which is obtained by GA, so the ability of the controller depends on the search ability of GA for the solutions of the swing-up control problem.

In GA, each individual is characterized by a sequence of genes, which is called a chromosome. The gene is the fundamental element which determines the character of the individuals. In order to code a sequence of the bang-bang with zero inputs to a chromosome, three types of genes shown in Table 2 are specified. Then, the input sequence to the system is coded into the string whose elements are 0, 1 or 2.

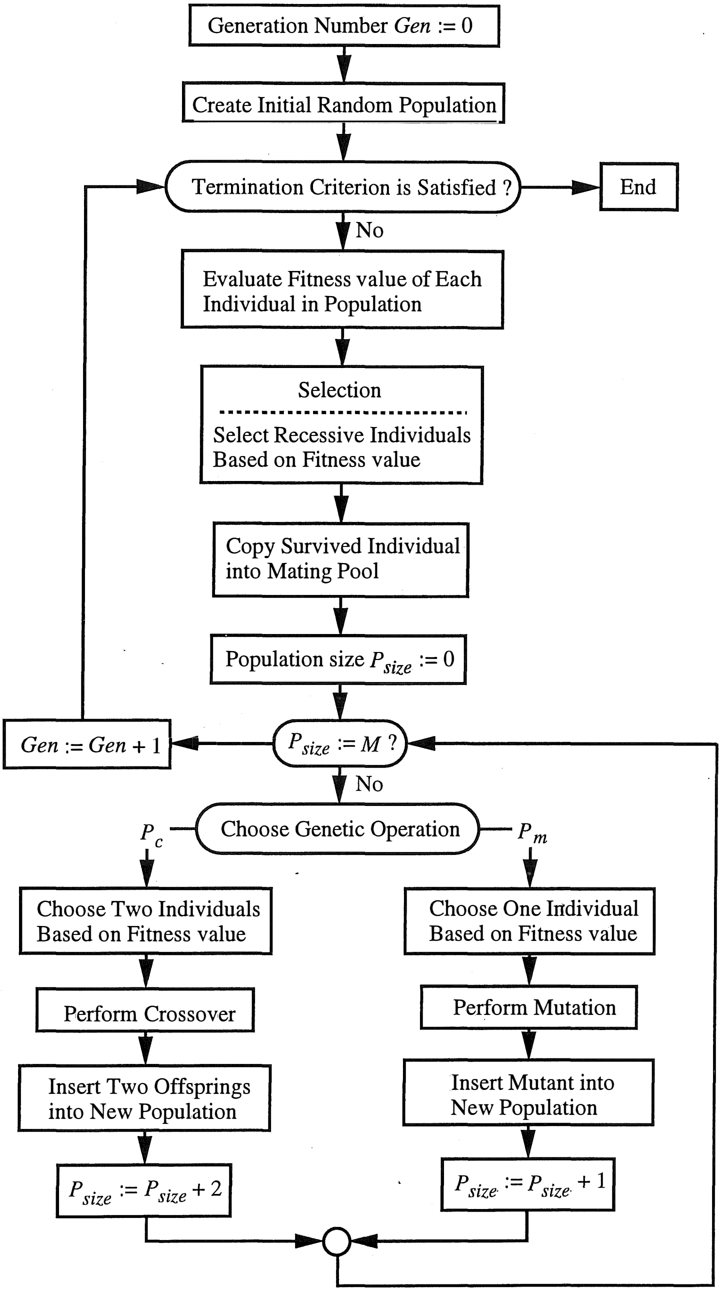

Fig.2. Search process of genetic algorithms

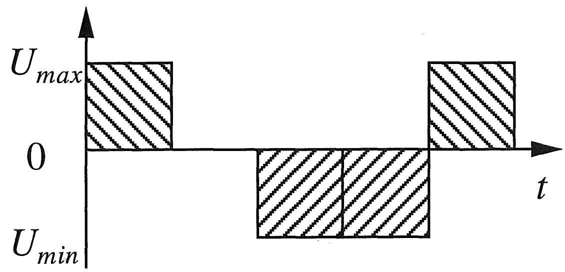

Fig.3. Control input to swing-up the pendulum

Type	body	property
Type 1	0	This gene represents 0 input with step width T_{bang} [sec].
Type 2	1	This gene represents U_{max} input with step width T_{bang} [sec].
Type 3	2	This gene represents U_{min} input with step width T_{bang} [sec].

