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On solvability of boundary value problems in elastoplasticity

by

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Abstract: In the paper the existence of a solution to the three-dimensional elastoplastic problem with the Prandtl-Reuss constitutive law and the Neumann boundary conditions is established. The proof is based on a suitable combination of the parabolic regularization of equations and the penalty method for the elastoplastic yield condition. The method is applied in the case of the domain with smooth boundary as well as in the case of an interior crack. It is shown that the weak solutions to the elastoplastic problem satisfying the variational inequality meet all boundary conditions.

Keywords: elastoplasticity, boundary problem, regularization, penalty, cracks

1. Introduction

In the standard approach to the elastoplastic problems the weak solutions are introduced in the form of solutions to the variational inequalities. We refer the reader to Temam (1983), Khludnev, Sokolowski (1997), Anzellotti (1983), Anzellotti, Giaquinta (1982), Demengel (1983), Khludnev (1988), Johnson (1976), Suquet (1981), Temam (1986) for the related results. The variational inequalities are derived under assumption that solutions to the elastoplastic problem are sufficiently smooth and the originally prescribed boundary conditions are satisfied. On the other hand, the original boundary conditions are not easily recovered for the solutions to the variational inequalities, even if such a weak solution is regular since the set of admissible stresses is not any linear space. As the result at least a part of boundary conditions prescribed for the elastoplastic problem is not obtained for the weak solutions. The problem has been solved in a satisfactory way only in one-dimensional cases; it has been shown that solutions of variational inequalities satisfy all boundary conditions one expects to obtain for the elastoplastic problem, Khludnev, Sokolowski (1997), Khludnev (1993a, b, 1992a, b).

In this paper we consider the three-dimensional elastoplastic Prandtl-Reuss model with the Neumann boundary conditions and prove that all the original boundary conditions hold. The proof is based on a suitable combination of the parabolic regularization and the penalty method for the constraints imposed upon the stresses. In Section 2 domains with smooth boundaries are considered, and in Section 3 the case of a domain obtained by removing a smooth two dimensional surface from its interior is analysed. The latter corresponds to elastoplastic problems for bodies with interior cracks.

We start with notations and preliminary remarks. Let $\Omega \subset \mathbb{R}^3$ be a bounded domain with a smooth boundary Γ having the exterior unit normal $n = (n_1, n_2, n_3)$. Denote $v = (v_1, v_2, v_3)$, $v_{i,j} = \frac{\partial v_i}{\partial x_j}$, $x = (x_1, x_2, x_3) \in \Omega$,

$$\varepsilon_{ij}(v) = \frac{1}{2}(v_{i,j} + v_{j,i}),$$

$i,j = 1,2,3$.

It is well known that if $\varepsilon_{ij}(v) = 0$ in Ω, $i,j = 1,2,3$, then $v_i(x) = c_i + b_{ij}x_j$, $i = 1,2,3$, where $c_i, b_{ij} \in \mathbb{R}$, $b_{ij} = -b_{ji}$, $i,j = 1,2,3$. Linear space of all vectors $v = (v_1, v_2, v_3)$, $v_i(x) = c_i + b_{ij}x_j$, is called the space of rigid displacements and denoted by $R(\Omega)$.

Summation convention over repeated indices $i, j, k, l = 1, 2, 3$ is used throughout the paper. All functions with two lower indices are assumed to be symmetric in those indices, eg. $\sigma_{ij} = \sigma_{ji}$, $i,j = 1, 2, 3$.

To simplify the formulae below the inclusion $v \in L^2(\Omega)$ for the vector function $v = (v_1, v_2, v_3)$ will mean $v_i \in L^2(\Omega), i = 1, 2, 3$, and the following notation is adopted for the scalar product of the vector functions u, v;

$$(v, u) = (v_i, u_i) = \int_\Omega v_i(x)u_i(x)dx .$$

The same convention is used for tensor functions

$$\sigma \in L^2(\Omega) \text{ for } \sigma_{ij} \in L^2(\Omega),\ i,j = 1, 2, 3,$$

$$\langle \sigma, \varepsilon \rangle = \langle \sigma_{ij}, \varepsilon_{ij} \rangle = \int_\Omega \sigma_{ij}(x)\varepsilon_{ij}(x)dx .$$

In view of the convention we use the notation $v \in L^2(\Omega)$ for the space of vector functions as well as for the space of tensor functions, i.e. we write

$$v \in L^2(\Omega) \text{ and } \sigma \in L^2(\Omega)$$

instead of $v \in [L^2(\Omega)]^3$ and $\sigma \in [L^2(\Omega)]^9$.

