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Impact of inflation and trade credit policy in an inventory model for imperfect quality items with allowable shortages*

by

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Abstract: Retail businesses must constantly evolve with new strategies introduced in order to meet and even to exceed customers' expectations. This effort can be enhanced by incorporating inspection processes in business routines that will maximize the effectiveness of attempts to sell quality products. Further, permissible delay in payments has certainly been a prominent strategy in today's business transactions, helping in gaining the financial advantage for both the retailers and the suppliers. Moreover, in order to recognize the proper and exact timing of cash flows associated with an inventory system, inflation and time value of money should also be incorporated.

Considering all the above described real life aspects and problems, a model is formulated here to study the combined effect of imperfect quality items, trade credit, shortages, inflation and time value of money on an inventory system. An analytical method is employed to jointly optimize the order quantity and the shortages. To study the behavior and application of the proposed model, a numerical example, including sensitivity analysis, has been analyzed. The potential applications, improving the decision making process of the model introduced can be found in industries like textile, footwear, plastics, electronics, etc.

Keywords: inventory, imperfect items, screening, shortages, permissible delay, inflation.

1. Introduction

In today's technology and competition driven world, firms must adopt high quality standards to dominate world markets. No company is perfect enough

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to achieve the truly supreme standards, and so, firms continuously make efforts to secure quality improvement. Despite the emergence of sophisticated production techniques and control systems, the items produced may still have a fraction of defectives, which need to be sorted out through a careful inspection process before reaching the customers. These defectives can also be used in specially devised inventory systems, which may be less restrictive. In all cases, there is substantial magnitude of cost involved, which cannot be ignored. In the field of imperfect items, the researchers, who initially developed the respective EOQ/EPQ models were Porteus (1986), Rosenblatt and Lee (1986), Lee and Rosenblatt (1987), Schwaller (1988), and Zhang and Gerchak (1990). Kim and Hong (2001) extended the problem of Rosenblatt and Lee (1986) by assuming that the time having elapsed until the process shift is arbitrarily distributed. Furthermore, Salameh and Jaber (2000) carried further the research by considering that the whole lot contains a random percentage of defective items with known p.d.f. They also assumed that the whole lot goes through 100% screening process and the defective items sorted out are sold as a single batch at a discounted price. Since then, several researchers have shown interest in elaborating upon the work of Salameh and Jaber (2000). Thus, in particular, Cárdenas-Barrón (2000) corrected the Salameh and Jaber's (2000) paper by adding a constant parameter in the optimum order size formula. Soon thereafter, Goyal and Cárdenas-Barrón (2002) formulated an EPQ model and compared their results with those of Salameh and Jaber (2000). Later, in 2007, Wee et al. (2007) extended the work of Salameh and Jaber (2000) by allowing for shortages. They showed that with an increase in backordering cost, the rate of change in annual profit decreases, as compared to the results for the Salameh and Jaber (2000) model. Further extension of Salameh and Jaber (2000) model was done by Maddeh and Jaber (2008) and Eroglu and Ozdemir (2007) by evaluating optimal order quantity and expected profit per unit time using renewal reward theorem, respectively, with both models allowing for shortages. Recently, Maddeh and Jaber (2011) suggested a practical approach for preventing shortages by ordering when the stock is just sufficient to satisfy the demand during the screening process, this demand being satisfied from the inventory of previous order. Later, Sarkar and Moon (2014) applied the concept of quality improvement and setup cost reduction to construct a distribution free inventory model. They used a variable backorder rate in an imperfect production system, which resulted in significant savings. Very recently, Sarkar and Saren (2016) considered product inspection policy along with screening errors and warranty costs in imperfect environment.

In order to increase the sales and enlarge the customer base, many firms take the policy of offering trade credit, since this enhances the demand for products. It is an unsecured credit that arises out of transfer of goods, whereby the buying firm receives supplies under delayed payment terms. This not only indicates the seller's faith in the buyer, but also reflects buyer's power to purchase now and pay later, which makes it the second most liquid asset after cash. When a supplier offers a credit period to his customer (i.e., retailer), he is actually providing him a loan without interest. During the credit period, the retailer can sell the items and generate revenue, and also earn interest on them. After the expiration of the term, if there are some unsold items, the retailer will have to finance and pay interest on them. Therefore, this makes it profitable and economically justified for the retailer to make the payment on the last day of the credit period. Haley and Higgins (1973) were the first to consider EOQ model under permissible delay in payment. Goyal (1985) considered a similar problem, including different interest rates before and after the expiration of credit periods. Aggarwal and Jaggi (1995) extended Goyal's (1985) model for deteriorating items. Kim et al. (1995) examined the effect of credit period on the increase of the whole seller's profits with demand as a function of price. Jamal et al. (1997) also generalized Goyal's (1985) model to allow for shortages. Teng (2002) further analyzed Goyal's (1985) model to conclude that it is more profitable to order less in terms of quantity, but more frequently. Furthermore, Jaggi et al. (2008) considered credit-linked demand function to derive the optimal replenishment policy. In recent years, Sarkar (2012a) constructed an EOQ model with delay in payments and stock dependent demand in the presence of imperfect production. In relation to this, Sarkar (2012b) formulated an inventory model for finite replenishment rate, where demand and deterioration rate are both time-dependent. He also applied trade credit policy in his model. Ouyang et al. (2013) explored a model under two levels of trade credit policy, in which the supplier offers to the whole seller a permissible delay period, and the whole seller also provides its retailers a permissible delay period. Further, Wu et al. (2014) developed an ordering lot size model for deteriorating items that deteriorate constantly but also have their expiration dates. In their model, both the optimal trade credit and the optimal cycle time not only exist, but are also unique. In the same year, Chen et al. (2014) revisited the economic order quantity model under conditionally permissible delay in payments, in which they proposed a simple arithmetic–geometric method to solve the problem, in contrast to the differential calculus method. Furthermore, Sarkar et al. (2014) considered trade credit policy along with the production of defective items and the inspection policy, where the order quantity and lead time are assumed to be the decision variables. Recently, Sarkar et al. (2015) considered supplier's and retailer's trade-credit policy for fixed lifetime products with time varying deterioration rates in an EOQ model.

But, before making any investment, it is crucial to understand inflation and time value of money. Inflation brings price rise and decreases the real value of money. To get the real estimate of all costs incurred, it is logical to incorporate the net effect of inflation and time value of money. In the literature, Buzacott (1975) and Misra (1975) were the first to elaborate on the significance of inflation and time value of money by developing inventory models with constant inflation rate. Misra (1979) continued the previous work for different inflation rates with respect to various associated costs. Bose (1995) developed a model under inflation and time value of money using the discounted cash flow (DCF) approach. Further, Jaggi et al. (1997) examined the effect of changing inflation rates and credit policies for non-deteriorating items. Thereafter, several interesting research papers have appeared, e.g. by Yang et al. (2001), Sarkar et al. (2000), Moon and Lee (2000). Recently, great amount of effort has been put to develop models including different combinations of trade credit, imperfect items, inflation and time value of money. For instance, Sarkar and Moon (2011) examined the effect of inflation on imperfect quantity items. In 2006, Jaggi et al. (2006) studied the effect of inflation-induced demand on order policies for deteriorating items. Later, Jaggi and Khanna (2009) developed retailer's procurement policy under inflationary conditions when the end demand is credit-linked, and Jaggi et al. (2010) examined the effect of imperfect quality and trade credit on economic ordering policies without shortages. Recently, Jaggi et al. (2013) further extended the work from Jaggi et al. (2010) by considering shortages at the end of the replenishment cycle.

In this paper, an attempt has been made to develop an inventory model for imperfect quality items under permissible delay in payments. Shortages are fully backlogged and are met parallel to the demand till the end of inspection process. The effect of inflation and time value of money has also been considered, and various costs are computed using the discounted cash flow (DCF) approach. The proposed model optimizes retailer's order quantity and shortages by maximizing his expected total profit per unit time. A numerical example is provided to demonstrate the applicability of the proposed model and a comprehensive sensitivity analysis has been conducted as well, in order to observe the effects of key model parameters on the optimal replenishment policy.

2. Assumptions

The fundamental assumptions of the model developed are listed below:

1. Demand rate is constant, uniform and deterministic.
2. Replenishment rate is instantaneous.
3. Shortages are allowed and are fully back-logged.
4. Lead time is negligible.
5. The effects of inflation and time value are considered.
6. Screening rate is greater than the demand rate.
7. The proportion of defectives, α, and its p.d.f., $f(\alpha)$, can be estimated using past data.
8. Rate of screening of good quality items is assumed to be $(1-\alpha)\lambda$ where λ is the screening rate.

3. Model formulation

This section discusses an inventory model for imperfect quality items, which undergo an inspection process, when a trade credit policy is being offered by the supplier, in the presence of inflationary conditions. Shortages are allowed at the beginning of the inventory cycle and are fully backordered.

Table 1. Notations

D	Demand rate in units per unit time
Q	Order size for each cycle
B	Maximum backorder level allowed
A_o	Fixed cost of placing an order at time t =0
C_o	Unit cost at t =0
P_o	Unit selling price of good quality items at t =0
h_o	Unit holding cost at t =0
β_o	Unit screening cost at t =0
C_s	Unit selling price of imperfect quality items at t =0, $C_s < P_o$
C_B	Unit backorder cost
$A(t)$	Fixed cost of placing an order at time t
$C(t)$	Unit cost at time t
$P(t)$	Unit selling price of good quality items at time t
$h(t)$	Unit holding cost at time t
$\beta(t)$	Unit screening cost at time t
$C_S(t)$	Unit selling price of imperfect quality items at time t
λ	Screening rate in units per unit time, $\lambda > D$
t_1	Time to build up backorder level of B units
t_2	Time to eliminate the backorder level of B units
t_3	Time to screen Q units ordered per cycle
T	Cycle length
z	Inventory level at t_2
z_1	Inventory level at t_3
$B(t)$	Backorder level during time interval $[0, t_1]$
$I_1(t)$	Inventory during time interval $[t_1, t_2]$
$I_2(t)$	Inventory during time interval $[t_2, t_3]$
$I_3(t)$	Inventory during time interval $[t_3, T]$
I_e	Interest earned per unit per unit time
I_p	Interest paid per unit per unit time
α	% of defective items in Q
$f(\alpha)$	p.d.f. of α
$E(\alpha)$	Expected value of α, which is equal to $= \int_a^b f(\alpha) d\alpha$
$E(.)$	Expected value operator
$(1\text{-}\alpha)\lambda$	Rate of good quality items during t_2
$(1\text{-}\alpha)\lambda - D$	Rate of good quality items to eliminate backorder, $(1\text{-}\alpha)\lambda - D > 0$
d	Discount rate, representing time value of money
i	Inflation rate
R	$d - i$; net discount rate of inflation, a constant
M	Retailer's credit period offered by supplier to settle the account
$\pi_j(Q, B)$	Retailer's profit which is a function of two variables, Q, B; $j = \{1, 2, 3, 4.\}$

The behavior of the present inventory system can be depicted using Fig. 1. At time A_1, Q units are procured and the whole lot goes through 100% screening process at the rate of λ units per unit time to separate good and defective items from time A_1 to A_3. From time A_1 to A_2, a fraction of good quality items fulfill the demand and the rest is used to eliminate backorders with the rate of $(1\text{-}\alpha)\lambda - D$. At A_3, the whole lot of defective items (αQ) is sold at a discounted price C_S. After the end of the screening process, inventory level gradually decreases, due to demand, and reaches zero at time A_4.

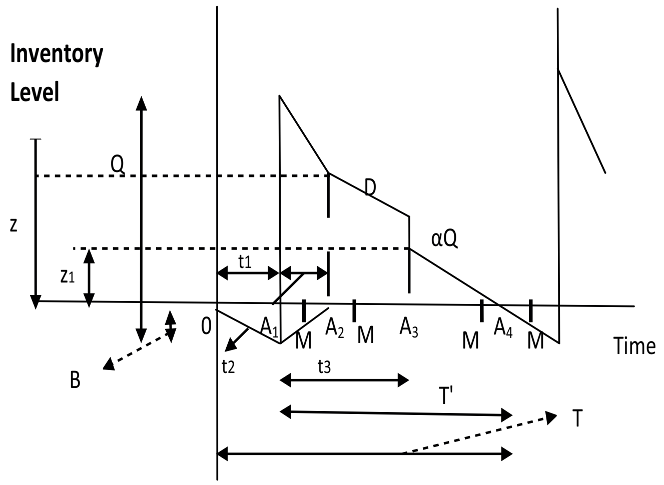

Figure 1. Behavior of the inventory system for all four cases

The following differential equations show the change in inventory level at any given time:

$$\frac{dB(t)}{dt} = -(-D)\,;\ 0 \leqslant t \leqslant t_1 \tag{1}$$

$$\frac{dI_1(t)}{dt} = -\{[(1-\alpha)\lambda - D] + D\}\,;\quad t_1 \leqslant t \leqslant t_1 + t_2 \tag{2}$$

$$\frac{dI_2(t)}{dt} = -D\ ;\quad t_1 + t_2 \leqslant t \leqslant t_1 + t_3 \tag{3}$$

$$\frac{dI_3(t)}{dt} = -D;\quad t_1 + t_3 \leqslant t \leqslant t_1 + T'. \tag{4}$$

Solutions of the above differential equations using initial and boundary conditions are as follows:

Using $B(0) = 0$ and $B(t_1) = B$, equation (1) is solved as

$$B(t) = Dt \tag{5}$$

and

$$B(t) = B - D(t_1 - t). \tag{6}$$

Using $I_1(t_1) = Q$, and $I_1(t_1 + t_2) = z$, equation (2) is solved as

$$I_1(t) = Q - (1-\alpha)\lambda(t - t_1) \tag{7}$$

and

$$z = Q - (1-\alpha)\lambda t_2. \tag{8}$$

Using $I_2(t_1 + t_2) = z$ and $I_2(t_1 + t_3) = z_1 + \alpha Q$, equation (3) is solved as

$$I_2(t) = z - D(t - t_1 - t_2) \tag{9}$$

and

$$z_1 = z - D(t_3 - t_2) - \alpha Q. \tag{10}$$

Using $I_3(t_3) = z_1$ and $I_3(t_1 + T') = 0$, equation (4) is solved as,

$$I_3(t) = z_1 - D(t - t_1 - t_3) \tag{11}$$

$$T' = \frac{z_1}{D} + t_3. \tag{12}$$

Using the inventory curve and the above solution of equations, the cycle length T is given by

$$T = \frac{(1-\alpha)Q}{D} \tag{13}$$

$$t_1 = \frac{B}{D} \tag{14}$$

$$t_2 = \frac{B}{(1-\alpha)\lambda - D} \tag{15}$$

$$t_3 = \frac{Q}{\lambda} \tag{16}$$

and

$$T = T' + t_1. \tag{17}$$

The present worth of retailer's profit function π**(Q,B)** consists of the following components:

π**(Q,B)**= Present Worth of Sales Revenue – Present Worth of Ordering Cost – Present Worth of Purchase Cost – Present Worth of Screening Cost – Present Worth of Shortage Cost – Present Worth of Holding Cost + Present Worth of Interest Earned –Present Worth of Interest Paid.