Table 2. Three types of gene

3.3. Design of the fitness function

The fitness value represents the degree of attainment of the control objective, and is assigned to the individual by the fitness function. The search process of GA proceeds using only the fitness vale of each individual, so it is important to design the proper fitness function for finding the optimum solution efficiently. From the objective of the swing-up control problem, the fitness function should be designed so that larger fitness value is assigned to the individual which moves the double pendulum more close to the upright position without violating the constraints such as

- The angular velocity of the pendulum at the end of the control period T_c is zero,
- The translation velocity of the cart at T_c is zero,

where T_c is the total control length of time and is specified by the designer, but it may have effect on the solution to the problem. Furthermore, the following constraints are added for checking the search ability of GA:

- The final position of the cart at T_c is same as its initial position,
- The displacement of the cart is within the region of ± 0.35 [m].

Based of these specifications, the fitness function is designed as follows, Kawaji (1994;1995).

$$F(gene_i, \mathbf{x}) = F_1(\mathbf{x}) + F_2(\mathbf{x}) \tag{2}$$

$$
\begin{aligned}
: F_1(\mathbf{x}) &= K_1(1+\cos\theta_1(T_c)) + K_2(1+\cos\theta_2(T_c)) - K_3|\theta_1(T_c)-\theta_2(T_c)| \\
&\quad -K_4|\theta_1(T_c)| - K_5|\dot{\theta}_1(T_c)| - K_6|\theta_2(T_c)| - K_7|\dot{\theta}_2(T_c)| \\
&\quad -K_8|r(T_c)| - K_9|\dot{r}(T_c)| \\
F_2(\mathbf{x}) &= F_{21}(|r|) + F_{22}(|\theta_1(T_c)|, |\theta_2(T_c)|) \\
F_{21}(|r|) &= \begin{cases} K_{10} & |r_{max}| \le r_\varepsilon[\mathrm{m}] \\ 0 & |r_{max}| > r_\varepsilon[\mathrm{m}] \end{cases} \\
F_{22}(|\theta_1(T_c)|, |\theta_2(T_c)|) &= \begin{cases} K_{11} & |\theta_1(T_c)|, |\theta_2(T_c)| \le \theta_\varepsilon[\mathrm{rad}] \\ 0 & |\theta_1(T_c)|, |\theta_2(T_c)| > \theta_\varepsilon[\mathrm{rad}] \end{cases}
\end{aligned}
$$

where K_i (i=1,2,$\cdots$,11), r_ε, and θ_ε are constant, r_{max} is the maximum displacement of the cart.

$F_1(\mathbf{x})$ in (2) evaluates the state of the double-pendulum system, in which $K_1(1+\cos\theta_1(T_c))$ and $K_2(1+\cos\theta_2(T_c))$ take the largest value when the pendulum is at the upright position. $-K_3|\theta_1(T_c)-\theta_2(T_c)|$ assigns larger value to the individual which makes double pendulum like a single pendulum. This constraint is added according to the control strategy which human operator uses when he swings up double pendulum. And the other terms indicate the penalty of failure to attain the control objective and of violating the constraints. Thus $F_1(\mathbf{x})$ takes larger value when the individual move the pendulum more close to the upright position without violating the constraints.

$F_{21}(|r|)$ in $F_2(\mathbf{x})$ gives the reward K_{10} to the individual satisfying the constraint that the maximum displacement r_{max} of the cart is within the region of $\pm r_\varepsilon$ [m]. And $F_{22}(|\theta_1(T_c)|, |\theta_2(T_c)|)$ gives the reward K_{11} to the individual moving the pendulum to the neighborhood of the upright position, $|\theta_i| < \theta_\varepsilon$ [rad]. That is, $F_2(\mathbf{x})$ in (2) represents a kind of inverted penalty barrier for searching the feasible solutions more efficiently.