Consider all vector functions with components from $L^2(\Omega)$ satisfying the following conditions

$$\int_\Omega v = 0, \quad \int_\Omega (v_i x_j - v_j x_i) = 0, \quad i,j = 1,2,3, \quad v = (v_1, v_2, v_3). \tag{1}$$

It is clear that if $p \in R(\Omega)$ satisfies (1) then $p \equiv 0$. In fact, we have $L^2(\Omega) = R(\Omega) \oplus R(\Omega)^\perp$, and $R(\Omega)^\perp$ coincides with the subspace of $L^2(\Omega)$ of vector functions with components from $L^2(\Omega)$ satisfying (1).

Let

$$H^1(\Omega) = \{v = (v_1, v_2, v_3) \mid v_i \in L^2(\Omega),\ i = 1,2,3,$$
$$v_{i,j} \in L^2(\Omega),\ i,j = 1,2,3\}.$$

In the space $H^1(\Omega)$ we consider the following equivalent norms,

$$\|v\|_1^2 = \|v\|_0^2 + \sum_{i,j=1}^{3} \|\varepsilon_{ij}(v)\|_0^2, \tag{2}$$

$$|v|_1 = \left|\int_\Omega v\right| + \sum_{i,j=1}^{3} \left|\int_\Omega (v_i x_j - v_j x_i)\right| + \sum_{i,j=1}^{3} \|\varepsilon_{ij}(v)\|_0. \tag{3}$$

Here, $\|\cdot\|_0$ is the norm in $L^2(\Omega)$. It can be shown by an application of the second Korn inequality that the norm (2) is equivalent to the usual norm in $H^1(\Omega)$. For the norm (3) it is easy to see that the first two terms in the right-hand side of (3) define the seminorm in $H^1(\Omega)$ being the norm in $R(\Omega)$, and the statement follows from Temam (1983).

We can consider the scalar product in $H^1(\Omega)$ inducing the norm (2),

$$((u,v)) = (u,v) + (\varepsilon_{ij}(u), \varepsilon_{ij}(v)), \quad u,v \in H^1(\Omega). \tag{4}$$

In this case $H^1(\Omega) = R(\Omega) \oplus H^1_R(\Omega)$, where

$$H^1_R(\Omega) = \{v = (v_1, v_2, v_3) \in H^1(\Omega) \mid v \text{ satisfies (1)}\}.$$

This means that in $H^1_R(\Omega)$ the following equivalent norm is defined

$$|v|_1 = \sum_{i,j=1}^{3} \|\varepsilon_{ij}(v)\|_0 .$$

Furthermore, the following notation is used. Consider the space

$$LD(\Omega) = \{v = (v_1, v_2, v_3) \mid v_i \in L^1(\Omega), i = 1,2,3,$$

$$\varepsilon_{ij}(v) \in L^1(\Omega),\ i,j = 1, 2, 3\}$$

equipped with the norm

$$\|v\|_{LD(\Omega)} = \|v\|_{L^1(\Omega)} + \sum_{i,j=1}^{3} \|\varepsilon_{ij}(v)\|_{L^1(\Omega)} . \tag{5}$$

Let

$$LDN(\Omega) = \{v \in LD(\Omega) \mid v \text{ satisfies } (1)\}.$$

Along with the usual norm (5) we shall consider the following norm in $LD(\Omega)$ (see Temam, 1983)

$$|v|_{LD(\Omega)} = \left|\int_\Omega v\right| + \sum_{i,j=1}^{3} \left|\int_\Omega (v_i x_j - v_j x_i)\right| + \sum_{i,j=1}^{3} \|\varepsilon_{ij}(v)\|_{L^1(\Omega)} .$$

Consequently, in the linear subspace $LDN(\Omega)$ of the space $LD(\Omega)$ the norm is defined as follows

$$\|v\|_{LDN(\Omega)} = \sum_{i,j=1}^{3} \|\varepsilon_{ij}(v)\|_{L^1(\Omega)} .$$