Therefore, by using the DCF approach, the present worth of various cost components for the first replenishment cycle is evaluated as follows:

1. Present Worth of Sales Revenue:

$$S_r = P(t_1)\int_{t_1}^{t_1+T} De^{-Rt}dt + C_s(t_1)\alpha Q e^{-Rt_3}$$
$$= \frac{P_o De^{-Rt_1}}{R}\left(e^{-Rt_1}te^{-R(t_1+T)}\right) + C_s\alpha Q e^{-R(t_1+t_3)}$$
$$= \frac{P_o De^{-2Rt_1}}{R}\left(1te^{-RT}\right) + C_s\alpha Q e^{-R(t_1+t_3)}. \tag{18}$$

2. Present Worth of Ordering Cost:

$$C_r = A(t_1) = A_0 e^{-Rt_1}. \tag{19}$$

3. Present Worth of Purchase Cost:

$$C_p = C(t_1)\,Q = C_0 e^{-Rt_1}Q. \tag{20}$$

4. Present Worth of Screening Cost:

$$C_{scr} = \beta(t_1)Q = Q\beta_0 e^{-Rt_1}. \tag{21}$$

5. Present Worth of Shortage Cost:

$$C_{sho} = C_B{}_{\int_0}^{t_1} Dte^{-Rt}dt = \frac{C_B D}{R^2}[1 - e^{-Rt_1}(1 + Rt_1)]. \tag{22}$$

6. Present Worth of Holding Cost:

$$C_h = h(t_1)\int_{t_1}^{t_1+T'} I(t)\,e^{-Rt}dt$$
$$= h(t_1)\left\{\int_{t_1}^{t_1+t_2} I_1(t)\,e^{-Rt}dt + \int_{t_1+t_2}^{t_1+t_3} I_2(t)\,e^{-Rt}dt + \int_{t_1+t_3}^{t_1+T'} I_3(t)\,e^{-Rt}dt\right\}$$

$$= h_0 e^{-2Rt_1}\left(\frac{Q}{R}\left(1 - e^{-Rt_2}\right) - \frac{(1-\alpha)\lambda}{R^2}\left(1 - e^{-Rt_2}\right) + \frac{(1-\alpha)\lambda t_2 e^{-Rt_2}}{R}\right)$$
$$+ h_0 e^{-2Rt_1}\left(+\frac{z}{R}\left(e^{-Rt_2} - e^{-Rt_3}\right) + \frac{z_1}{R}\left(e^{-Rt_3} - e^{-RT'}\right)\right)$$
$$+ h_0 e^{-2Rt_1}\left(\frac{D}{R^2}\left(e^{-RT'} - e^{-Rt_2}\right) + \frac{De^{-RT'}}{R}(T' - t_3) + \frac{De^{-Rt_3}}{R}(t_3 - t_2)\right). \tag{23}$$

Depending upon the value of M, T and T', the present worth of interest earned and interest paid is calculated for four distinct possible cases $\pi_j(Q, B)$; $j = 1, 2, 3, 4$, namely:

Case (i): $t_1 \leqslant t_1 + M \leqslant t_1 + t_2 \leqslant t_1 + t_3 \leqslant t_1 + T'$

Case (ii): $t_1 + t_2 \leqslant t_1 + M \leqslant t_1 + t_3 \leqslant t_1 + T'$

Case (iii): $t_1 + t_2 \leqslant t_1 + t_3 \leqslant t_1 + M \leqslant t_1 + T'$, and

Case (iv): $t_1 + T' \leqslant t_1 + M \leqslant T$.

Case(i): $\mathbf{t_1 \leqslant t_1{+}M \leqslant t_1{+}t_2 \leqslant t_1{+}\ t_3 \leqslant t_1{+}T'}$

From Fig. 2 it can be clearly concluded that the interest earning period is from A_1 to $t_1 + M$, generated by selling the items as per demand. At $t_1 + M$, the account is settled and interest is charged on the remaining unsold items for the time $t_1 + M$ to A_4, i.e. finances are to be arranged to make the payment to the supplier for the remaining stock.

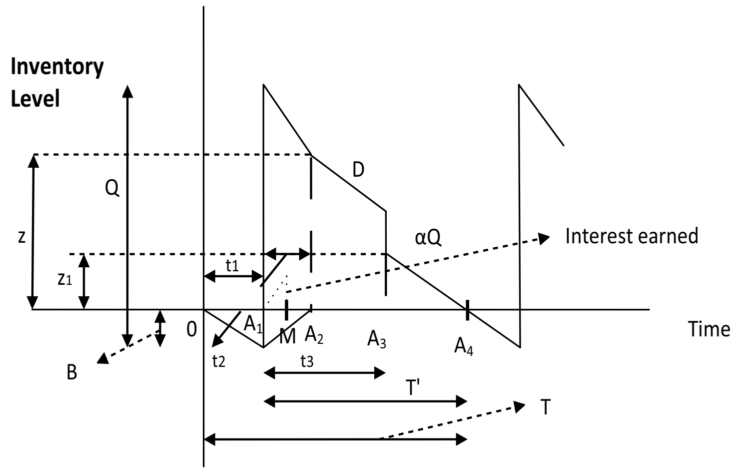

Figure 2. Inventory system for Case (i): $t_1 \leqslant t_1{+}M \leqslant t_1{+}t_2 \leqslant t_1{+}t_3 \leqslant t_1{+}T'$

7(i) Present Worth of Interest Earned:

$$C_{Ie1} = P\left(t_1\right) I_e \left(\int_{t_1}^{t_1+M} \left[\left(1-\alpha\right)\lambda - D\right]\left(t-t_1\right)e^{-Rt}dt + \int_{t_1}^{t_1+M} D(t{-}t_1)e^{-Rt}dt \right)$$

$$= I_e P_o e^{-2Rt_1} \left(-\frac{\left(1-\alpha\right)\lambda M}{R} e^{-RM} - \frac{(1-\alpha)\lambda}{R^2} e^{-RM} + \frac{(1-\alpha)\lambda}{R^2} \right). \qquad (24)$$

8(i) Present Worth of Interest Paid:

$$
\begin{aligned}
&C_{Ip1} = \\
&C\left(t_1\right) I_p\left(\int_{t_1+M}^{t_1+t_2} I_1\left(t\right) e^{-Rt} dt + \int_{t_1+t_2}^{t_1+t_3} I_2\left(t\right) e^{-Rt} dt + \int_{t_1+t_3}^{t_1+T'} I_3\left(t\right) e^{-Rt} dt\right) \\
&+C_s(t_1) I_p \int_{t_1+M}^{t_1+t_3} \alpha Q e^{-Rt} dt \\
&= I_p C_0 e^{-2Rt_1}\left(\frac{Q}{R}\left(e^{-RM} - e^{-Rt_2}\right) + \frac{\left(1-\alpha\right)\lambda}{R^2}\left(e^{-Rt_2} - e^{-RM}\right)\right. \\
&\left.+\frac{\left(1-\alpha\right)\lambda}{R}\left(t_2 e^{-Rt_2} - M e^{-RM}\right)\right) \\
&+I_p C_0 e^{-2Rt_1}\left(\frac{z}{R}\left(e^{-Rt_2} - e^{-Rt_3}\right) + \frac{z_1}{R}\left(e^{-Rt_3} - e^{-RT'}\right)\right) \\
&+I_p C_0 e^{-2Rt_1}\left(\frac{D}{R^2}\left(e^{-RT'} - e^{-Rt_2}\right) + \frac{De^{-RT'}}{R}\left(T' - t_3\right) + \frac{De^{-Rt_3}}{R}\left(t_3 - t_2\right)\right) \\
&+\frac{C_s \alpha Q I_p e^{-2Rt_1}}{R}\left(e^{-RM} - e^{-Rt_3}\right). \qquad (25)
\end{aligned}
$$

Therefore, the present worth of retailer's profit function for **Case (i)** using equations (18), (19), (20), (21), (22), (23),(24), and (25) is given by

$$
\begin{aligned}
&\pi_1\left(Q, B\right) = S_r - C_r - C_p - C_{scr} - C_s - C_h + C_{Ie1} - C_{Ip1} \\
&= \frac{P_o D e^{-Rt_1}}{R}\left(e^{-Rt_1} - e^{-RT}\right) + C_s \alpha Q e^{-R(t_1+t_3)} - A_0 e^{-Rt_1} \\
&-C_0 e^{-Rt_1} Q - Q\beta_0 e^{-Rt_1} - \frac{C_B D}{R^2}\left[1 - e^{-Rt_1}\left(1 + Rt_1\right)\right] \\
&-h_0 e^{-2Rt_1}\left(\frac{Q}{R}\left(1 - e^{-Rt_2}\right) - \frac{\left(1-\alpha\right)\lambda}{R^2}\left(1 - e^{-Rt_2}\right) + \frac{\left(1-\alpha\right)\lambda t_2 e^{-Rt_2}}{R} + \right. \\
&\frac{z}{R}\left(e^{-Rt_2} - e^{-Rt_3}\right) + \frac{z_1}{R}\left(e^{-Rt_3} - e^{-RT'}\right) + \frac{D}{R^2}\left(e^{-RT'} - e^{-Rt_2}\right) + \\
&\left.\frac{De^{-RT'}}{R}\left(T' - t_3\right) + \frac{De^{-Rt_3}}{R}\left(t_3 - t_2\right)\right) +
\end{aligned}
$$

$$
\begin{aligned}
&I_eP_oe^{-2Rt_1}\left(-\frac{(1-\alpha)\lambda M}{R}e^{-RM}-\frac{(1-\alpha)\lambda}{R^2}e^{-RM}+\frac{(1-\alpha)\lambda}{R^2}\right)-\\
&I_pC_0e^{-2Rt_1}\left(\frac{Q}{R}\left(e^{-RM}-e^{-Rt_2}\right)+\frac{(1-\alpha)\lambda}{R^2}\left(e^{-Rt_2}-e^{-RM}\right)+\right.\\
&\frac{(1-\alpha)\lambda}{R}\left(t_2e^{-Rt_2}-Me^{-RM}\right)+\\
&\frac{z}{R}\left(e^{-Rt_2}-e^{-Rt_3}\right)+\frac{z_1}{R}\left(e^{-Rt_3}-e^{-RT'}\right)\\
&\left.+\frac{D}{R^2}\left(e^{-RT'}-e^{-Rt_2}\right)+\frac{De^{-RT'}}{R}\left(T'-t_3\right)+\frac{De^{-Rt_3}}{R}\left(t_3-t_2\right)\right)+\\
&\frac{C_s\alpha QI_pe^{-2Rt_1}}{R}\left(e^{-RM}-e^{-Rt_3}\right). \qquad (26)
\end{aligned}
$$

Case (ii): $\mathbf{t_1+t_2\leqslant t_1+M\leqslant t_1+t_3\leqslant t_1+T'}$

As this can be seen in Fig. 3, the retailer earns interest on the revenue, which is generated from selling the items up to time t_1+M as per demand. Since backorders are completely satisfied till time A_2, therefore the retailer earns additional interest from B units (i.e. the maximum backordered quantity) for the time period (A_2, t_1+M). After the account is settled at t_1+M, the retailer finances the unsold items at a specified interest rate for the time period (t_1+M, A_4).

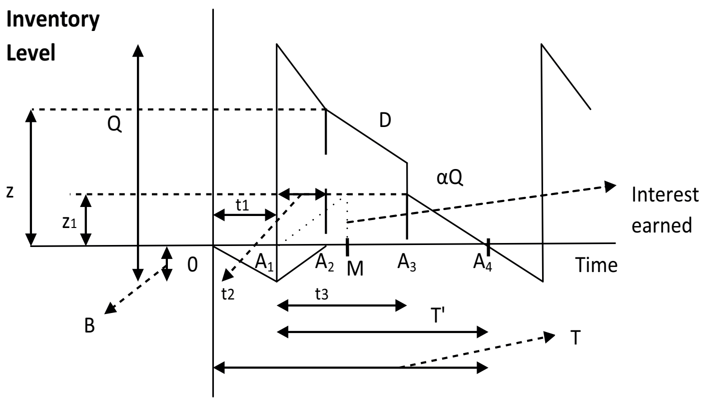

Figure 3. Inventory system for Case (ii): $t_1+t_2\leqslant t_1+M\leqslant t_1+t_3\leqslant t_1+T'$

7(ii) Present Worth of Interest Earned:

$$C_{Ie2} =$$
$$P(t_1) I_e \left(\int_{t_1}^{t_1+t_2} [(1-\alpha)\lambda - D](t-t_1) e^{-Rt} dt \right.$$
$$\left. + \int_{t_1}^{t_1+M} D(t-t_1) e^{-Rt} dt + \int_{t_1+t_2}^{t_1+M} B e^{-Rt} dt \right)$$
$$= I_e P_o e^{-2Rt_1} \left(-\frac{[(1-\alpha)\lambda - D]}{R^2} \left[(1+Rt_2) e^{-Rt_2} \right] \right.$$
$$\left. + \frac{(1-\alpha)\lambda}{R^2} - \frac{D}{R^2} \left[(1+RM) e^{-RM} \right] + \frac{B}{R} \left(e^{-Rt_2} - e^{-RM} \right) \right). \tag{27}$$

8(ii) Present Worth of Interest Paid:

$$C_{Ip2} =$$
$$C(t_1) I_p \left(\int_{t_1+M}^{t_1+t_3} I_2(t) e^{-Rt} dt + \int_{t_1+t_3}^{t_1+T'} I_3(t) e^{-Rt} dt \right)$$
$$+ C_s(t_1) I_p \int_{t_1+M}^{t_1+t_3} \alpha Q e^{-Rt} dt$$
$$= I_p C_0 e^{-2Rt_1} \left(\frac{z}{R} \left(e^{-RM} - e^{-Rt_3} \right) + \frac{z_1}{R} \left(e^{-Rt_3} - e^{-RT'} \right) \right.$$
$$+ \frac{D}{R^2} \left(e^{-RT'} - e^{-RM} \right)$$
$$\left. + \frac{D}{R} (t_2 - M) e^{-RM} + \frac{D}{R} (T' - t_3) e^{-RT'} + \frac{D}{R} (t_3 - t_2) e^{-Rt_3} \right)$$
$$+ \frac{C_s \alpha Q I_p e^{-2Rt_1}}{R} \left(e^{-RM} - e^{-Rt_3} \right). \tag{28}$$

Therefore, the present worth of retailer's profit function for **Case (ii)** using equations (18), (19), (20), (21), (22), (23), (27) and (28) is given by

$$\pi_2(Q,B) = S_r - C_r - C_p - C_{scr} - C_s - C_h + C_{Ie2} - C_{Ip2}$$
$$= \frac{P_o D e^{-Rt_1}}{R}\left(e^{-Rt_1} - e^{-RT}\right) + C_s \alpha Q e^{-R(t_1+t_3)} - A_0 e^{-Rt_1} - C_0 e^{-Rt_1} Q$$
$$-Q\beta_0 e^{-Rt_1} - \frac{C_B D}{R^2}\left[1 - e^{-Rt_1}(1+Rt_1)\right] - h_0 e^{-2Rt_1}$$
$$\left(\frac{Q}{R}\left(1-e^{-Rt_2}\right) - \frac{(1-\alpha)\lambda}{R^2}\left(1-e^{-Rt_2}\right) + \frac{(1-\alpha)\lambda t_2 e^{-Rt_2}}{R}\right.$$
$$+\frac{z}{R}\left(e^{-Rt_2} - e^{-Rt_3}\right) + \frac{z_1}{R}\left(e^{-Rt_3} - e^{-RT'}\right) + \frac{D}{R^2}\left(e^{-RT'} - e^{-Rt_2}\right)$$
$$\left.+\frac{De^{-RT'}}{R}\left(T' - t_3\right) + \frac{De^{-Rt_3}}{R}\left(t_3 - t_2\right)\right)$$
$$+I_e P_o e^{-2Rt_1}\left(-\frac{((1-\alpha)\lambda - D)}{R^2}\left((1+Rt_2)e^{-Rt_2}\right) + \frac{(1-\alpha)\lambda}{R^2}\right.$$
$$\left.-\frac{D}{R^2}\left((1+RM)e^{-RM}\right) + \frac{B}{R}\left(e^{-Rt_2} - e^{-RM}\right)\right)$$
$$-I_p C_0 e^{-2Rt_1}\left[\frac{z}{R}\left(e^{-RM} - e^{-Rt_3}\right) + \frac{z_1}{R}\left(e^{-Rt_3} - e^{-RT'}\right)\right.$$
$$+\frac{D}{R^2}\left(e^{-RT'} - e^{-RM}\right) + \frac{D}{R}(t_2 - M)e^{-RM}$$
$$\left.+\frac{D}{R}(T' - t_3)e^{-RT'} + \frac{D}{R}(t_3 - t_2)e^{-Rt_3}\right)$$
$$-\frac{C_s \alpha Q I_p e^{-2Rt_1}}{R}\left(e^{-RM} - e^{-Rt_3}\right). \tag{29}$$

Case (iii): $\mathbf{t_1{+}t_2 \leqslant t_1{+}t_3 \leqslant t_1{+}M \leqslant t_1{+}T'}$

Here, the retailer earns interest on revenue, generated by selling the items up to $t_1 + M$ as per demand. Since the defective lot has been sold by this time, therefore he earns additional interest from the sale of defective lot for time $(A_3, t_1 + M)$. Interest is also earned from the shortages, which are backlogged during $(A_2, t_1 + M)$. After the settlement of account at M, interest is charged on the remaining unsold items, as this is shown in Fig. 4.