3.4. Genetic operators

In this study a GA in its simplest form is employed, i.e., three kinds of genetic operators of *selection, crossover*, and *mutation* are used.

As for *selection*, the *roulette selection* and the *elite preserving* strategy are used. In roulette selection, each individual is assigned the probability in proportion to its fitness value, and according to the probability, the individual is weeded out or reproduced. That is, the individual with larger fitness value can contribute more to produce offsprings in the next generation. The elite preserving strategy is that the individual with largest fitness value in every generation remains in the next generation without any action of the genetic operators. Surviving individuals after selection form a temporary population called the mating pool for producing the offsprings of the next generation.

By *crossover* operator, a part of the chromosome exchanges between any two individuals in the mating pool with the probability P_c which is called crossover rate, and as a result, two offsprings are produced. In the operation, only one cutting point can be specified so that the length of a cut off parts of each chromosome is same. Therefore, the length of the chromosome of the offsprings are same as of the parents. Usually, this crossover is called *one point crossover.*

Parameters	Value
Total length of control time (T_c)	3 [sec]
Step width of bang-bang with zero input (T_{bang})	100 [msec]
Population size	300
Number of generations	150
Crossover rate (P_c)	0.5
Mutation rate (P_m)	0.1
K_1, K_2, K_{11}	100
K_3, K_4, K_5, K_6, K_7	20
K_8, K_9	10
K_{10}	50
r_ε	0.35 [m]
θ_ε	0.5 [rad]

Table 3. Parameters in simulations

The *mutation* operator is applied to the individuals in the mating pool with the probability P_m which is called mutation rate. In the operation, genes which are arranged at some positions of the chromosome are changed into other genes. For example of our swing-up control problem, a gene arranged at a position of the chromosome is changed into 0, 1, or 2.

4. Simulation results

In this section, we show the simulation results which are obtained by applying the GA-designed swing-up controller to the double-pendulum system of (1). The parameters used in simulations are shown in Table 3. Sampling time is 10 [msec]. The parameters K_1 through K_{11} in (2) are determined experimentally. Let us define the optimized input sequence as the individual with the largest fitness value in the last generation.

Figs. 4 through Fig. 7 show the examples of the swing-up motion. They are realized by using the input sequence calculated by GA from different initial populations. The initial populations are produced randomly, so no underlying structure of the search space is supposed during a search process. Thus, the quality of the obtained optimum solutions varies depending on not only the fitness function, but also the initial population. From the figures, it can be found that the pendulum is controlled so as to move close to the upright position after the several swingings, and various swing-up motions can be realized by GA. This variety is caused by the property of GA that in the search process, only the final state of the system at time T_c is evaluated, so the intermediated state of the system is not restricted at all. This means that the process is able to proceed without using deep knowledge about the control objective. Thus, it is expected that the property of the designed controller is independent of the skills of the designer.

In the simulations, double pendulum is moved into the region $-1.5 \leq \theta_1 \leq 0.7$ [rad], $-1.9 \leq \dot{\theta}_1 \leq 4.8$ [rad/sec], $-0.77 \leq \theta_2 \leq 0.57$ [rad], $-7.8 \leq \dot{\theta}_2 \leq 7.4$ [rad/sec]. This result shows that GA does not necessarily search for an optimum solution with desired quality. But, it is confirmed that without using deep knowledge about the control objective, it is possible to obtain solutions with desired quality by using GA as shown in Figs. 4 through 7. This feature is an advantage for solving the nonlinear dynamic control problem in which control strategies are not found out due to the complexity of the control objective. So, in order to make GA a more useful method for solving the nonlinear dynamic control problems, it is necessary to make some improvement in its search process so that the optimum solution with desired quality may be obtained. This is the subject for a future study, see Kawaji (1995a;b).

5. Conclusion

In this paper, the feasibility of the application of Genetic Algorithms (GAs) for solving the nonlinear dynamic control problems was discussed. The swing-up control of double pendulum was considered as an example. In the computer simulations, the pendulum is moved into the neighborhood of the upright position by using input sequences obtained by GA without referring to a priori knowledge about the motion. Thus, we can say that GA has the ability to solve the nonlinear control problems. But, optimum solutions with desired quality are not necessarily found by GA. Improvement of this defect of GA is the subject of a future study, and authors will propose a new search method, Kawaji (1995a;b).