The space of bounded measures on Ω is denoted by $M^1(\Omega)$; $M^1(\Omega) = (C_0(\Omega))^*$ is the dual space of the normed space $C_0(\Omega)$ of continuous functions with compact supports, equipped with the uniform convergence topology. It is known that any bounded set in $M^1(\Omega)$ is relatively compact in the (*)-weak topology, i.e. every bounded sequence in $M^1(\Omega)$ contains a subsequence which is (*)-weak convergent. We recall that $h_m \in M^1(\Omega)$ is (*)-weak convergent to an element $h \in M^1(\Omega)$ as $m \to \infty$ if

$$h_m(\varphi) \to h(\varphi) \quad \forall \varphi \in C_0(\Omega).$$

Introduce the Banach space of vector functions of bounded deformations

$$BD(\Omega) = \{v = (v_1, v_2, v_3) \mid v_i \in L^1(\Omega),\ i = 1,2,3,$$

$$\varepsilon_{ij}(v) \in M^1(\Omega),\ i,j = 1,2,3\}$$

endowed with the norm

$$\|v\|_{BD(\Omega)} = \|v\|_{L^1(\Omega)} + \sum_{i,j=1}^{3} \|\varepsilon_{ij}(v)\|_{M^1(\Omega)} .$$

In the same way as above,

BDN(D) = {v E BD(D)I v satisfies (1)}.

In the sequel we need the appropriate Green formula. Let erij E L^2 (D), i, j = 1, 2, 3, erij,j E L^2(D), i = 1, 2, 3. Then, erijnj, i = 1, 2, 3, can be considered as elements of the space $H^{-1/2}$(I') on the boundary r and, moreover, the following formula holds, Temam (1983)

-(erij,j, 0) = (erij, 0,j) - (erijnj, 0)i;2,r \/0 E H^1 (D), i = 1, 2, 3, (6)

where (·, -)i;2,r denotes a duality pairing between the spaces $H^{-1/2}$(r) and Hl/2(r).

2. Domain with a smooth boundary

Let D c IR^3 be a bounded domain with smooth boundary I', Q = O x (0, T), x = (x_1, x_2, x_3) E n, t E (0, T). The elastoplastic problem for a body occupying the domain n in the nondeformable state is formulated as follows.

Find the functions v = (v_1, v_2, v_3), er = {erij},'T/ij,i,j = 1,2,3, defined in Q and satisfying the following equations and inequalities

-erij,j = fi, i = 1, 2, 3, (7)

Eij(v) = Cijkz!Ykl + 'T/ij, i, j = 1, 2, 3, (8)

$$\Phi(\sigma) \leq 0, \quad \eta_{ij}(\bar{\sigma}_{ij} - \sigma_{ij}) \leq 0 \quad \forall\bar{\sigma}, \ \Phi(\bar{\sigma}) \leq 0, \tag{9}$$

erijnj = 0, i = 1, 2, 3, on r x (0, T), (10)

er= 0, t = 0. (11)

The functions v, erij, Eij (v) represent the velocity, components of the stress tensor and components of the strain tensor velocity, respectively. The dot denotes the derivative with respect *tot,* e.g. ijkz = d t·. The convex and continuous function 1> defines the plasticity yield condition. It is assumed that the set

k ={er= {erij} E IR^6 I 1>(er) O} (12)

contains zero as its interior point, 0 Eintk. We assume that the symmetry conditions Cijkz(x) = Cjikz(x) = Cklij(x) for i,j,k,l = 1,2,3 are satisfied and there exist two positive constants c_1, c_2 such that

$$c_1|\sigma|^2 \leq c_{ijkl}\sigma_{kl}\sigma_{ij} \leq c_2|\sigma|^2 \quad \forall\sigma = \{\sigma_{ij}\}. \tag{13}$$

To simplify the formulae below we assume that Cijkl = oto{' o; is the Kronecker symbol. Nevertheless, all the results obtained in the paper are valid in general case (13).

The functions *T/ij*, i,j = 1, 2, 3, can be eliminated from (8), (9). In fact, multiply (8) by *i'fij* - *aij* and sum up in i,j. This leads to

$$\Phi(\sigma) \leq 0, \ (\dot{\sigma}_{ij} - \varepsilon_{ij}(v))(\bar{\sigma}_{ij} - \sigma_{ij}) \geq 0 \quad \forall\bar{\sigma}, \ \Phi(\bar{\sigma}) \leq 0. \tag{14}$$

Inequality (14) is used as the definition of a weak solution to the problem (7)-(11).