7(iii) Present Worth of Interest Earned:

$$C_{Ie3} =$$
$$P(t_1) I_e\left(\int_{t_1}^{t_1+t_2}\left[(1-\alpha)\lambda - D\right](t-t_1)e^{-Rt}dt + \right.$$
$$\left.\int_{t_1}^{t_1+M} D(t-t_1)e^{-Rt}dt + \int_{t_1+t_2}^{t_1+M} Be^{-Rt}dt\right) + C_{s(t_1)} I_e \int_{t_1+t_3}^{t_1+M} \alpha Q e^{-Rt}dt$$

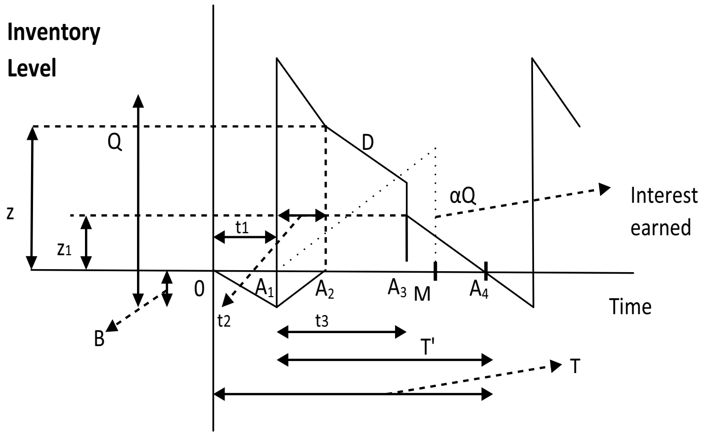

Figure 4. Inventory system for Case (iii): $t_1 + t_2 \leqslant t_1 + t_3 \leqslant t_1 + M \leqslant t_1 + T'$

$$= I_e P_o e^{-2Rt_1} \left(-\frac{((1-\alpha)\lambda - D)}{R^2} \left((1 + Rt_2) e^{-Rt_2} \right) + \frac{(1-\alpha)\lambda}{R^2} \right.$$
$$\left. -\frac{D}{R^2} \left((1 + RM) e^{-RM} \right) + \frac{B}{R} \left(e^{-Rt_2} - e^{-RM} \right) \right)$$
$$+\frac{C_s \alpha Q I_e e^{-2Rt_1}}{R} \left(e^{-RM} - e^{-Rt_3} \right). \tag{30}$$

8(iii) Present Worth of Interest Paid:

$$C_{Ip3} = C(t_1) I_p \int_{t_1+M}^{t_1+T'} I_3(t) e^{-Rt} dt$$
$$= I_p C_0 e^{-2Rt_1} \left(\frac{z_1}{R} \left(e^{-RM} - e^{-RT'} \right) + \right.$$
$$\left. \frac{D}{R^2} \left(e^{-RT'} - e^{-RM} \right) + \frac{D}{R} (t_3 - M) e^{-RM} + \frac{D}{R} (T' - t_3) e^{-RT'} \right). \tag{31}$$

Therefore, the present worth of retailer's profit function for **Case (iii)**, obtained by using equations (18), (19), (20), (21), (22), (23), (30) and (31), is given by

$$
\begin{aligned}
&\pi_3(Q,B) = \\
&S_r - C_r - C_p - C_{scr} - C_s - C_h + C_{Ie3} - C_{Ip3} \\
&= \frac{P_o D e^{-Rt_1}}{R}\left(e^{-Rt_1} - e^{-RT}\right) + C_s \alpha Q e^{-R(t_1+t_3)} - A_0 e^{-Rt_1} \\
&-C_0 e^{-Rt_1} Q - Q\beta_0 e^{-Rt_1} - \frac{C_B D}{R^2}\left[1 - e^{-Rt_1}(1+Rt_1)\right] \\
&-h_0 e^{-2Rt_1}\left(\frac{Q}{R}\left(1 - e^{-Rt_2}\right) - \frac{(1-\alpha)\lambda}{R^2}\left(1 - e^{-Rt_2}\right)\right. \\
&+\frac{(1-\alpha)\lambda t_2 e^{-Rt_2}}{R} + \frac{z}{R}\left(e^{-Rt_2} - e^{-Rt_3}\right) \\
&+\frac{z_1}{R}\left(e^{-Rt_3} - e^{-RT'}\right) + \frac{D}{R^2}\left(e^{-RT'} - e^{-Rt_2}\right) \\
&\left.+\frac{De^{-RT'}}{R}(T' - t_3) + \frac{De^{-Rt_3}}{R}(t_3 - t_2)\right) \\
&+I_e P_o e^{-2Rt_1}\left(-\frac{((1-\alpha)\lambda - D)}{R^2}\left((1+Rt_2)e^{-Rt_2}\right) + \frac{(1-\alpha)\lambda}{R^2}\right. \\
&\left.-\frac{D}{R^2}\left((1+RM)e^{-RM}\right) + \frac{B}{R}\left(e^{-Rt_2} - e^{-RM}\right)\right) \\
&+\frac{C_s \alpha Q I_e e^{-2Rt_1}}{R}\left(e^{-RM} - e^{-Rt_3}\right) \\
&-I_p C_0 e^{-2Rt_1}\left(\frac{z_1}{R}\left(e^{-RM} - e^{-RT'}\right)\right. \\
&\left.+\frac{D}{R^2}\left(e^{-RT'} - e^{-RM}\right) + \frac{D}{R}(t_3 - M)e^{-RM} + \frac{D}{R}(T' - t_3)e^{-RT'}\right). \qquad (32)
\end{aligned}
$$

Case (iv): $\mathbf{t_1}$+T'$\leqslant$ $\mathbf{t_1}$+M $\leqslant$ T

As this is explained in Fig. 5, we deal here with the case, in which no interest is paid by the retailer. The retailer only earns interest on the sales revenue, generated by the selling of items as per demand from time t_1 till time $t_1 + M$. Additional interest is earned from the sale of the defective lot for the time period $(A_3, t_1 + M)$ and also from the shortages, which are backordered for the time period $(A_2, t_1 + M)$.

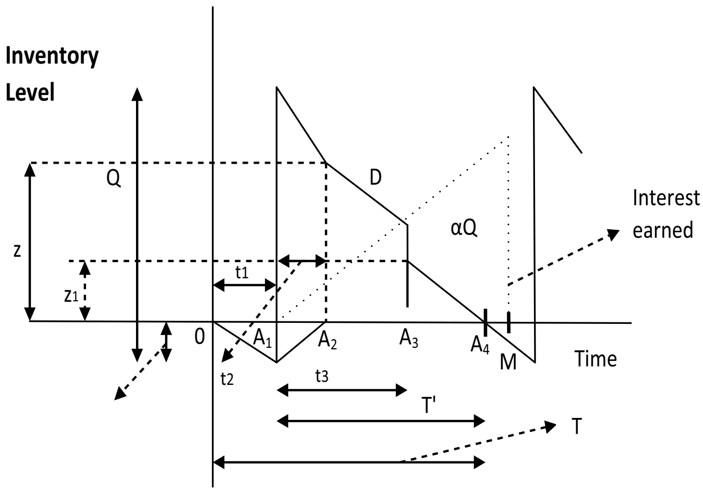

Figure 5. Inventory system for Case (iv): $t_1 + T' \leqslant t_1 + M \leqslant T$

7(iv) Present Worth of Interest Earned:

$$\begin{aligned}
C_{Ie4} &= P\left(t_1\right) I_e \left(\int_{t_1}^{t_1+t_2} \left[(1-\alpha)\lambda - D\right]\left(t-t_1\right) e^{-Rt} dt \right. \\
&\left. + \int_{t_1}^{t_1+T'} D\left(t-t_1\right) e^{-Rt} dt + \int_{t_1+T'}^{t_1+M} DT_1 e^{-Rt} + \int_{t_1+t_2}^{t_1+M} Be^{-Rt} dt \right) \\
&+ C_s\left(t_1\right) I_e \int_{t_1+t_3}^{t_1+M} \alpha Q e^{-Rt} dt \\
&= I_e P_o e^{-2Rt_1} \left(-\frac{(1-\alpha)\lambda - D}{R}\left(t_2 e^{-Rt_2}\right) - \frac{(1-\alpha)\lambda - D}{R^2} e^{-Rt_2} \right. \\
&\left. + \frac{(1-\alpha)\lambda}{R^2} - \frac{D}{R^2} e^{-RT'} + \frac{B}{R}\left(e^{-Rt_2} - e^{-RM}\right) - \frac{DT'}{R} e^{-RM} \right) \\
&+ \frac{C_s \alpha Q I_e e^{-2Rt_1}}{R} e^{-RM} - e^{-Rt_3}
\end{aligned} \tag{33}$$

8(iv) Present Worth of Interest Paid:

$$C_{Ip4} = 0. \tag{34}$$

Therefore, the present worth of retailer's profit function for **Case (iv)**, using equations (18), (19), (20), (21), (22), (23), (33) and (34) is given by

$$\pi_4(Q,B) = S_r - C_r - C_p - C_{scr} - C_s - C_h + C_{Ie4} - C_{Ip4}$$
$$= \frac{P_o D e^{-Rt_1}}{R}\left(e^{-Rt_1} - e^{-RT}\right) + C_s \alpha Q e^{-R(t_1+t_3)} - A_0 e^{-Rt_1} - C_0 e^{-Rt_1} Q$$
$$-Q\beta_0 e^{-Rt_1} - \frac{C_B D}{R^2}$$
$$\left(1 - e^{-Rt_1}(1+Rt_1)\right) - h_0 e^{-2Rt_1}\left(\frac{Q}{R}\left(1-e^{-Rt_2}\right) - \frac{(1-\alpha)\lambda}{R^2}\left(1-e^{-Rt_2}\right)\right.$$
$$+\frac{(1-\alpha)\lambda t_2 e^{-Rt_2}}{R} + \frac{z}{R}\left(e^{-Rt_2} - e^{-Rt_3}\right) + \frac{z_1}{R}\left(e^{-Rt_3} - e^{-RT'}\right)$$
$$\left.+\frac{D}{R^2}\left(e^{-RT'} - e^{-Rt_2}\right) + \frac{De^{-RT'}}{R}(T'-t_3) + \frac{De^{-Rt_3}}{R}(t_3-t_2)\right)$$
$$+I_e P_o e^{-2Rt_1}\left(-\frac{[(1-\alpha)\lambda - D]}{R}\left(t_2 e^{-Rt_2}\right) - \frac{(1-\alpha)\lambda - D}{R^2} e^{-Rt_2}\right.$$
$$+\frac{(1-\alpha)\lambda}{R^2} - \frac{D}{R^2} e^{-RT'} + \frac{B}{R}\left(e^{-Rt_2} - e^{-RM}\right)$$
$$\left.-\frac{DT'}{R} e^{-RM}\right) + \frac{C_s \alpha Q I_e e^{-2Rt_1}}{R}\left(e^{-RM} - e^{-Rt_3}\right). \qquad (35)$$

Case (v): $\mathbf{t_1+T' \leqslant t_1+M}$

This case is similar to **Case (iv)** and all of its expressions coincide with those for the previous case.

Hence, effectively we have four different cases for the present worth of retailer's total profit per cycle (Q,B), which can be expressed as:

$$\pi(Q,\, B) = \begin{cases} \pi_1(Q,B), & t_1 \leqslant t_1+M \leqslant t_1+t_2 \leqslant t_1+t_3 \leqslant t_1+T', \\ \pi_2(Q,B), & t_1+t_2 \leqslant t_1+M \leqslant t_1+t_3 \leqslant t_1+T', \\ \pi_3(Q,B), & t_1+t_2 \leqslant t_1+t_3 \leqslant t_1+M \leqslant t_1+T', \\ \pi_4(Q,B), & t_1+T' \leqslant t_1+M. \end{cases} \qquad (36)$$

Since all the parameters in the above profit function $\pi(Q,B)$ are deterministic, except for α, which is a random variable with p.d.f., $f(\alpha)$, hence, the case wise expected value of the total profit function per cycle is given by:

$$E[\pi(Q,\, B)] = E[\pi_1(Q,B)], \text{ for } t_1 \leqslant t_1+M \leqslant t_1+t_2 \leqslant t_1+t_3 \leqslant t_1+T', \qquad (37)$$

$$E[\pi(Q,\, B)] = E[\pi_2(Q,B)], \text{ for } t_1+t_2 \leqslant t_1+M \leqslant t_1+t_3 \leqslant t_1+T', \qquad (38)$$

$$E[\pi(Q,\, B)] = E[\pi_3(Q,B)], \text{ for } t_1+t_2 \leqslant t_1+t_3 \leqslant t_1+M \leqslant t_1+T', \qquad (39)$$

$$E[\pi(Q,\, B)] = E[\pi_4(Q,B)], \text{ for } t_1+T' \leqslant t_1+M, \qquad (40)$$

and the expected duration of the ordering cycle is: $E[T] = \frac{(1-E[\alpha])Q}{D}$.

Thus, by using the renewal-reward theorem, the expected value of present worth of the total profit per unit time for all the above cases can be calculated as:

$$E[\pi^T(Q,\,B)] = \begin{cases} E\left[\pi_1^T(Q,B)\right] = \frac{E[\pi_1(Q,B)]}{E[T]} = \frac{E[\pi_1(Q,B)]*D}{\{1-E[\alpha]\}*Q}, \\ E\left[\pi_2^T(Q,B)\right] = \frac{E[\pi_2(Q,B)]}{E[T]} = \frac{E[\pi_2(Q,B)]*D}{\{1-E[\alpha]\}*Q}, \\ E\left[\pi_3^T(Q,B)\right] = \frac{E[\pi_3(Q,B)]}{E[T]} = \frac{E[\pi_3(Q,B)]*D}{\{1-E[\alpha]\}*Q}, \\ E\left[\pi_4^T(Q,B)\right] = \frac{E[\pi_4(Q,B)]}{E[T]} = \frac{E[\pi_4(Q,B)]*D}{\{1-E[\alpha]\}*Q}. \end{cases} \tag{41}$$

Therefore,

$$\begin{aligned} E\left[\pi_1^T(Q,B)\right] &= \frac{P_o e^{-\frac{RB}{D}}}{R}\frac{D^2}{Q(1-E[\alpha])}\left(e^{-\frac{RB}{D}} - e^{-\frac{(1-E[\alpha])RQ}{D}}\right) \\ &+C_s D\frac{E[\alpha]}{(1-E[\alpha])}e^{-R\left(\frac{B}{D}+\frac{Q}{\lambda}\right)} - A_0\frac{D}{Q(1-E[\alpha])}e^{-\frac{RB}{D}} - C_0\frac{D}{(1-E[\alpha])}e^{-\frac{RB}{D}} \\ &-\beta_0\frac{D}{(1-E[\alpha])}e^{-\frac{RB}{D}} - \frac{C_B}{R^2}\frac{D^2}{Q(1-E[\alpha])}\left(1-e^{-\frac{RB}{D}}\left(1+\frac{RB}{D}\right)\right) \\ &-h_0 e^{-\frac{2RB}{D}}\left(\frac{D}{R(1-E[\alpha])}\left(1-e^{-\frac{RB}{(1-E(\alpha))\lambda-D}}\right) - \frac{\lambda D}{QR^2}\left(1-e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}\right)\right. \\ &+\frac{\lambda DB}{QR\{(1-E[\alpha])\lambda-D\}}e^{-\frac{RB}{(1-E[\alpha])\lambda-D}} \\ &+\frac{D}{QR(1-E[\alpha])}\left(Q-\frac{\lambda B(1-E[\alpha])}{(1-E[\alpha])\lambda-D}\right)\left(e^{-\frac{RB}{(1-E[\alpha])\lambda-D}} - e^{-\frac{RQ}{\lambda}}\right) \\ &+\frac{D}{QR(1-E[\alpha])}\left(Q-\frac{\lambda B(1-E[\alpha])}{(1-E[\alpha])\lambda-D} - \left(\frac{DQ}{\lambda} - \frac{BD}{(1-E[\alpha])\lambda-D}\right)\right. \\ &\left.-E[\alpha]Q\right)\left(e^{-\frac{RQ}{\lambda}} - e^{-\left[\frac{(1-E[\alpha])RQ}{D}-\frac{RB}{D}\right]}\right) \\ &+\frac{D^2}{QR^2(1-E[\alpha])}\left(e^{-\left[\frac{(1-E[\alpha])RQ}{D}-\frac{RB}{D}\right]} - e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}\right) \\ &+\frac{D}{R}e^{-\left[\frac{(1-E[\alpha])RQ}{D}-\frac{RB}{D}\right]}\left(e^{-\left[\frac{(1-E[\alpha])RQ}{D}-\frac{RB}{D}\right]} - \frac{Q}{\lambda}\right) \\ &\left.+e^{-\frac{RQ}{\lambda}}\frac{D^2}{QR(1-E[\alpha])}\left(\frac{RQ}{\lambda} - \frac{B}{(1-E[\alpha])\lambda-D}\right)\right) \\ &+I_e P_o e^{-\frac{2RB}{D}}\left(-\frac{\lambda DM}{QR}e^{-RM} - \frac{\lambda D}{QR^2}e^{-RM} + \frac{\lambda D}{QR^2}\right) \end{aligned}$$