In the search process of GA, the designer only evaluates to what extent each candidate solution attains the control objective of the problem, and needs no information about the structure of the search space. Because of this property of GA, various swing-up motions were realized in the simulations. From these results, it can be expected that the properties of the controller designed by GA are independent of the designer's skill, and that GA is a useful method to design a controller for the plant whose qualitative characteristics are unknown.

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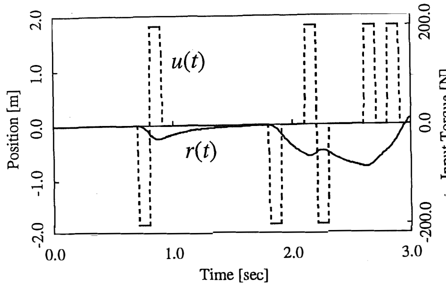
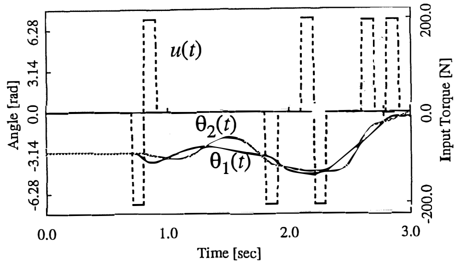
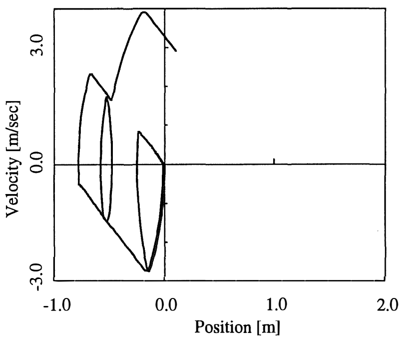