Consider the set of admissible stresses

K = { *a* = {ail}1 *aij* E *L*² *(D), i,j* = 1, 2, 3, *iJ?(a(x)):::;* 0 a.e. in D}

and denote by p the penalty operator related to the set *K* The operator is given by the formula *p(a)* = *a* - *lra,* where I r: L² (D) —,*KC L² (D)* is the operator of orthogonal projection with respect to the scalar product in L² (D). Recall that the operator p : L² (D) —,L² (D) is bounded, monotone and continuous, and we use the convention to denote L² (D) for [L² (D)]⁹.

Let the brackets(-,•) denote the scalar product in *L² (Q),*

(u,v) = *l* ^T^ *(u (t), v (t))dt*

for vector functions *u, v* E *L² (Q), f* = *(fi, h, !J).*

The space of statically admissible stresses is denoted by

Vo(D) = { *a* = {ail}1 *aij* E *L² (D), i,j* = 1,2,3; *aij,j* E L³(D), i = 1,2,3; *aijnj* = 0, i = 1, 2, 3, on r}.

Suppose that there exists a function = {fo}, i,j = 1, 2, 3, satisfying the equation (7) such that E C² *(Q),* (0) = t(o) = 0 and (1 + "") (t) E K n V0(D), where "' E IR,"" > 0, *t* E [0, T].

Now we can prove the following existence theorem for the problem (7)-(11).

THEOREM 2.1 *Assume that f* E *L³ (Q),* j E L² (Q), *f(0)* = 0, (f(t), p) = 0 \Ip E *R(D), t* E *[O, T], and the above assumption on is satisfied.*

Then there exist functions *v* = *(v1, v2, v3), a* = *{ai1}* such that

v E £² (0,T; BDN(D)), a E £² (0,T; Vo(D)),

& E *L² (Q), a(t)* E K, *t* E (0, *T),*

(ai1, E1(v)) = *(f, v)* *Vv* E £² (0,T; H¹ (D)), (15)

$$(\dot{\sigma}_{ij}, \bar{\sigma}_{ij} - \sigma_{ij}) + (v_i, \bar{\sigma}_{ij,j} - \sigma_{ij,j}) \geq 0 \quad \forall\bar{\sigma} \in L^2(0,T;V_0(\Omega)), \tag{16}$$

with *i'f(t)* E *K* a.e. in (0, *T),*

a = 0, *t* = 0. (17)

Proof. First, an approximation of the problem (7)-(11) by penalization of the plastic yield condition is introduced. Then, the parabolic regularization of the penalized problem is considered. The auxiliary boundary value problem constructed in such a way depends on two positive parameters α, δ. The parameter α is used for the parabolic regularization, the parameter δ is used for the penalty term. We prove the existence of solutions for the fixed parameters $\alpha > 0$, $\delta > 0$ and justify the passage to limits as $\alpha, \delta \to 0$. A priori estimates uniform with respect to α, δ are needed to perform the passage to the limits. All necessary estimates (29), (31), (32) are derived below.

We consider in the domain Q the following auxiliary boundary value problem: find the functions $v = (v_1, v_2, v_3)$, $\sigma = \{\sigma_{ij}\}$ such that

$$\alpha \dot{v}_i - \alpha\varepsilon_{ij}(v)_{,j} - \sigma_{ij,j} = f_i, \quad i = 1,2,3, \tag{18}$$

$$\dot{\sigma}_{ij} + \alpha\sigma_{ij} - \varepsilon_{ij}(v) + \frac{1}{\delta}p(\sigma)_{ij} = 0, \quad i,j = 1,2,3, \tag{19}$$

$$\sigma_{ij}n_j + \alpha\varepsilon_{ij}(v)n_j = 0, \quad i = 1,2,3, \quad \text{on } \Gamma \times (0,T), \tag{20}$$

$$v = 0, \quad \sigma = 0, \quad t = 0. \tag{21}$$

The dependence of solutions to (18)-(21) on the parameters α, δ is not indicated in (20)-(21) in order to simplify the formulae. Note that boundary conditions (20) do not coincide with (10); the conditions (20) can be viewed as the regularization of (10) connected with the proposed regularization of the equilibrium equations (7). Also, the artificial initial condition for v is introduced.