$$
\begin{aligned}
&-I_pC_0e^{-2\frac{RB}{D}}\left[\frac{D}{R\left(1-E[\alpha]\right)}\left(e^{-RM}-e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}\right)\right.\\
&+\frac{\lambda D}{QR^2}\left(e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}-e^{-RM}\right)\\
&+\frac{\lambda D}{QR}\left(\frac{B}{\left(1-E\left[\alpha\right]\right)\lambda-D}e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}-Me^{-RM}\right)\\
&+\frac{D}{QR\left(1-E[\alpha]\right)}\left[Q-\frac{\lambda B\left(1-E[\alpha]\right)}{\left(1-E[\alpha]\right)\lambda-D}\right]\left(e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}-e^{-\frac{RQ}{\lambda}}\right)\\
&+\frac{D}{QR\left(1-E[\alpha]\right)}\left(Q-\frac{\lambda B\left(1-E[\alpha]\right)}{\left(1-E[\alpha]\right)\lambda-D}\right.\\
&\left.-\left(\frac{DQ}{\lambda}-\frac{BD}{\left(1-E[\alpha]\right)\lambda-D}\right)-E[\alpha]Q\right)\left(e^{-\frac{RQ}{\lambda}}-e^{-\left[\frac{(1-E[\alpha])RQ}{D}-\frac{RB}{D}\right]}\right)\\
&+\frac{D^2}{QR^2\left(1-E[\alpha]\right)}\left(e^{-\left[\frac{(1-E[\alpha])RQ}{D}-\frac{RB}{D}\right]}-e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}\right)\\
&+\frac{D^2}{QR\left(1-E[\alpha]\right)}e^{-\left[\frac{(1-E[\alpha])RQ}{D}-\frac{RB}{D}\right]}\left(e^{-\left[\frac{(1-E[\alpha])Q}{D}-\frac{B}{D}\right]}-\frac{Q}{\lambda}\right)\\
&\left.+\frac{D^2}{QR\left(1-E[\alpha]\right)}e^{-\frac{RQ}{\lambda}}\left(\frac{Q}{\lambda}-\frac{B}{\left(1-E\left[\alpha\right]\right)\lambda-D}\right)\right]\\
&-\frac{C_sDI_pe^{-2\frac{RB}{D}}}{R}\frac{E[\alpha]}{\left(1-E[\alpha]\right)}\left(e^{-RM}-e^{-\frac{RQ}{\lambda}}\right)
\end{aligned}
\tag{42}
$$

$$
\begin{aligned}
&E\left[\pi_2^T\left(Q,B\right)\right]=\\
&\frac{P_oe^{-\frac{RB}{D}}}{R}\frac{D^2}{Q\left(1-E[\alpha]\right)}\left(e^{-\frac{RB}{D}}-e^{-\frac{(1-E[\alpha])RQ}{D}}\right)\\
&+C_sD\frac{E[\alpha]}{\left(1-E[\alpha]\right)}e^{-R\left(\frac{B}{D}+\frac{Q}{\lambda}\right)}-A_0\frac{D}{Q\left(1-E[\alpha]\right)}e^{-\frac{RB}{D}}-C_0\frac{D}{\left(1-E[\alpha]\right)}e^{-\frac{RB}{D}}\\
&-\beta_0\frac{D}{\left(1-E[\alpha]\right)}e^{-\frac{RB}{D}}-\frac{C_B}{R^2}\frac{D^2}{Q\left(1-E[\alpha]\right)}\left[1-e^{-\frac{RB}{D}}\left(1+\frac{RB}{D}\right)\right]\\
&-h_0e^{-\frac{2RB}{D}}\left[\frac{D}{R\left(1-E[\alpha]\right)}\left(1-e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}\right)-\left(\frac{\lambda D}{QR^2}\left(1-e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}\right)\right.\right.\\
&+\frac{\lambda DB}{QR\left\{\left(1-E\left[\alpha\right]\right)\lambda-D\right\}}e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}\\
&+\frac{D}{QR\left(1-E[\alpha]\right)}\left\{Q-\frac{\lambda B\left(1-E\left[\alpha\right]\right)}{\left(1-E\left[\alpha\right]\right)\lambda-D}\right\}\left\{e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}-e^{-\frac{RQ}{\lambda}}\right\}
\end{aligned}
$$

$$+\frac{D}{QR\,(1-E[\alpha])}\left(Q-\frac{\lambda B\,(1-E[\alpha])}{(1-E[\alpha])\,\lambda-D}\right.$$
$$\left.-\left(\frac{DQ}{\lambda}-\frac{BD}{(1-E[\alpha])\,\lambda-D}\right)-E[\alpha]Q\right)\left(e^{-\frac{RQ}{\lambda}}-e^{-\left[\frac{(1-E[\alpha])RQ}{D}-\frac{RB}{D}\right]}\right)$$
$$+\frac{D^2}{QR^2\,(1-E[\alpha])}\left(e^{-\left[\frac{(1-E[\alpha])RQ}{D}-\frac{RB}{D}\right]}-e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}\right)$$
$$+\frac{D}{R}e^{-\left[\frac{(1-E[\alpha])RQ}{D}-\frac{RB}{D}\right]}\left(e^{-\left[\frac{(1-E[\alpha])RQ}{D}-\frac{RB}{D}\right]}-\frac{Q}{\lambda}\right)$$
$$\left.+e^{-\frac{RQ}{\lambda}}\frac{D^2}{QR\,(1-E[\alpha])}\left(\frac{RQ}{\lambda}-\frac{B}{(1-E\,[\alpha])\,\lambda-D}\right)\right]$$
$$+I_eP_oe^{-\frac{2RB}{D}}\left(-\frac{((1-E[\alpha])\,\lambda-D]}{R^2}\frac{D}{Q\,(1-E[\alpha])}\left((1+\right.\right.$$
$$\left.\left.\frac{RB}{(1-E[\alpha])\,\lambda-D}\right)e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}\right)$$
$$+\frac{\lambda D}{QR^2}e^{-\frac{RB}{D}}-\frac{D^2}{QR^2\,(1-E[\alpha])}\left[(1+RM)\,e^{-RM}\right]+$$
$$\left.\frac{BD}{QR\,(1-E[\alpha])}\left(e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}-e^{-RM}\right)\right)$$
$$-I_pC_0e^{-\frac{2RB}{D}}\left(\frac{D}{QR\,(1-E[\alpha])}\left\{Q-\frac{\lambda B\,(1-E[\alpha])}{(1-E[\alpha])\,\lambda-D}\right\}\left(e^{-RM}-e^{-\frac{RQ}{\lambda}}\right)+\right.$$
$$\frac{D}{QR\,(1-E[\alpha])}\left(Q-\frac{\lambda B\,(1-E[\alpha])}{(1-E[\alpha])\,\lambda-D}-\left(\frac{DQ}{\lambda}-\frac{BD}{(1-E[\alpha])\,\lambda-D}\right)\right.$$
$$\left.-E[\alpha]Q\right)\left(e^{-\frac{RQ}{\lambda}}-e^{-\left[\frac{(1-E[\alpha])RQ}{D}-\frac{RB}{D}\right]}\right)$$
$$+\frac{D^2}{QR^2\,(1-E[\alpha])}\left(e^{-\left[\frac{(1-E[\alpha])RQ}{D}-\frac{RB}{D}\right]}-e^{-RM}\right)$$
$$+\frac{D^2}{QR\,(1-E[\alpha])}\left(\frac{B}{(1-E[\alpha])\,\lambda-D}-M\right)e^{-RM}$$
$$+\frac{D^2}{QR\,(1-E[\alpha])}\left(\left(\frac{(1-E\,[\alpha])\,Q}{D}-\frac{B}{D}\right)-\frac{Q}{\lambda}\right)$$
$$\left.e^{-\left[\frac{(1-E[\alpha])RQ}{D}-\frac{RB}{D}\right]}+\frac{D^2}{QR\,(1-E[\alpha])}\left(\frac{Q}{\lambda}-\frac{B}{(1-E[\alpha])\,\lambda-D}\right)e^{-\frac{RQ}{\lambda}}\right)$$
$$-\frac{C_sDI_pe^{-2\frac{RB}{D}}}{R}\frac{E[\alpha]}{(1-E[\alpha])}\left(e^{-RM}-e^{-\frac{RQ}{\lambda}}\right)\tag{43}$$

$$
\begin{aligned}
&E\left[\pi_3^T(Q,B)\right]=\frac{P_o e^{-\frac{RB}{D}}}{R}\frac{D^2}{Q(1-E[\alpha])}\left(e^{-\frac{RB}{D}}-e^{-\frac{(1-E[\alpha])RQ}{D}}\right)\\
&+C_sD\frac{E[\alpha]}{(1-E[\alpha])}e^{-R\left(\frac{B}{D}+\frac{Q}{\lambda}\right)}-A_0\frac{D}{Q(1-E[\alpha])}e^{-\frac{RB}{D}}-C_0\frac{D}{(1-E[\alpha])}e^{-\frac{RB}{D}}\\
&-\beta_0\frac{D}{(1-E[\alpha])}e^{-\frac{RB}{D}}-\frac{C_B}{R^2}\frac{D^2}{Q(1-E[\alpha])}\left[1-e^{-\frac{RB}{D}}\left(1+\frac{RB}{D}\right)\right]\\
&-h_0e^{-\frac{2RB}{D}}\left[\frac{D}{R(1-E[\alpha])}\left(1-e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}\right)-\frac{\lambda D}{QR^2}\left(1-e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}\right)\right.\\
&+\frac{\lambda DB}{QR\{(1-E[\alpha])\lambda-D\}}e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}\\
&+\frac{D}{QR(1-E[\alpha])}\left(Q-\frac{\lambda B(1-E[\alpha])}{(1-E[\alpha])\lambda-D}\right)\left(e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}-e^{-\frac{RQ}{\lambda}}\right)\\
&+\frac{D}{QR(1-E[\alpha])}\left(Q-\frac{\lambda B(1-E[\alpha])}{(1-E[\alpha])\lambda-D}-\left(\frac{DQ}{\lambda}-\frac{BD}{(1-E[\alpha])\lambda-D}\right)\right.\\
&\left.-E[\alpha]Q\right)\left(e^{-\frac{RQ}{\lambda}}-e^{-\left[\frac{(1-E[\alpha])RQ}{D}-\frac{RB}{D}\right]}\right)\\
&+\frac{D^2}{QR^2(1-E[\alpha])}\left(e^{-\left[\frac{(1-E[\alpha])RQ}{D}-\frac{RB}{D}\right]}-e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}\right)\\
&+\frac{D}{R}e^{-\left[\frac{(1-E[\alpha])RQ}{D}-\frac{RB}{D}\right]}\left(e^{-\left[\frac{(1-E[\alpha])RQ}{D}-\frac{RB}{D}\right]}-\frac{Q}{\lambda}\right)\\
&\left.+e^{-\frac{RQ}{\lambda}}\frac{D^2}{QR(1-E[\alpha])}\left(\frac{RQ}{\lambda}-\frac{B}{(1-E[\alpha])\lambda-D}\right)\right]\\
&+I_eP_oe^{-\frac{2RB}{D}}\left\{-\frac{[(1-E[\alpha])\lambda-D]}{R^2}\frac{D}{Q(1-E[\alpha])}\left[\left(1+\right.\right.\right.\\
&\left.\left.\frac{RB}{(1-E[\alpha])\lambda-D}\right)e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}\right]+\frac{\lambda D}{QR^2}-\frac{D^2}{QR^2(1-E[\alpha])}\left[(1+RM)e^{-RM}\right]\\
&\left.+\frac{BD}{QR(1-E[\alpha])}\left(e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}-e^{-RM}\right)\right\}\\
&+\frac{C_sDI_ee^{-2\frac{RB}{D}}}{R}\frac{E[\alpha]}{(1-E[\alpha])}\left(e^{-RM}-e^{-\frac{RQ}{\lambda}}\right)\\
&-I_pC_0e^{-\frac{2RB}{D}}\left(\frac{D}{QR(1-E[\alpha])}\left(Q-\frac{\lambda B(1-E[\alpha])}{(1-E[\alpha])\lambda-D}-\left(\frac{DQ}{\lambda}-\frac{BD}{(1-E[\alpha])\lambda-D}\right)\right.\right.\\
&\left.-E[\alpha]Q\right)\left(e^{-RM}-e^{-\left[\frac{(1-E[\alpha])RQ}{D}-\frac{RB}{D}\right]}\right)+\frac{D^2}{QR^2(1-E[\alpha])}\left(e^{-\left[\frac{(1-E[\alpha])RQ}{D}-\frac{RB}{D}\right]}-e^{-RM}\right)\\
&+\frac{D^2}{QR(1-E[\alpha])}\left(\frac{Q}{\lambda}-M\right)e^{-RM}\\
&\left.+\frac{D^2}{QR(1-E[\alpha])}\left(\left(\frac{(1-E[\alpha])Q}{D}-\frac{B}{D}\right)-\frac{Q}{\lambda}\right)e^{-\left[\frac{(1-E[\alpha])RQ}{D}-\frac{RB}{D}\right]}\right) \qquad (44)
\end{aligned}
$$

$$
\begin{aligned}
&E\left[\pi_4^T(Q,B)\right] = \\
&\frac{P_o e^{-\frac{RB}{D}}}{R}\frac{D^2}{Q(1-E[\alpha])}\left(e^{-\frac{RB}{D}} - e^{-\frac{(1-E[\alpha])RQ}{D}}\right) + C_s D\frac{E[\alpha]}{(1-E[\alpha])}e^{-R\left(\frac{B}{D}+\frac{Q}{\lambda}\right)} \\
&-A_0\frac{D}{Q(1-E[\alpha])}e^{-\frac{RB}{D}} - C_0\frac{D}{(1-E[\alpha])}e^{-\frac{RB}{D}} - \beta_0\frac{D}{(1-E[\alpha])}e^{-\frac{RB}{D}} \\
&-\frac{C_B}{R^2}\frac{D^2}{Q(1-E[\alpha])}\left[1-e^{-\frac{RB}{D}}\left(1+\frac{RB}{D}\right)\right] \\
&-h_0 e^{-\frac{2RB}{D}}\left[\frac{D}{R(1-E[\alpha])}\left(1-e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}\right) - \frac{\lambda D}{QR^2}\left(1-e^{-\frac{RB}{(1-E[\alpha))\lambda-D}}\right)\right. \\
&+\frac{\lambda DB}{QR((1-E[\alpha])\lambda-D)}e^{-\frac{RB}{(1-E[\alpha])\lambda-D}} \\
&+\frac{D}{QR(1-E[\alpha])}\left(Q-\frac{\lambda B(1-E[\alpha])}{(1-E[\alpha])\lambda-D}\right)\left(e^{-\frac{RB}{(1-E[\alpha])\lambda-D}} - e^{-\frac{RQ}{\lambda}}\right) \\
&+\frac{D}{QR(1-E[\alpha])}\left\{Q-\frac{\lambda B(1-E[\alpha])}{(1-E[\alpha])\lambda-D} - \left(\frac{DQ}{\lambda} - \frac{BD}{(1-E[\alpha])\lambda-D}\right)\right. \\
&\left.-E[\alpha]Q\right\}\left(e^{-\frac{RQ}{\lambda}} - e^{-\left[\frac{(1-E[\alpha])RQ}{D}-\frac{RB}{D}\right]}\right) \\
&+\frac{D^2}{QR^2(1-E[\alpha])}\left(e^{-\left[\frac{(1-E[\alpha])RQ}{D}-\frac{RB}{D}\right]} - e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}\right) \\
&+\frac{D}{R}e^{-\left[\frac{(1-E[\alpha])RQ}{D}-\frac{RB}{D}\right]}\left(e^{-\left[\frac{(1-E[\alpha])RQ}{D}-\frac{RB}{D}\right]} - \frac{Q}{\lambda}\right) \\
&\left.+e^{-\frac{RQ}{\lambda}}\frac{D^2}{QR(1-E[\alpha])}\left(\frac{RQ}{\lambda} - \frac{B}{(1-E[\alpha])\lambda-D}\right)\right] \\
&+I_e P_o e^{-\frac{2RB}{D}}\left(-\frac{[(1-E[\alpha])\lambda-D]}{R}\frac{D}{Q(1-E[\alpha])}\left(\frac{B}{(1-E[\alpha])\lambda-D}e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}\right)\right. \\
&-\frac{[(1-E[\alpha])\lambda-D]}{R^2}\frac{D}{Q(1-E[\alpha])}e^{-\frac{RB}{(1-E[\alpha])\lambda-D}} \\
&+\frac{\lambda D}{QR^2} - \frac{D^2}{QR^2(1-E[\alpha])}e^{-\left[\frac{(1-E[\alpha])RQ}{D}-\frac{RB}{D}\right]} \\
&\left.+\frac{BD}{QR(1-E[\alpha])}\left(e^{-\frac{RB}{(1-E[\alpha])\lambda-D}} - e^{-RM}\right) - \frac{D}{R}\left(1-\frac{B}{Q(1-E[\alpha])}\right)e^{-RM}\right) \\
&+\frac{C_s D I_e e^{-\frac{2RB}{D}}}{R}\frac{E[\alpha]}{(1-E[\alpha])}\left(e^{-RM} - e^{-\frac{RQ}{\lambda}}\right). \qquad (45)
\end{aligned}
$$