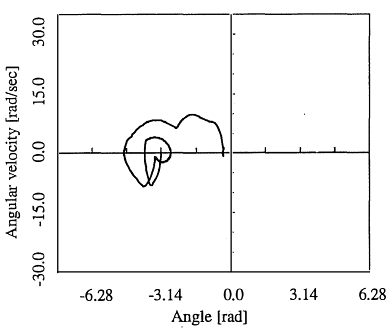
(a) Response of the cart



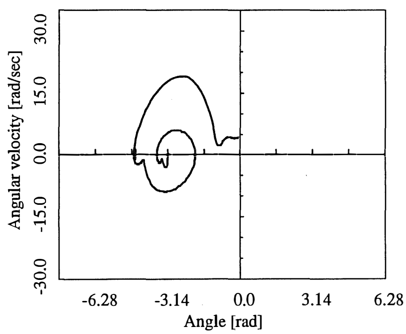
(b) Responses of the pendulum



(c) Response of the cart in the phase plane

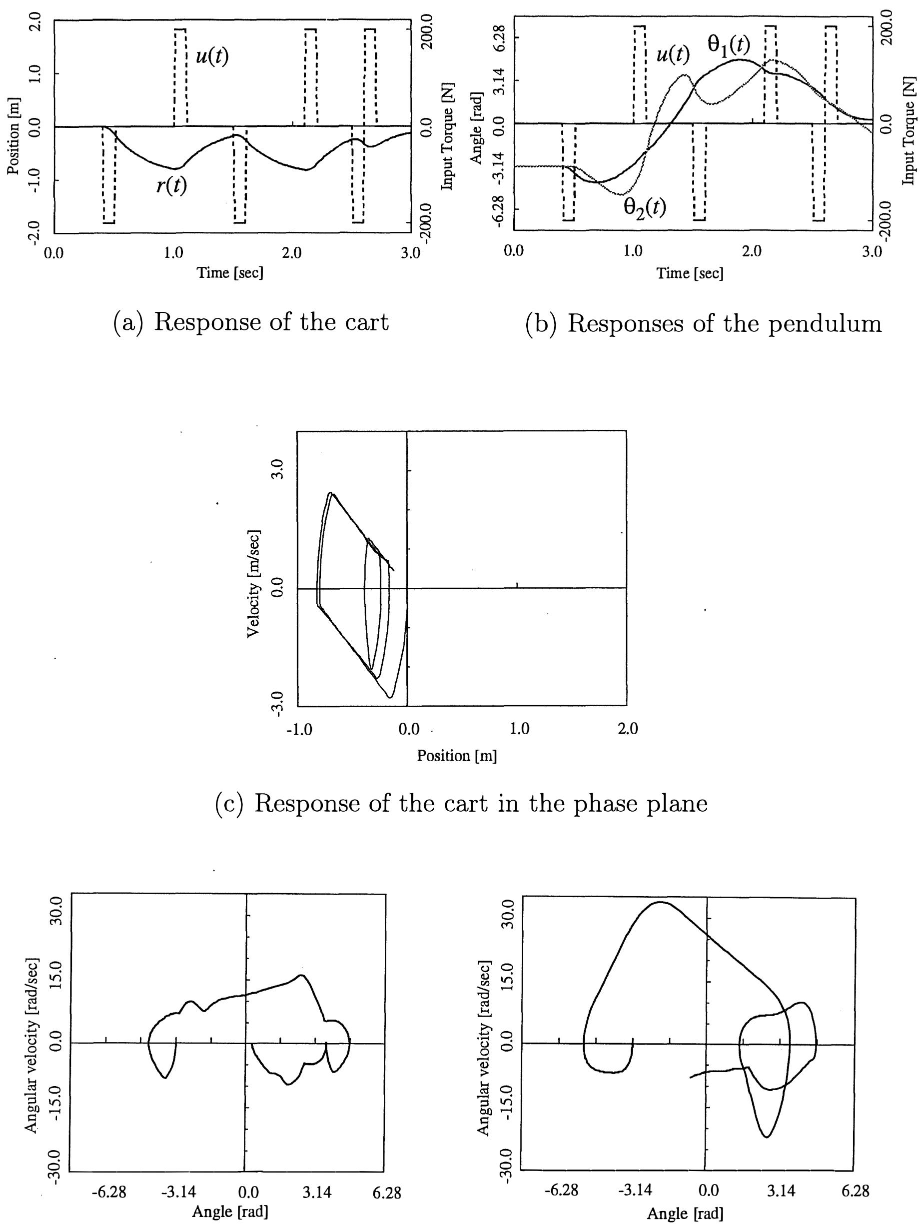


(d) Response of the 1st pendulum in the phase plane



(e) Response of the 2nd pendulum in the phase plane

Fig.4. Swing-up motion realized by the optimum solution (Example 1)



(a) Response of the cart

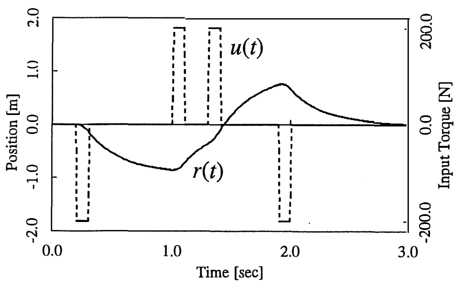
(b) Responses of the pendulum

(c) Response of the cart in the phase plane

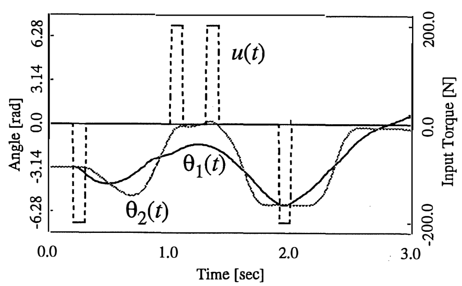
(d) Response of the 1st pendulum in the phase plane

(e) Response of the 2nd pendulum in the phase plane

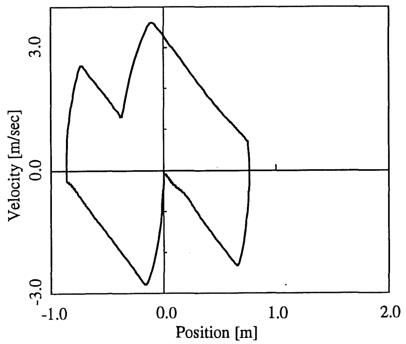
Fig.5. Swing-up motion realized by the optimum solution (Example 2)