Also, the present worth of the expected total profit at $T = M$ is given by:

$$
E\left[\pi^M(Q,B)\right] = \frac{E[\pi(Q,B)]}{E[M]} = \frac{E[\pi(Q,B)] * D}{(1-E[\alpha])Q}.
$$

Since

$$E\left[M\right]=\frac{\left(1-E[\alpha]\right)Q}{D},$$

then

$$\begin{aligned}
&E\left[\pi^{M}\left(Q,B\right)\right]=\frac{P_{o}e^{-\frac{RB}{D}}}{R}\frac{D^{2}}{Q\left(1-E[\alpha]\right)}\left(e^{-\frac{RB}{D}}-e^{-\frac{(1-E[\alpha])RQ}{D}}\right)\\
&+C_{s}D\frac{E[\alpha]}{(1-E[\alpha])}e^{-R\left(\frac{B}{D}+\frac{Q}{\lambda}\right)}-A_{0}\frac{D}{Q\left(1-E[\alpha]\right)}e^{-\frac{RB}{D}}-C_{0}\frac{D}{(1-E[\alpha])}e^{-\frac{RB}{D}}\\
&-\beta_{0}\frac{D}{(1-E[\alpha])}e^{-\frac{RB}{D}}-\frac{C_{B}}{R^{2}}\frac{D^{2}}{Q\left(1-E[\alpha]\right)}\left[1-e^{-\frac{RB}{D}}\left(1+\frac{RB}{D}\right)\right]\\
&-h_{0}e^{-\frac{2RB}{D}}\left[\frac{D}{R\left(1-E[\alpha]\right)}\left(1-e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}\right)-\frac{\lambda D}{QR^{2}}\left(1-e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}\right)\right.\\
&+\frac{\lambda DB}{QR\left\{\left(1-E\left[\alpha\right]\right)\lambda-D\right\}}e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}\\
&+\frac{D}{QR\left(1-E[\alpha]\right)}\left\{Q-\frac{\lambda B\left(1-E\left[\alpha\right]\right)}{\left(1-E\left[\alpha\right]\right)\lambda-D}\right\}\left\{e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}-e^{-\frac{RQ}{\lambda}}\right\}\\
&+\frac{D}{QR\left(1-E[\alpha]\right)}\left(Q-\frac{\lambda B\left(1-E[\alpha]\right)}{\left(1-E[\alpha]\right)\lambda-D}-\left(\frac{DQ}{\lambda}-\frac{BD}{\left(1-E[\alpha]\right)\lambda-D}\right)\right.\\
&\left.-E[\alpha]Q\right)\left(e^{-\frac{RQ}{\lambda}}-e^{-\left[\frac{(1-E[\alpha])RQ}{D}-\frac{RB}{D}\right]}\right)\\
&+\frac{D^{2}}{QR^{2}\left(1-E[\alpha]\right)}\left(e^{-\left[\frac{(1-E[\alpha])RQ}{D}-\frac{RB}{D}\right]}-e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}\right)\\
&+\frac{D}{R}e^{-\left[\frac{(1-E[\alpha])RQ}{D}-\frac{RB}{D}\right]}\left(e^{-\left[\frac{(1-E[\alpha])RQ}{D}-\frac{RB}{D}\right]}-\frac{Q}{\lambda}\right)\\
&\left.+e^{-\frac{RQ}{\lambda}}\frac{D^{2}}{QR\left(1-E[\alpha]\right)}\left(\frac{RQ}{\lambda}-\frac{B}{\left(1-E\left[\alpha\right]\right)\lambda-D}\right)\right]\\
&+I_{e}P_{o}e^{-\frac{2RB}{D}}\left(-\frac{\left[\left(1-E[\alpha]\right)\lambda-D\right]}{R}\frac{D}{Q\left(1-E[\alpha]\right)}\left(\frac{B}{\left(1-E[\alpha]\right)\lambda-D}e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}\right)\right.\\
&-\frac{\left[\left(1-E[\alpha]\right)\lambda-D\right]}{R^{2}}\frac{D}{Q\left(1-E[\alpha]\right)}e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}+\frac{\lambda D}{QR^{2}}\\
&-\frac{D^{2}}{QR^{2}\left(1-E[\alpha]\right)}e^{-\left[\frac{(1-E[\alpha])RQ}{D}-\frac{RB}{D}\right]}+\frac{BD}{QR\left(1-E[\alpha]\right)}\left(e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}-e^{-RM}\right)\\
&\left.-\frac{D}{R}\left(1-\frac{B}{Q\left(1-E[\alpha]\right)}\right)e^{-RM}\right)+\frac{C_{s}DI_{e}e^{-\frac{2RB}{D}}}{R}\frac{E[\alpha]}{(1-E[\alpha])}\left[e^{-RM}-e^{-\frac{RQ}{\lambda}}\right].
\end{aligned} \tag{46}$$

4. Optimality

In this model, in order to find the optimal values of Q and B that maximize the expected value of the present worth of total profit function, i.e. $E[\pi^T(Q,B)]$, the necessary conditions for optimality to be fulfilled are:

$$\frac{\partial E[\pi^T(Q,B)]}{\partial Q} = 0 \quad and \quad \frac{\partial E[\pi^T(Q,B)]}{\partial B} = 0,$$

this being equivalent, case wise, to the following sets of conditions, formulated for the consecutive four cases that we consider throughout the paper:

Case (i) $\mathbf{t_1 \leqslant t_1{+}M \leqslant t_1{+}t_2 \leqslant t_1{+}\ t_3 \leqslant t_1{+}T'}$

Let the values of Q and B which maximize $E\left[\pi_1^T(Q,B)\right]$, be Q_1 and B_1. These are obtained by solving

$$\frac{E\left[\pi_1^T(Q,B)\right]}{\partial Q} = 0 \text{ and } \frac{E\left[\pi_1^T(Q,B)\right]}{\partial B} = 0, \text{ respectively}$$

where

$$\begin{aligned}
&\frac{\partial E\left[\pi_1^T(Q,B)\right]}{\partial Q} = \\
&X_5 + \left(I_P C_0\left(1+\frac{RB}{(1-E[\alpha])\lambda - D}\right)\right)X_3 e^{-\frac{RB}{(1-E[\alpha])\lambda-D}} \\
&+(I_e P_0 - I_P C_0)X_3 MRe^{-RM} + (I_e P_0 + I_P C_0)X_3 e^{-RM} \\
&+\frac{I_P C_0 X_2\left(e^{-\frac{(1-E[\alpha])RQ}{D}} - e^{-\frac{RQ}{\lambda}}\right)}{\lambda RQ} + \frac{I_P C_0 X_2\left(e^{-\frac{RB}{(1-E[\alpha])\lambda-D}} - e^{-\frac{RQ}{\lambda}}\right)}{RDQ} \\
&+\frac{I_P C_0 X_2 e^{-\frac{RQ}{\lambda}}\left(\frac{Q}{\lambda} - \frac{B}{(1-E[\alpha])\lambda-D}\right)}{RQ^2} + \frac{I_P C_0 X_2 e^{-\frac{RQ}{\lambda}}\left(\frac{Q}{\lambda} - \frac{B}{(1-E[\alpha])\lambda-D}\right)}{\lambda Q} \\
&-\frac{I_P C_0 X_4\left(e^{-\frac{RB}{(1-E[\alpha])\lambda-D}} - e^{-\frac{RQ}{\lambda}}\right)}{RQ^2} + \frac{I_P C_0 X_4\left(e^{-\frac{RQ}{\lambda}} - e^{-\frac{(1-E[\alpha])RQ}{D}}\right)}{RQ^2} \\
&-\frac{I_P C_0 X_2 e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}}{R^2Q^2} + \frac{I_P C_0 X_2 e^{-\frac{(1-E[\alpha])RQ}{D}}\left(\frac{(1-E[\alpha])Q}{D} - \frac{Q}{\lambda}\right)}{RQ^2} - I_P X_1;
\end{aligned} \tag{47}$$

$$\frac{\partial E\left[\pi_1^T(Q,B)\right]}{\partial B}$$
$$= Y_5 - \frac{2I_eP_0e^{-\frac{2RB}{D}}\lambda}{RQ} + \frac{2I_eP_0e^{-\frac{2RB}{D}}\lambda M e^{-RM}}{Q}$$
$$+\frac{2I_eP_0e^{-\frac{2RB}{D}}\lambda e^{-RM}}{RQ} + 2I_PC_0Y_4\left(e^{-\frac{RQ}{\lambda}} - e^{-\frac{(1-E[\alpha])RQ}{D}}\right)$$
$$+\frac{2I_pC_0e^{-\frac{2RB}{D}}\left(e^{-RM} - e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}\right)}{1-E(\alpha)}$$
$$+\frac{2I_PC_0\lambda e^{-\frac{2RB}{D}}\left(\frac{Be^{-\frac{RB}{(1-E[\alpha])\lambda-D}}}{(1-E[\alpha])\lambda-D} - Me^{-RM}\right)}{Q} - \frac{I_pC_0Y_1Qe^{-\frac{RB}{(1-E[\alpha])\lambda-D}}}{((1-E(\alpha))\lambda-D)}$$
$$+2Y_3I_Pe^{-\frac{RB}{D}}\left(e^{-RM} - e^{-\frac{RQ}{\lambda}}\right) + \frac{2I_PC_0\lambda e^{-\frac{2RB}{D}}\left(e^{-\frac{RB}{(1-E[\alpha])\lambda-D}} - e^{-RM}\right)}{RQ}$$
$$+\frac{I_PC_0D\lambda Be^{-\frac{2RB}{D}}Re^{-\frac{RB}{(1-E[\alpha])\lambda-D}}}{RQ\left((1-E[\alpha])\lambda-D\right)\left((1-E(\alpha))\lambda-D\right)}$$
$$+\frac{2I_PC_0Y_1\left(e^{-\frac{(1-E[\alpha])RQ}{D}} - e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}\right)}{R}$$
$$+\frac{I_pC_0D\lambda e^{-\frac{2RB}{D}}\left(e^{-RM} - e^{-\frac{RQ}{\lambda}}\right)}{((1-E(\alpha))\lambda-D)RQ} - I_PC_0Y_2e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}$$
$$+2I_PC_0Y_1e^{-\frac{RQ}{\lambda}}\left(\frac{Q}{\lambda} - \frac{B}{(1-E[\alpha])\lambda-D}\right) + I_PC_0Y_2e^{-\frac{RQ}{\lambda}}$$
$$+2I_PC_0Y_4\left(e^{-\frac{RQ}{\lambda}} - e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}\right)$$
$$+2I_PC_0Y_1e^{-\frac{(1-E[\alpha])RQ}{D}}\left(\frac{(1-E[\alpha])Q}{D} - \frac{Q}{\lambda}\right). \tag{48}$$

All X_i $(i = 1,2,3,4,5)$ are explicitly provided in **APPENDIX A** and all Y_i $(i = 1,2,3,4,5)$ in **APPENDIX B**.

Case (ii) $\mathbf{t_1+t_2 \leqslant t_1+M \leqslant t_1+t_3 \leqslant t_1+T'}$

Let the values of Q and B, which maximize $E\left[\pi_2^T(Q,B)\right]$, be Q_2 and B_2. These are obtained by solving

$$\frac{E\left[\pi_2^T(Q,B)\right]}{\partial Q} = 0 \text{ and } \frac{E\left[\pi_2^T(Q,B)\right]}{\partial B} = 0, \text{ respectively}$$

where

$$\frac{\partial E\left[\pi_2^T(Q,B)\right]}{\partial Q}=$$
$$X_5+\frac{I_eP_0X_2\left[(1-E(\alpha))\lambda-D\right]\left(1+\frac{RB}{(1-E[\alpha])\lambda-D}\right)e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}}{RDQ^2}$$
$$-\frac{I_eP_0BX_2\left(e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}-e^{-RM}\right)}{RDQ^2}+\frac{I_eP_0X_2(1+RM)e^{-RM}}{R^2Q^2}$$
$$+\frac{I_PC_0X_2e^{-\frac{(1-E[\alpha])RQ}{D}}\left(\frac{(1-E[\alpha])Q}{D}-\frac{Q}{\lambda}\right)}{RQ^2}+\frac{I_PC_0X_2\left(e^{-\frac{RQ}{\lambda}}-e^{-RM}\right)}{RDQ}$$
$$+\frac{I_PC_0X_2e^{-\frac{RQ}{\lambda}}\left(\frac{Q}{\lambda}-\frac{B}{(1-E[\alpha])\lambda-D}\right)}{RQ^2}+\frac{I_PC_0X_2e^{-\frac{RQ}{\lambda}}\left(\frac{Q}{\lambda}-\frac{B}{(1-E[\alpha])\lambda-D}\right)}{\lambda Q}$$
$$+\frac{I_PC_0X_2\left(e^{-\frac{(1-E[\alpha])RQ}{D}}-e^{-\frac{RQ}{\lambda}}\right)}{\lambda RQ}-\frac{I_PC_0X_2e^{-RM}}{Q^2R^2}$$
$$+\frac{I_PC_0X_2e^{-RM}\left(\frac{B}{(1-E[\alpha])\lambda-D}-M\right)}{RQ^2}+\frac{I_PC_0X_4\left(e^{-RM}-e^{-\frac{(1-E[\alpha])RQ}{D}}\right)}{RQ^2}$$
$$-I_PX_1; \tag{49}$$

$$\frac{\partial E[\pi_2^T(Q,B)]}{\partial B}=Y_5-\frac{2I_eP_0BY_1\left(e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}-e^{-RM}\right)}{D}$$
$$+\frac{2I_eP_0Y_1(1+RM)e^{-RM}}{R}$$
$$+\frac{2I_eP_0Y_1\left[(1-E(\alpha))\lambda-D\right](1+\frac{RB}{(1-E[\alpha])\lambda-D})e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}}{RDQ}$$
$$+\frac{I_eP_0Y_1B}{(1-E[\alpha])\lambda-D}e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}$$
$$+I_PC_0Y_2e^{-\frac{RQ}{\lambda}}+2I_PC_0Y_4\left(e^{-RM}-e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}\right)$$
$$+2I_PC_0Y_1e^{-\frac{(1-E[\alpha])RQ}{D}}\left(\frac{(1-E[\alpha])Q}{D}-\frac{Q}{\lambda}\right)$$
$$+2I_PC_0Y_4\left(e^{-\frac{RQ}{\lambda}}-e^{-\frac{(1-E[\alpha])RQ}{D}}\right)$$
$$+2I_PY_3e^{-\frac{RB}{D}}\left(e^{-RM}-e^{-\frac{RQ}{\lambda}}\right)+2I_PC_0Y_1e^{-\frac{RQ}{\lambda}}\left(\frac{Q}{\lambda}-\frac{B}{(1-E[\alpha])\lambda-D}\right)$$

$$+\frac{2I_PC_0Y_1\left(e^{-\frac{(1-E[\alpha])RQ}{D}}-e^{-RM}\right)}{R}+\frac{I_pC_0D\lambda e^{-\frac{2RB}{D}}\left(e^{-RM}-e^{-\frac{RQ}{\lambda}}\right)}{\left(\left(1-E\left(\alpha\right)\right)\lambda-D\right)RQ}$$

$$+\frac{I_eP_0Y_1\left(e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}-e^{-RM}\right)}{R}-\frac{I_eP_0BY_1e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}}{\left(1-E\left(\alpha\right)\right)\lambda-D}$$

$$-I_PC_0Y_2e^{-RM}+2I_PC_0Y_1e^{-RM}\left(\frac{B}{\left(1-E[\alpha]\right)\lambda-D}-M\right). \tag{50}$$

All $X_i\,(i=1,2,3,4,5)$ are explicitly calculated in **APPENDIX A** and all of $Y_i\,(i=1,2,3,4,5)$ in **APPENDIX B**.

Case (iii) $\mathbf{t_1+t_2 \leqslant t_1+t_3 \leqslant t_1+M \leqslant t_1+T'}$

Let the values of Q and B, which maximize $E\left[\pi_3^T\left(Q,B\right)\right]$, be Q_3 and B_3. These are obtained by solving

$$\frac{E\left[\pi_3^T\left(Q,B\right)\right]}{\partial Q}=0 \text{ and } \frac{E\left[\pi_3^T\left(Q,B\right)\right]}{\partial B}=0,\text{ respectively}$$

where

$$\frac{\partial E\left[\pi_3^T\left(Q,B\right)\right]}{\partial Q}=X_5$$

$$+\frac{I_eP_0X_2\left[\left(1-E\left(\alpha\right)\right)\lambda-D\right]\left(1+\frac{RB}{(1-E[\alpha])\lambda-D}\right)e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}}{RDQ^2}$$

$$+\frac{I_eP_0X_2\left(1+RM\right)e^{-RM}}{R^2Q^2}-\frac{I_eP_0BX_2\left(e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}-e^{-RM}\right)}{RDQ^2}-I_eX_1$$

$$+\frac{I_PC_0X_2e^{-\frac{(1-E[\alpha])RQ}{D}}\left(\frac{(1-E[\alpha])Q}{D}-\frac{Q}{\lambda}\right)}{RQ^2}+\frac{I_PC_0X_2\left(e^{-\frac{(1-E[\alpha])RQ}{D}}-e^{-RM}\right)}{\lambda RQ}$$

$$+\frac{I_PC_0X_4\left(e^{-RM}-e^{-\frac{(1-E[\alpha])RQ}{D}}\right)}{RQ^2}+\frac{I_PC_0X_2e^{-RM}\left(\frac{Q}{\lambda}-\frac{B}{(1-E[\alpha])\lambda-D}\right)}{RQ^2}$$

$$-\frac{I_PC_0X_2e^{-RM}}{R^2Q^2}; \tag{51}$$

$$\frac{\partial E\left[\pi_3^T\left(Q,B\right)\right]}{\partial B}=Y_5-2I_eY_3e^{-\frac{RB}{D}}\left(e^{-RM}-e^{-\frac{RQ}{\lambda}}\right)-\frac{2I_eP_0e^{-\frac{RB}{D}}\lambda}{RQ}$$

$$+\frac{2I_eP_0Y_1\left[\left(1-E\left(\alpha\right)\right)\lambda-D\right]\left(1+\frac{RB}{(1-E[\alpha])\lambda-D}\right)e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}}{RDQ}$$

$$+\frac{I_eP_0Y_1Be^{-\frac{RB}{(1-E[\alpha])\lambda-D}}}{(1-E[\alpha])\,\lambda-D}-\frac{2I_eP_0BY_1\left(e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}-e^{-RM}\right)}{D}$$
$$+\frac{2I_eP_0Y_1\,(1+RM)\,e^{-RM}}{R}+\frac{I_eP_0Y_1\left(e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}-e^{-RM}\right)}{R}$$
$$-\frac{I_eP_0BY_1e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}}{(1-E\,(\alpha))\,\lambda-D}+\frac{2I_PC_0Y_1\left(e^{-\frac{(1-E[\alpha])RQ}{D}}-e^{-RM}\right)}{R}$$
$$+2I_PC_0Y_4\left(e^{-RM}-e^{-\frac{(1-E[\alpha])RQ}{D}}\right)+2I_PC_0Y_1e^{-RM}\left(\frac{Q}{\lambda}-M\right)$$
$$+2I_PC_0Y_1e^{-\frac{(1-E[\alpha])RQ}{D}}\left(\frac{(1-E\,[\alpha])\,Q}{D}-\frac{Q}{\lambda}\right). \quad (52)$$

All X_i $(i=1,2,3,4,5)$ are explicitly demonstrated in **APPENDIX A** and all of Y_i $(i=1,2,3,4,5)$ in **APPENDIX B**.

Case (iv) t_1+T'$\leqslant$ t_1+M $\leqslant$ T

Let the values of Q and B, which maximize $E\left[\pi_4^T\,(Q,B)\right]$ be Q_4 and B_4. These are obtained by solving

$$\frac{E\left[\pi_4^T\,(Q,B)\right]}{\partial Q}=0 \text{ and } \frac{E\left[\pi_4^T\,(Q,B)\right]}{\partial B}=0, \text{ respectively}$$

where,

$$\frac{\partial E[\pi_4^T\,(Q,B)]}{\partial Q}=X_5+\frac{I_eP_0BX_2e^{-RM}}{RDQ^2}+\frac{I_eP_0\,(1-E\,[\alpha])\,QX_2e^{-RM}}{DRQ^2}$$
$$+\frac{I_eP_0X_2\,[(1-E\,(\alpha))\,\lambda-D]\,(1+\frac{RB}{(1-E[\alpha])\lambda-D})e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}}{RDQ^2}-I_eX_1 \quad (53)$$

$$\frac{E\left[\pi_4^T\,(Q,B)\right]}{\partial B}=Y_5+\frac{2I_eP_0BY_1e^{-RM}}{D}-\frac{2I_eP_0e^{-\frac{2RB}{D}}\lambda}{RQ}$$
$$+\frac{2I_eP_0Y_1\,[(1-E\,(\alpha))\,\lambda-D]\,e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}}{RD}-2I_eY_3e^{-\frac{RB}{D}}\left(e^{-RM}-e^{-\frac{RQ}{\lambda}}\right)$$
$$+\frac{2I_eP_0Y_1e^{-\frac{(1-E[\alpha])RQ}{D}}}{R}+\frac{I_eP_0Y_1\left(e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}-e^{-RM}\right)}{R}$$
$$+\frac{2I_eP_0Y_1\,(1-E\,[\alpha])\,RQe^{-RM}}{D}. \quad (54)$$

All of X_i $(i=1,2,3,4,5)$ are explicitly calculated in **APPENDIX A**, and all of Y_i $(i=1,2,3,4,5)$ in **APPENDIX B**.

To prove the concavity of the expected profit function, the fulfillment of the following sufficient conditions must be established:

$$\left(\frac{\partial^2 E[\pi^T(Q,B)]}{\partial Q \partial B}\right)^2 - \left(\frac{\partial^2 E[\pi^T(Q,B)]}{\partial Q^2}\right)\left(\frac{\partial^2 E[\pi^T(Q,B)]}{\partial B^2}\right) \leqslant 0$$

and

$$\frac{\partial^2 E[\pi^T(Q,B)]}{\partial Q^2} \leqslant 0 \ , \ \frac{\partial^2 E[\pi^T(Q,B)]}{\partial B^2} \leqslant 0.$$

All the second order derivatives have been calculated as this is shown in **APPENDIX C.** Due to the complexity of these derivatives, it becomes difficult to prove the concavity mathematically; so the concavity of all the expected profit functions have been established graphically, as this is shown in Fig. 6.

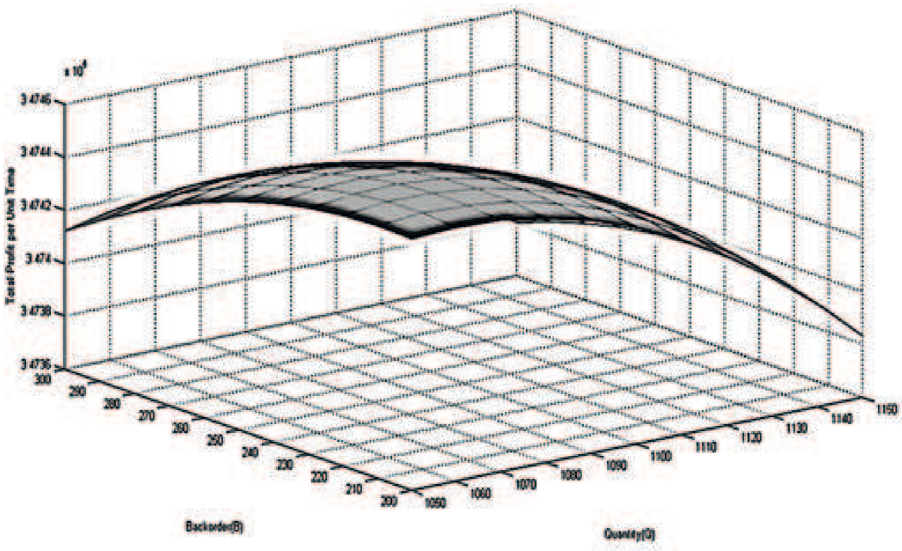

Figure 6. Concavity of the expected total profit function

5. The algorithm

In order to find the optimal values of Q and B, which maximize the expected total profit function, $E[\pi^T(Q,B)]$, the following algorithm is followed.

Step 1: The optimal values of $Q = Q_1$ (*say*) and $B = B_1$ (*say*) are determined by solving the equations (47) and (48) simultaneously. Using these, the values of t_1, t_2, t_3 and T are calculated from equations (14), (15), (16) and (17). If $t_1 \leqslant M \leqslant t_2 \leqslant t_3 \leqslant T'$, the value of the expected total profit, which is optimal, is given by equation (37), else go to Step 2.

Step 2: The values of $Q^* = Q_2\,(say)$ and $B^* = B_2\,(say)$ are determined from equations in (49) and (50). Using these, the values of t_1, t_2, t_3, and T are calculated from equations (14), (15), (16) and (17). If $t_2 \leqslant M \leqslant t_3 \leqslant T'$, the value of the expected total profit, which is optimal, is given by equation (38); else go to Step 3.

Step 3: The values of $Q^* = Q_3\,(say)$ and $B^* = B_3\,(say)$ are determined from equations in (51) and (52). Using these, the values of t_1, t_2, t_3 , and T are calculated from equations(14), (15), (16) and (17). If $t_2 \leqslant t_3 \leqslant M \leqslant T'$, the value of the expected total profit, which is optimal, is given by equation (39); else go to Step 4.

Step 4: The values of $Q^* = Q_4\,(say)$ and $B^* = B_4\,(say)$ are determined from equations in (53) and (54). Using these, the values of t_1, t_2, t_3 and T are calculated from equations (14), (15), (16) and (17). If $T' \leqslant M$, the value of the expected total profit, which is optimal, is given by equation (40); else go to Step 5.

Step 5: The value of the expected total profit, which is optimal, is calculated at $T = M$ from equation (46).

6. Numerical example

To validate the model, a numerical example with the following data has been used:
$A = Rs\,400$ per order; $D = 15,000$ units per year; $h_o = Rs\,4$ per unit; $C_2 = Rs\,6$ per unit per year; $\lambda = 60,000$ units per year; $\beta_0 = Rs$ 1 per unit; $C_0 = Rs\,35$ per unit; $P_0 = Rs\,60$ per unit; $C_S = Rs\,25$ per unit; $M = 0.0273$ years; $I_e = 0.08$ per Rs per year; $I_p = 0.14$ per Rs per year; $R = 0.06; \alpha = 0.05$.

Using the proposed algorithm, the optimal results obtained are as follows: $Q_3* =$ 1,053 and $B_3* =$ 215 units, hence $E\left[\pi_3^T(Q*,\, B*)\right] =$ 3,41,486/-.

7. Sensitivity analysis

The change in the values of parameters may happen due to uncertainties in any decision-making situation, therefore, sensitivity analysis has also been performed in order to analyze the impact of permissible delay (M), interest earned (I_e), interest payable (I_p), inflation and time value of money (R) and expected number of imperfect quality items (α) on the lot size ($Q*$) and backorders ($B*$) and the retailers expected total profit per unit time, $E[\pi^T(Q*, B*)]$.

Results are summarized in Tables 2-6 using the data of the above outlined numerical example.

Based on the above computational results, we can formulate the observations and insights, which are provided in the consecutive section.

Table 2. Impact of M on the optimal replenishment policy, taking $R = 0.06$, $\alpha = 0.05$, $I_P = 0.14$, I_e =0.08 as constant.

M (in days)	**T (in days)**	**Q***	**B***	**Total Cost**	**E[$\pi^{\mathbf{T}}$(Q*, B*)]**
10	24.3	1,053	215	38,417	3,41,486
20	24.4	1055	212	38,449	3,43,638
30	24.4	1058	210	38,589	3,45,770
40	24.5	1,063	208	38,833	3,47,883

Table 3. Impact of R on the optimal replenishment policy, taking $M = 10$ days, $\alpha = 0.05$, $I_P = 0.14$, I_e =0.08 as constant.

R	**T (in days)**	**Q***	**B***	**Total Cost**	**E[$\pi^{\mathbf{T}}$(Q*, B*)]**
0.05	25.4	1,100	273	40,098	3,41,996
0.07	23.4	1,015	163	37,041	3,41,033
0.09	22.1	957	71	34,965	3,40,277
0.10	21.6	935	29	34,184	3,39,964

Table 4. Impact of I_p on the optimal replenishment policy, taking $R = 0.06$, $M = 10$ days, $\alpha = 0.05$, I_e =0.08 as constant.

Ip	**T (in days)**	**Q***	**B***	**Total Cost**	**E[$\pi^{\mathbf{T}}$(Q*, B*)]**
0.12	24.7	1,069	203	39,005	3,41,529
0.13	24.5	1,061	210	38,700	3,41,507
0.14	24.3	1,053	215	38,417	3,41,486
0.15	24.1	1,046	221	38,148	3,41,467

Table 5. Impact of I_e on the optimal replenishment policy, taking $R = 0.06$, $M = 10$ days, $\alpha = 0.05$, I_p =0.14 as constant.

Ie	**T (in days)**	**Q***	**B***	**Total Cost**	**E[$\pi^{\mathbf{T}}$(Q*, B*)]**
0.07	24.4	1,058	205	38,590	3,41,389
0.08	24.3	1,053	215	38,417	3,41,486
0.09	24.2	1,049	226	38,230	3,41,586
0.10	24.4	1,043	235	38,035	3,41,688

Table 6. Impact of α on the optimal replenishment policy, taking $R = 0.06$, $M = 15$ days , I_p =0.14,I_e =0.08 as constant.

α	**T (in days)**	**Q***	**B***	**Total Cost**	**E[π^T(Q*, B*)]**
0.01	24.6	1,024	219	37,367	3,48,513
0.08	24.0	1,076	212	39,218	3,35,807
0.10	23.8	1,091	210	39,759	3,31,807
0.20	22.7	1,168	198	42,518	3,08,746

8. Observations

Characteristics of the optimal solutions

- It is evident from **Table 1** that upon increasing M, the order quantity (Q) increases along with the expected total profit per unit time E[$\pi^T(Q*, B*)$], while the number of shortages (B) decreases.
- **Table 2** implies that when the net discount rate of inflation (R) increases (i.e. inflation rate (i) is decreasing), the optimal order quantity (Q), the shortages (B) and the expected total profit per unit time $E[\pi^T(Q*, B*)]$ decrease.
- From **Table 3** it is clear that the total profit per unit time $E[\pi^T(Q*, B*)]$ decreases along with the order quantity (Q), while the shortages (B) increase, with increase in interest payable rates (I_p).
- **Table 4** suggests that as (I_e) increases, (Q) decreases, but shortages (B) increase along with the total profit per unit time $E[\pi^T(Q*, B*)]$.
- In **Table 5** it is shown that the order quantity (Q) increases, but the expected total profit E[$\pi^T(Q*, B*)$] and the number of shortages (B) decrease with the increase of (α).

Managerial insights

- The larger the delay in payment (within the credit limit), the longer is the duration of revenue to be put in an interest bearing account by the retailer. Hence, this boosts up his profit value considerably. To make use of the trade credit policy in a competent way, the retailer orders more, which also helps him to satisfy the shortages in an efficient way. In this manner, the policy also becomes cost effective as the inventory holding cost gets reduced by the fast replenishment of items. This clearly suggests that permissible delay in payment is economically favorable for both the supplier and the retailer, as it increases the sales for both of them.
- Analysis regarding R shows that the influence of inflation should be considered even if it is small. The rise in prices decreases the purchasing power of people; hence, the retailer needs to order less, as otherwise unwanted increase of inventory charges on the retailer may take place. The inflated prices prohibit the retailer to achieve maximum sales, so profit

values show decreasing trend. In order to achieve higher sales during inflationary conditions, the retailer should order less with higher frequency.

- When interest payable rate is increasing, total costs get elevated as interest charges are added to the costs incurred, leading to declining trend of profit values. So as to minimize the losses/costs, the retailer should order less, but more frequently. The lowering of order quantity also leads to the increase of shortages.
- As interest earning rate rises, the retailer should prefer to order less but more frequently, so as to make the maximum use of higher interest rates. Shortages are showing increasing trend on increase of (I_e) and therefore contributing to more revenue from the very beginning of the cycle. So, it is evident that profit increases upon the raise of interest earning rates (I_e).
- When the percentage of defective items increases, the retailer should look into the source of imperfection from the received lot by taking appropriate measures, such as defect tracking, and analyzing complaint reasons. Since, with increasing number of defectives, the count of perfect items decreases, so a larger order quantity is needed to satisfy the demand. Even though resolving quality control issues does come at a cost, in the long run it helps the firm to grow and build a good customer base.