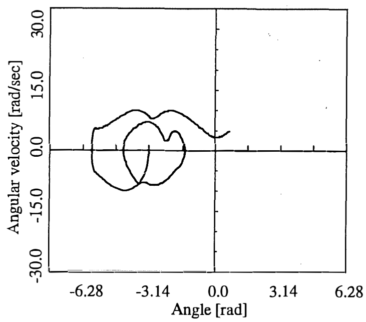
(a) Response of the cart



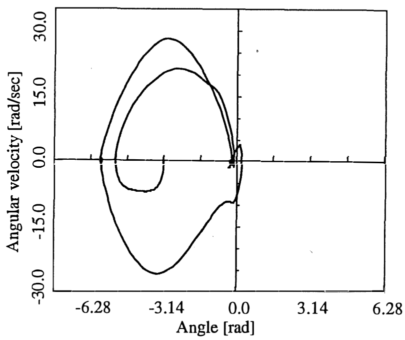
(b) Responses of the pendulum



(c) Response of the cart in the phase plane

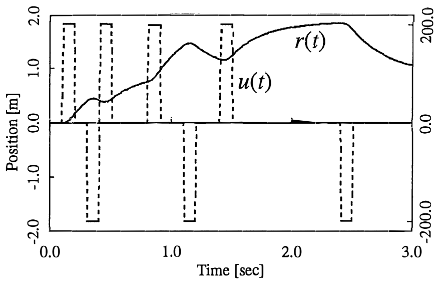


(d) Response of the 1st pendulum in the phase plane

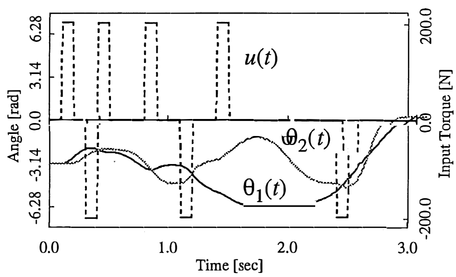


(e) Response of the 2nd pendulum in the phase plane

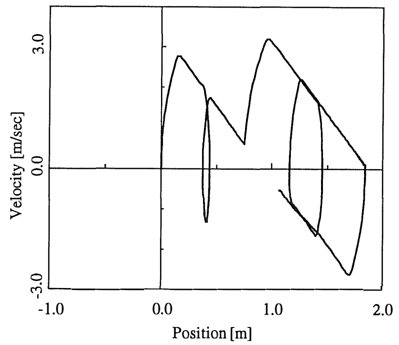
Fig.6. Swing-up motion realized by the optimum solution (Example 3)



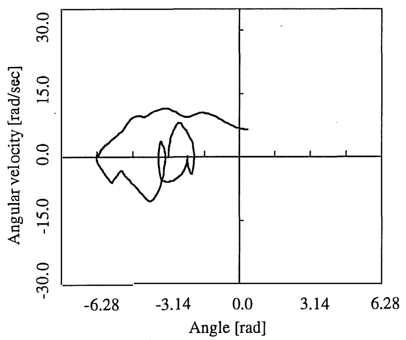
(a) Response of the cart



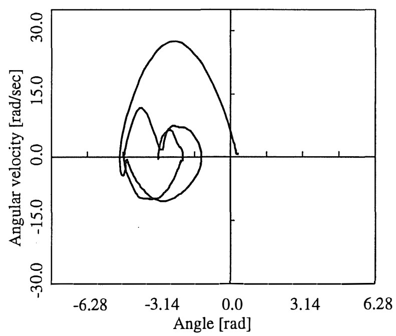
(b) Responses of the pendulum



(c) Response of the cart in the phase plane



(d) Response of the 1st pendulum in the phase plane



(e) Response of the 2nd pendulum in the phase plane

Fig.7. Swing-up motion realized by the optimum solution (Example 4)

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