9. Conclusion

In the present paper, an inventory model for imperfect quality items for determining the optimal order quantity and shortages has been investigated in the presence of trade credit and allowable shortages. Since the financial decisions can mislead and undermine the performance due to the uncertainty attached to the future price values, therefore, the effects of inflation and time value of money are also incorporated while determining optimal ordering policies. An algorithm has been employed which jointly optimizes the order quantity and the backorder level, resulting in maximization of profit. Numerical example, along with the sensitivity analysis, has been presented to validate the model. One of the most significant conclusions of the model is the combined effect of trade credit and inflation on retailer's sales and revenue. This suggests for the operational manager to order more under inflationary conditions and also make use of the trade credit more frequently. Moreover, the findings suggest to the retailer to take corrective measures to reduce the defective fraction in the ordered lot, so as to achieve customer satisfaction and higher profits.

For future research, it would be interesting to study the present model under different practical parameters, like partially backlogged shortages, deterioration, and single or two warehousing, fuzzy modeling etc. Another potential direction may be taken by integrating different forms of trade credit decisions, as implied by the present study. The research may be extended over different demand functions viz., price and time dependent demand, stock dependent demand or both.

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APPENDIX A

$$X_1 = \frac{C_s E(\alpha) D e^{-\frac{RB}{D}} e^{-\frac{RQ}{\lambda}}}{(1-E(\alpha))\lambda};$$

$$X_2 = \frac{D^2 e^{-\frac{2RB}{D}}}{(1-E(\alpha))};$$

$$X_3 = \frac{D\lambda e^{-\frac{2RB}{D}}}{R^2Q^2};$$

$$X_4 = \frac{De^{-\frac{2RB}{D}}\left(Q - \frac{B(1-E(\alpha))\lambda}{(1-E[\alpha])\lambda - D}\right)}{(1-E(\alpha))};$$

$$\begin{aligned}
X_5 = {} & \frac{A_0 D e^{-\frac{RB}{D}}}{Q^2(1-E(\alpha))} + \frac{P_0 D e^{-\frac{2RB}{D}} e^{-\frac{(1-E[\alpha])RQ}{D}}}{Q} - \frac{P_0 X_2\left(1-e^{-\frac{(1-E[\alpha])RQ}{D}}\right)}{RQ^2} \\
& + \frac{C_B X_2\left(e^{\frac{2RB}{D}} - e^{\frac{RB}{D}}\left(1+\frac{RB}{D}\right)\right)}{R^2Q^2} + h_0 X_3\left(1 - e^{-\frac{RB}{(1-E[\alpha])\lambda - D}}\right) \\
& + \frac{h_0 X_4\left(e^{-\frac{RB}{(1-E[\alpha])\lambda-D}} - e^{-\frac{(1-E[\alpha])RQ}{D}}\right)}{RQ^2} + \frac{h_0 BRX_3 e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}}{[(1-E(\alpha))\lambda - D]} \\
& + \frac{h_0 X_2 (e^{-\frac{(1-E[\alpha])RQ}{D}} - e^{-\frac{RB}{(1-E[\alpha])\lambda-D}})}{Q^2R^2} + \frac{h_0 X_2 e^{-\frac{(1-E[\alpha])RQ}{D}}\left(\frac{(1-E[\alpha])Q}{D} - \frac{Q}{\lambda}\right)}{RQ^2} \\
& + \frac{h_0 X_2\left(e^{-\frac{(1-E[\alpha])RQ}{D}} - e^{-\frac{RQ}{\lambda}}\right)}{\lambda RQ} - \frac{h_0 X_2\left(e^{-\frac{RB}{(1-E[\alpha])\lambda-D}} - e^{-\frac{RQ}{\lambda}}\right)}{RDQ} \\
& + \frac{h_0 X_2 e^{-\frac{RQ}{\lambda}}\left(\frac{Q}{\lambda} - \frac{B}{(1-E[\alpha])\lambda-D}\right)}{RQ^2} + \frac{h_0 X_2 e^{-\frac{RQ}{\lambda}}\left(\frac{Q}{\lambda} - \frac{B}{(1-E[\alpha])\lambda-D}\right)}{\lambda Q} \\
& - I_e P_0 X_3 + \frac{I_P C_0 X_2 e^{-\frac{(1-E[\alpha])RQ}{D}}}{R^2Q^2} - RX_1.
\end{aligned}$$

APPENDIX B

$$Y_1 = \frac{De^{-\frac{2RB}{D}}}{Q\left(1-E\left(\alpha\right)\right)};$$

$$Y_2 = \frac{D^2 e^{-\frac{2RB}{D}}}{R\left(\left(1-E\left(\alpha\right)\right)\lambda - D\right)Q\left(1-E\left(\alpha\right)\right)};$$

$$Y_3 = \frac{C_S E\left(\alpha\right)e^{-\frac{RB}{D}}}{1-E\left(\alpha\right)};$$

$$Y_4 = \frac{\left(Q - \frac{B(1-E(\alpha))\lambda}{(1-E[\alpha])\lambda - D}\right)e^{-\frac{2RB}{D}}}{Q\left(1-E\left(\alpha\right)\right)};$$

$$\begin{aligned}
Y_5 =& \left(C_0 + \beta_0 + \frac{A_0}{Q}\right)\frac{Re^{-\frac{RB}{D}}}{1-E\left(\alpha\right)} - RY_3 e^{-\frac{RQ}{\lambda}} - 2P_0 Y_1\left(1 - e^{-\frac{(1-E[\alpha])RQ}{D}}\right) \\
&- \frac{C_B B Y_1}{D}e^{\frac{RB}{D}} + 2h_0 Y_4\left(e^{-\frac{RB}{(1-E[\alpha])\lambda - D}} - e^{-\frac{RQ}{\lambda}}\right) \\
&+ \frac{2h_0 Y_1\left(e^{-RT} - e^{-\frac{RB}{(1-E[\alpha])\lambda - D}}\right)}{R} \\
&+ 2h_0 Y_1 e^{-\frac{(1-E[\alpha])RQ}{D}}\left(\frac{\left(1-E\left[\alpha\right]\right)Q}{D} - \frac{Q}{\lambda}\right) \\
&+ 2h_0 Y_1 e^{-\frac{RQ}{\lambda}}\left(\frac{Q}{\lambda} - \frac{B}{\left(1-E[\alpha]\right)\lambda - D}\right) \\
&+ \frac{2h_0\lambda Be^{-\frac{2RB}{D}}e^{-\frac{RB}{(1-E[\alpha])\lambda - D}}}{\left(\left(1-E\left(\alpha\right)\right)\lambda - D\right)Q} + \frac{2h_0 e^{-\frac{2RB}{D}}\left(1 - e^{-\frac{RB}{(1-E[\alpha])\lambda - D}}\right)}{1-E\left(\alpha\right)} \\
&- \frac{2h_0\lambda e^{-\frac{2RB}{D}}\left(1 - e^{-\frac{RB}{(1-E[\alpha])\lambda - D}}\right)}{RQ} - h_0 Y_2 e^{-\frac{RB}{(1-E[\alpha])\lambda - D}} \\
&- \frac{h_0 Q Y_1 e^{-\frac{RB}{(1-E[\alpha])\lambda - D}}}{\left(\left(1-E\left(\alpha\right)\right)\lambda - D\right)} \\
&+ \frac{h_0 D Y_4 e^{-\frac{RB}{(1-E[\alpha])\lambda - D}}}{\left(\left(1-E\left(\alpha\right)\right)\lambda - D\right)} + 2h_0 Y_4\left(e^{-\frac{RQ}{\lambda}} - e^{-\frac{(1-E[\alpha])RQ}{D}}\right) + h_0 Y_2 e^{-\frac{RQ}{\lambda}} \\
&+ \frac{h_0 D\lambda Be^{-\frac{2RB}{D}}e^{-\frac{RB}{(1-E[\alpha])\lambda - D}}}{\left(\left(1-E\left(\alpha\right)\right)\lambda - D\right)^2 D} + \frac{h_0 D\lambda e^{-\frac{2RB}{D}}\left(e^{-\frac{RB}{(1-E[\alpha])\lambda - D}} - e^{-\frac{RQ}{\lambda}}\right)}{\left(\left(1-E\left(\alpha\right)\right)\lambda - D\right)RQ}.
\end{aligned}$$

APPENDIX C

We provide here the second derivatives of the expected total profit functions with respect to Q for all four cases:

Case (i) $\mathbf{t_1 \leqslant t_1 + M \leqslant t_1 + t_2 \leqslant t_1 + t_3 \leqslant t_1 + T'}$

$$
\begin{aligned}
&\frac{\partial^2 E[\pi_1^T(Q,B)]}{\partial Q^2} = \\
&V_3 - 2I_eP_0V_2RMe^{-RM} - 2I_eP_0V_2e^{-RM} \\
&+I_PRV_1e^{-\frac{RB}{D}} - 2I_PC_0RV_2\left(\frac{Be^{-\frac{RB}{(1-E[\alpha])\lambda-D}}}{(1-E[\alpha])\lambda - D} - Me^{-RM}\right) \\
&-\frac{I_PC_0X_4Re^{-\frac{RQ}{\lambda}}}{\lambda^2 Q} - 2I_PC_0V_2\left(e^{-\frac{RB}{(1-E[\alpha])\lambda-D}} - e^{-RM}\right) \\
&-\frac{2I_PC_0X_2\left(e^{-\frac{(1-E(\alpha))RQ}{D}} - e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}\right)}{R^2Q^3} + \frac{2I_PC_0X_2e^{-\frac{RQ}{\lambda}}}{\lambda RQ^2} \\
&+\frac{2I_PC_0X_2e^{-\frac{RQ}{\lambda}}}{\lambda^2 Q} - \frac{2I_PC_0X_2e^{-\frac{RQ}{\lambda}}\left(\frac{Q}{\lambda} - \frac{B}{(1-E[\alpha])\lambda-D}\right)}{RQ^3} \\
&-\frac{2I_PC_0X_2e^{-\frac{RQ}{\lambda}}\left(\frac{Q}{\lambda} - \frac{B}{(1-E[\alpha])\lambda-D}\right)}{\lambda Q^2} \\
&-\frac{I_PC_0X_2e^{-\frac{RQ}{\lambda}}\left(\frac{Q}{\lambda} - \frac{B}{(1-E[\alpha])\lambda-D}\right)}{\lambda^2 Q} \\
&-\frac{2I_PC_0X_2e^{-\frac{(1-E[\alpha])RQ}{D}}\left(\frac{(1-E[\alpha])Q}{D} - \frac{Q}{\lambda}\right)}{RQ^3} \\
&-\frac{2I_PC_0X_2e^{-\frac{(1-E[\alpha])RQ}{D}}}{\lambda RQ^2} + \frac{2I_PC_0X_2\left(e^{-\frac{RQ}{\lambda}} - e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}\right)}{RDQ^2} \\
&+\frac{2I_PC_0X_2e^{-\frac{RQ}{\lambda}}}{\lambda DQ} - \frac{2I_PC_0X_4\left(e^{-\frac{RQ}{\lambda}} - e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}\right)}{RQ^3} \\
&-\frac{2I_PC_0X_4e^{-\frac{RQ}{\lambda}}}{\lambda Q^2} - \frac{I_PC_0X_4e^{-\frac{RQ}{\lambda}}}{\lambda^2 Q} \\
&-\frac{2I_PC_0X_4\left(e^{-\frac{RQ}{\lambda}} - e^{-\frac{(1-E[\alpha])RQ}{D}}\right)}{RQ^3} - \frac{2I_PC_0X_4e^{-\frac{RQ}{\lambda}}}{\lambda Q^2}.
\end{aligned}
$$

Case (ii) $\mathbf{t_1+t_2 \leqslant t_1+M \leqslant t_1+t_3 \leqslant t_1+T'}$

$$\begin{aligned}
&\frac{\partial^2 E[\pi_2^T(Q,B)]}{\partial Q^2}\\
&= V_3 + \frac{2I_eP_0BX_2\left(e^{-\frac{RB}{(1-E[\alpha])\lambda-D}} - e^{-RM}\right)}{RDQ^3}\\
&-\frac{2I_eP_0X_2\left((1-E(\alpha))\lambda - D\right)\left((1+\frac{RB}{(1-E[\alpha])\lambda-D})\right)e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}}{R^2DQ^3}\\
&-\frac{2I_eP_0X_2(1+RM)e^{-RM}}{R^2Q^3}\\
&-\frac{2I_PC_0X_2e^{-RM}\left(\frac{B}{(1-E[\alpha])\lambda-D} - M\right)}{RQ^3}\\
&-\frac{2I_PC_0X_2e^{-\frac{(1-E[\alpha])RQ}{D}}\left(\frac{(1-E[\alpha])Q}{D} - \frac{Q}{\lambda}\right)}{RQ^3}\\
&+\frac{I_PC_0(R-1)X_4e^{-\frac{RQ}{\lambda}}}{\lambda^2Q}\\
&-\frac{2I_PC_0X_4\left(e^{-RM} - e^{-\frac{(1-E[\alpha])RQ}{D}}\right)}{RQ^3}\\
&+I_PRV_1e^{-\frac{RB}{D}}\\
&-\frac{2I_PC_0X_2\left(e^{-\frac{(1-E[\alpha])RQ}{D}} - e^{-RM}\right)}{R^2Q^3}\\
&+\frac{2I_PC_0X_2\left(e^{-\frac{RQ}{\lambda}} - e^{-\frac{(1-E[\alpha])RQ}{D}}\right)}{\lambda RQ^2}\\
&+\frac{2I_PC_0X_2e^{-\frac{RQ}{\lambda}}}{\lambda^2Q} - \frac{2I_PC_0X_2e^{-\frac{RQ}{\lambda}}\left(\frac{Q}{\lambda} - \frac{B}{(1-E[\alpha])\lambda-D}\right)}{RQ^3}\\
&-\frac{2I_PC_0X_2e^{-\frac{RQ}{\lambda}}\left(\frac{Q}{\lambda} - \frac{B}{(1-E[\alpha])\lambda-D}\right)}{\lambda Q^2}\\
&-\frac{I_PC_0X_2e^{-\frac{RQ}{\lambda}}\left(\frac{Q}{\lambda} - \frac{B}{(1-E[\alpha])\lambda-D}\right)}{\lambda^2Q}\\
&+\frac{2I_PC_0X_2\left(e^{-RM} - e^{-\frac{RQ}{\lambda}}\right)}{RDQ^2} - \frac{2I_PC_0X_2e^{-\frac{RQ}{\lambda}}}{\lambda DQ}.
\end{aligned}$$

Case (iii) $\mathbf{t_1+t_2 \leqslant t_1+t_3 \leqslant t_1+M \leqslant t_1+T'}$

$$\frac{\partial^2 E\left[\pi_3^T(Q,B)\right]}{\partial Q^2}$$
$$= V_3 - \frac{2I_eP_0X_2\left[(1-E(\alpha))\lambda - D\right](1+\frac{RB}{(1-E[\alpha])\lambda-D})e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}}{R^2DQ^3}$$
$$-\frac{2I_eP_0X_2(1+RM)e^{-RM}}{R^2Q^3} + \frac{2I_eP_0BX_2\left(e^{-\frac{RB}{(1-E[\alpha])\lambda-D}} - e^{-RM}\right)}{RDQ^3}$$
$$-I_eRV_1e^{-\frac{RB}{D}} - \frac{2I_PC_0X_4\left(e^{-RM} - e^{-\frac{(1-E[\alpha])RQ}{D}}\right)}{RQ^3}$$
$$-\frac{2I_PC_0X_2\left(e^{-\frac{(1-E[\alpha])RQ}{D}} - e^{-RM}\right)}{R^2Q^3} + \frac{2I_PC_0X_2e^{-RM}}{\lambda RQ^2}$$
$$-\frac{2I_PC_0X_2e^{-RM}\left(\frac{Q}{\lambda} - M\right)}{RQ^3}$$
$$-\frac{2I_PC_0X_2e^{-\frac{(1-E[\alpha])RQ}{D}}\left(\frac{(1-E[\alpha])Q}{D} - \frac{Q}{\lambda}\right)}{RQ^3}.$$

Case (iv) $\mathbf{t_1+T' \leqslant t_1+M \leqslant T}$

$$\frac{\partial^2 E\left[\pi_4^T(Q,B)\right]}{\partial Q^2} = V_3 - \frac{2I_eP_0BX_2e^{-RM}}{RDQ^3}$$
$$-\frac{2I_eP_0X_2\left[(1-E(\alpha))\lambda - D\right]e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}}{R^2DQ^3}$$
$$-\frac{2I_eP_0X_2e^{-\frac{(1-E[\alpha])RQ}{D}}}{R^2Q^3} - \frac{2I_eP_0X_2(1-E[\alpha])Qe^{-RM}}{DRQ^3} - I_eRV_1e^{-\frac{RB}{D}};$$

where

$$V_1 = \frac{C_sE(\alpha)De^{-\frac{RB}{D}}e^{-\frac{RQ}{\lambda}}}{(1-E(\alpha))\lambda^2};$$

$$V_2 = \frac{D\lambda e^{-\frac{2RB}{D}}}{R^2Q^3};$$

$$V_3 = -\frac{2A_0De^{-\frac{RB}{D}}}{Q^3\left(1-E\left(\alpha\right)\right)} - \frac{P_0Re^{-\frac{2RB}{D}}\left(1-E\left(\alpha\right)\right)e^{-\frac{(1-E(\alpha))RQ}{D}}}{Q}$$
$$+\frac{2P_0D^2e^{-\frac{2RB}{D}}\left(1-e^{-\frac{(1-E(\alpha))RQ}{D}}\right)}{RQ^3\left(1-E\left(\alpha\right)\right)} - \frac{2P_0De^{-\frac{2RB}{D}}e^{-\frac{(1-E[\alpha])RQ}{D}}}{Q^2}$$
$$-\frac{2C_BD^2\left(1-e^{-\frac{RB}{D}}\left(1+\frac{RB}{D}\right)\right)}{R^2Q^3\left(1-E\left(\alpha\right)\right)} + 2h_0V_2\left(1-e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}\right)$$
$$-\frac{2h_0X_2\left(e^{-\frac{(1-E[\alpha])RQ}{D}}-e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}\right)}{R^2Q^3}$$
$$-\frac{2h_0X_2e^{-\frac{(1-E[\alpha])RQ}{D}}\left(\frac{(1-E[\alpha])Q}{D}-\frac{Q}{\lambda}\right)}{RQ^3} + \frac{2h_0X_2e^{-\frac{RQ}{\lambda}}}{\lambda^2Q}$$
$$+\frac{2h_0X_2\left(e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}-e^{-\frac{RQ}{\lambda}}\right)}{RDQ^2} - \frac{2h_0X_2e^{-\frac{RQ}{\lambda}}}{D\lambda Q}$$
$$-\frac{2h_0X_4\left(e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}-e^{-\frac{(1-E[\alpha])RQ}{D}}\right)}{RQ^3}$$
$$+\frac{h_0X_4\left(1-R\right)e^{-\frac{RQ}{\lambda}}}{\lambda^2Q}$$
$$+\frac{2h_0X_2\left(e^{-\frac{RQ}{\lambda}}-e^{-\frac{(1-E[\alpha])RQ}{D}}\right)}{\lambda RQ^2}$$
$$-\frac{2h_0X_2e^{-\frac{RQ}{\lambda}}\left(\frac{Q}{\lambda}-\frac{B}{(1-E[\alpha])\lambda-D}\right)}{RQ^3}$$
$$-\frac{2h_0X_2e^{-\frac{RQ}{\lambda}}\left(\frac{Q}{\lambda}-\frac{B}{(1-E[\alpha])\lambda-D}\right)}{\lambda Q^2}$$
$$-\frac{h_0X_2\,R\,e^{-\frac{RQ}{\lambda}}\left(\frac{Q}{\lambda}-\frac{B}{(1-E[\alpha])\lambda-D}\right)}{\lambda^2Q}$$
$$-\frac{2h_0V_2BRe^{-\frac{RB}{(1-E[\alpha])\lambda-D}}}{\left[\left(1-E\left(\alpha\right)\right)\lambda-D\right]} + 2I_eP_0V_2 + R^2V_1.$$

APPENDIX D

In this appendix we provide the second derivatives of the expected total profit functions with respect to B for all four cases:

Case (i) $\mathbf{t_1 \leqslant t_1+M \leqslant t_1+t_2 \leqslant t_1+ t_3 \leqslant t_1+T'}$

$$
\begin{aligned}
&\frac{\partial^2 E[\pi_1^T(Q,B)]}{\partial B^2} = \\
&W_9 + 4I_eP_0W_5 - 4I_eP_0W_5e^{-RM} - 4I_eP_0RW_5Me^{-RM} \\
&+I_PC_0W_2e^{-\frac{RB}{(1-E[\alpha])\lambda-D}} \\
&+4I_PC_0W_7W_5\left(1-\frac{RB}{(1-E[\alpha])\lambda-D}\right) \\
&+4I_PC_0W_7W_6 - 4I_PC_0W_6\left(e^{-RM} - e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}\right) \\
&+I_PC_0W_8W_6 - 4I_PC_0W_5\left(e^{-\frac{RB}{(1-E[\alpha])\lambda-D}} - e^{-RM}\right) - 4I_PC_0W_7W_5 \\
&-4I_PC_0RW_5\left(\frac{B}{(1-E[\alpha])\lambda-D}e^{-\frac{RB}{(1-E[\alpha])\lambda-D}} - Me^{-RM}\right) \\
&-4I_PC_0W_1\left(e^{-\frac{(1-E[\alpha])RQ}{D}} - e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}\right) \\
&-\frac{4I_PC_0W_4\left(e^{-\frac{RQ}{\lambda}} - e^{-\frac{(1-E[\alpha])RQ}{D}}\right)}{D} \\
&+I_PC_0DW_8W_6 - I_PC_0DW_8W_5\left(\frac{RB}{(1-E[\alpha])\lambda-D} - 2\right) \\
&+I_PC_0DW_8W_5 + 4I_PC_0W_7W_1 \\
&-4W_3I_PRe^{-\frac{RB}{D}}\left(e^{-RM} - e^{-\frac{RQ}{\lambda}}\right) \\
&-4I_PC_0RW_1e^{-\frac{RQ}{\lambda}}\left(\frac{Q}{\lambda} - \frac{B}{(1-E[\alpha])\lambda-D}\right) \\
&-\frac{4I_PC_0DW_1e^{-\frac{RQ}{\lambda}}}{(1-E(\alpha))\lambda-D} \\
&-\frac{4I_PC_0W_4\left(e^{-\frac{RQ}{\lambda}} - e^{-\frac{(1-E[\alpha])RQ}{D}}\right)}{D} \\
&+\frac{4I_PC_0W_7W_4}{D} \\
&-\frac{4I_PC_0\lambda e^{-\frac{2RB}{D}}\left(e^{-\frac{RQ}{\lambda}} - e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}\right)}{((1-E(\alpha))\lambda-D)Q} \\
&-4I_PC_0RW_1e^{-\frac{(1-E[\alpha])RQ}{D}}\left(\frac{(1-E[\alpha])Q}{D} - \frac{Q}{\lambda}\right) \\
&+I_PC_0W_8W_4.
\end{aligned}
$$

Case (ii) $\mathbf{t_1+t_2 \leqslant t_1+M \leqslant t_1+t_3 \leqslant t_1+T'}$

$$
\begin{aligned}
&\frac{\partial^2 E[\pi_2^T(Q,B)]}{\partial B^2} = \\
&W_9 - 4I_eP_0W_1\left(e^{-\frac{RB}{(1-E[\alpha])\lambda-D}} - e^{-RM}\right) \\
&-4I_eP_0W_1\left(1+\frac{RB}{(1-E[\alpha])\lambda-D}\right)e^{-\frac{RB}{(1-E[\alpha])\lambda-D}} \\
&+4I_eP_0W_1e^{-\frac{RB}{(1-E[\alpha])\lambda-D}} - 4I_eP_0W_1(1+RM)e^{-RM} \\
&+I_eP_0BRW_1W_8 + \frac{4I_eP_0BRW_1\left(e^{-\frac{RB}{(1-E[\alpha])\lambda-D}} - e^{-RM}\right)}{D} \\
&-\frac{I_eP_0DW_1\left(1+\frac{RB}{(1-E[\alpha])\lambda-D}\right)e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}}{((1-E(\alpha))\lambda-D)} \\
&-\frac{4I_eP_0((1-E(\alpha))\lambda-D)W_1\left(1+\frac{RB}{(1-E[\alpha])\lambda-D}\right)e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}}{D} \\
&+\frac{4I_eP_0BRW_1e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}}{((1-E(\alpha))\lambda-D)} \\
&-\frac{4I_PC_0DW_1e^{-\frac{RQ}{\lambda}}}{(1-E(\alpha))\lambda-D} \\
&-\frac{4I_PC_0W_4\left(e^{-RM} - e^{-\frac{RQ}{\lambda}}\right)}{D} \\
&-4I_PC_0RW_1e^{-\frac{(1-E[\alpha])RQ}{D}}\left(\frac{(1-E[\alpha])Q}{D} - \frac{Q}{\lambda}\right) \\
&-\frac{4I_PC_0W_4\left(e^{-\frac{RQ}{\lambda}} - e^{-\frac{(1-E[\alpha])RQ}{D}}\right)}{D} \\
&+\frac{4I_PC_0DW_1e^{-RM}}{(1-E(\alpha))\lambda-D} \\
&-\frac{4I_PC_0\lambda e^{-\frac{2RB}{D}}\left(e^{-RM} - e^{-\frac{RQ}{\lambda}}\right)}{((1-E(\alpha))\lambda-D)Q} \\
&-4I_PC_0RW_1e^{-RM}\left(\frac{Q}{\lambda} - M\right) \\
&-4W_3I_PRe^{-\frac{RB}{D}}\left(e^{-RM} - e^{-\frac{RQ}{\lambda}}\right) \\
&-4I_PC_0RW_1e^{-\frac{RQ}{\lambda}}\left(\frac{Q}{\lambda} - \frac{B}{(1-E[\alpha])\lambda-D}\right).
\end{aligned}
$$

Case (iii) $\mathbf{t_1+t_2 \leqslant t_1+t_3 \leqslant t_1+M \leqslant t_1+T'}$

$$\frac{\partial^2 E[\pi_3^T(Q,B)]}{\partial B^2} = W_9 - 4I_eP_0W_1\left(1+\frac{RB}{(1-E[\alpha])\lambda - D}\right)e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}$$
$$-4I_eP_0W_1\left(e^{-\frac{RB}{(1-E[\alpha])\lambda-D}} - e^{-RM}\right) - 4I_eP_0W_1(1+RM)e^{-RM}$$
$$-\frac{4I_PC_0W_4\left(e^{-RM}-e^{-\frac{(1-E[\alpha])RQ}{D}}\right)}{D} + 4W_3I_eRe^{-\frac{RB}{D}}\left(e^{-RM}-e^{-\frac{RQ}{\lambda}}\right)$$
$$-\frac{4I_eP_0\left((1-E(\alpha))\lambda - D\right)W_1\left(1+\frac{RB}{(1-E[\alpha])\lambda-D}\right)e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}}{D}$$
$$+4I_eP_0W_1e^{-\frac{RB}{(1-E[\alpha])\lambda-D}} - \frac{I_eP_0DW_1\left(1+\frac{RB}{(1-E[\alpha])\lambda-D}\right)e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}}{((1-E(\alpha))\lambda - D)}$$
$$+\frac{4I_eP_0BRW_1\left(e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}-e^{-RM}\right)}{D} + \frac{4I_eP_0BRW_1e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}}{((1-E(\alpha))\lambda - D)}$$
$$+I_eP_0BRW_1W_8 - 4I_PC_0RW_1e^{-RM}\left(\frac{Q}{\lambda}-M\right)$$
$$-4I_PC_0RW_1e^{-\frac{(1-E[\alpha])RQ}{D}}\left(\frac{(1-E[\alpha])Q}{D}-\frac{Q}{\lambda}\right).$$

Case (iv) $\mathbf{t_1+T' \leqslant t_1+M \leqslant T}$

$$\frac{\partial^2 E[\pi_4^T(Q,B)]}{\partial B^2} = W_9 + 4I_eP_0W_5 - 4I_eP_0W_1e^{-\frac{(1-E[\alpha])RQ}{D}}$$
$$-4I_eP_0W_1\left(e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}-e^{-RM}\right) - \frac{4I_eP_0BRW_1e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}}{D}$$
$$+\frac{4I_eP_0BRW_1\left(e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}-e^{-RM}\right)}{D}$$
$$-\frac{4I_eP_0\left((1-E(\alpha))\lambda-D\right)W_1e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}}{D}$$
$$-\frac{I_eP_0DW_1e^{-\frac{RB}{(1-E[\alpha])\lambda-D}}}{((1-E(\alpha))\lambda-D)} - \frac{4I_eP_0RW_1e^{-RM}(1-E[\alpha])Q}{D};$$

where

$$W_1 = \frac{e^{-\frac{2RB}{D}}}{Q\left(1-E\left(\alpha\right)\right)};$$

$$W_2 = \frac{D^2 e^{-\frac{2RB}{D}}}{\left(\left(1-E\left(\alpha\right)\right)\lambda - D\right)^2 Q\left(1-E\left(\alpha\right)\right)};$$

$$W_3 = \frac{C_S E\left(\alpha\right) e^{-\frac{RB}{D}}}{D\left(1-E\left(\alpha\right)\right)};$$

$$W_4 = \frac{\left(Q - \frac{(1-E(\alpha))\lambda B}{(1-E[\alpha])\lambda - D}\right) R e^{-\frac{2RB}{D}}}{Q\left(1-E\left(\alpha\right)\right)};$$

$$W_5 = \frac{\lambda e^{-\frac{2RB}{D}}}{DQ};$$

$$W_6 = \frac{R e^{-\frac{2RB}{D}}}{D\left(1-E\left(\alpha\right)\right)};$$

$$W_7 = \frac{D e^{-\frac{RB}{(1-E[\alpha])\lambda - D}}}{\left(1-E\left(\alpha\right)\right)\lambda - D};$$

$$W_8 = \frac{W_7}{\left(1-E\left(\alpha\right)\right)\lambda - D};$$

$$\begin{aligned}
W_9 = &-\left(C_0 + \beta_0 + \frac{A_0}{Q}\right)\frac{R^2 e^{-\frac{RB}{D}}}{D\left(1-E\left(\alpha\right)\right)} - C_B\left(1 - \frac{RB}{D}\right) W_1 e^{\frac{RB}{D}} \\
&+ W_3 R^2 e^{-\frac{RQ}{\lambda}} + 4P_0 R W_1\left(1 - e^{-\frac{(1-E[\alpha])RQ}{D}}\right) - 4h_0 W_6\left(1 - e^{-\frac{RB}{(1-E[\alpha])\lambda - D}}\right) \\
&+ 4h_0 W_6 W_7 + 4h_0 W_7 W_1 + 4h_0 W_5\left(1 - e^{-\frac{RB}{(1-E[\alpha])\lambda - D}}\right) \\
&- 4h_0 W_7 W_1 e^{-R\left(\frac{Q}{\lambda} - \frac{B}{(1-E[\alpha])\lambda - D}\right)} \\
&- 4h_0 R W_1 e^{-\frac{(1-E[\alpha])RQ}{D}}\left(\frac{\left(1-E\left[\alpha\right]\right)Q}{D} - \frac{Q}{\lambda}\right) \\
&- 4h_0 R W_1 e^{-\frac{RQ}{\lambda}}\left(\frac{Q}{\lambda} - \frac{B}{\left(1-E[\alpha]\right)\lambda - D}\right) + h_0 W_2 e^{-\frac{RB}{(1-E[\alpha])\lambda - D}} \\
&+ \frac{h_0 RQ W_2 e^{-\frac{RB}{(1-E[\alpha])\lambda - D}}}{D} - \frac{4h_0 W_4\left(e^{-\frac{RB}{(1-E[\alpha])\lambda - D}} - e^{-\frac{RQ}{\lambda}}\right)}{D} \\
&- \frac{4h_0 W_4 W_7}{D} - \frac{4h_0 BRW_7 W_5}{D} - 4h_0 BRW_5 W_8 \\
&- 4h_0 W_1\left(e^{-\frac{(1-E[\alpha])RQ}{D}} - e^{-\frac{RB}{(1-E[\alpha])\lambda - D}}\right) \\
&- \frac{4h_0 W_4\left(e^{-\frac{RQ}{\lambda}} - e^{-\frac{(1-E[\alpha])RQ}{D}}\right)}{D} - h_0 W_4 W_8 - h_0 D W_8 W_5 \\
&- \frac{h_0 DBRW_5 W_8}{\left(1-E\left(\alpha\right)\right)\lambda - D} - \frac{4h_0 \lambda e^{-\frac{2RB}{D}}\left(e^{-\frac{RB}{(1-E[\alpha])\lambda - D}} - e^{-\frac{RQ}{\lambda}}\right)}{\left(\left(1-E\left(\alpha\right)\right)\lambda - D\right)Q}.
\end{aligned}$$

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