A solution to the problem (18)-(21) is defined in the following sense

$$v \in L^2(0,T;H^1(\Omega)), \quad \dot{v}, \sigma, \dot{\sigma} \in L^2(Q), \tag{22}$$

$$\alpha(\dot{v}, \bar{v}) + \alpha(\varepsilon_{ij}(v), \varepsilon_{ij}(\bar{v})) + (\sigma_{ij}, \varepsilon_{ij}(\bar{v})) = \mathcal{F}(\bar{v}) \quad \forall \bar{v} \in L^2(0,T;H^1(\Omega)), \tag{23}$$

$$\dot{\sigma}_{ij} + \alpha\sigma_{ij} - \varepsilon_{ij}(v) + \frac{1}{\delta}p(\sigma)_{ij} = 0, \quad i,j = 1,2,3, \tag{24}$$

$$v = 0, \quad \sigma = 0, \quad t = 0. \tag{25}$$

In this case the boundary conditions (20) are included in (23). At the first step of the proof we derive a priori estimates. To this end it is assumed that the solutions to (18)-(21) are sufficiently smooth. Multiply (18), (19) by v_i, $\sigma_{ij} - f_0$, respectively, and integrate over Ω. Taking into account that the penalty term is nonnegative this provides the inequality

$$\frac{1}{2}\frac{d}{dt}\{\alpha\|v\|_0^2 + \|\sigma\|_0^2\} + \alpha\|\sigma\|_0^2 - (\alpha\varepsilon_{ij}(v)_{,j} + \sigma_{ij,j}, v_i) - \tag{26}$$

$$-(\varepsilon_{ij}(v), \sigma_{ij} - f_0) - \mathcal{F}(v) \le (\alpha\sigma + \dot{\sigma}, f_0).$$

For the sake of simplicity we do not show in (26) the dependence of v, σ, · f on t. The integration by parts in the third term of the left-hand side in (26) can be performed. Recall that satisfies the equation (7). As a result the following inequality is obtained

$$\frac{1}{2}\{\alpha\|v(t)\|_0^2+\|\sigma(t)\|_0^2\}+\alpha\sum_{i,j=1}^{3}\int_0^t\|\varepsilon_{ij}(v)\|_0^2d\tau\le \tag{27}$$

$$\le\langle\sigma(t),\xi(t)\rangle-\int_0^t\langle\sigma(\tau),\dot{\xi}(\tau)\rangle d\tau+\frac{\alpha}{2}\int_0^t\|\xi(\tau)\|_0^2d\tau.$$

Since (O',) ::::; ¼110'116 + 11116 the integration of (27) leads to the estimate

$$\sup_{0\le t\le T}\|\sigma(t)\|_0^2+\alpha\|v\|_{L^2(Q)}^2+\alpha\sum_{i,j=1}^{3}\int_0^T\|\varepsilon_{ij}(v)\|_0^2d\tau\le c \tag{28}$$

with the constant c being uniform in a, 8, for a::::; a 0. Hence

$$\sup_{0\le t\le T}\|\sigma(t)\|_0^2+\alpha\|v\|_{L^2(0,T;H^1(\Omega))}^2\le c. \tag{29}$$

The derivation of the following estimate requires the (a, 8)-uniform boundedness of }p(O') in the L¹ (Q) norm. By (29) it is easy to see that

$$\frac{1}{\delta}\int_0^T\langle p(\sigma),\sigma-\xi\rangle dt\le c$$

uniformly in a, 8, provided that the penalty term is not neglected when deriving (26). Due to the monotonicity of p

$$\frac{1}{\delta}\int_0^T\langle p(\sigma),\bar{\sigma}-\sigma\rangle dt\le 0\quad\forall\bar{\sigma}\in L^2(Q),\ \bar{\sigma}(t)\in K.$$

Summing up the two last inequalities we dotain

3/4 (p(O''), fj - Odt::::: c

We can take here O'= + [, ll[IIL=(Q) ::::; μ, μ > 0 is sufficiently small. By the hypothesis imposed on the inclusions O''(t) E K, t E (0, T), hold, hence

$$\frac{1}{\delta}\int_0^T\langle p(\sigma),\bar{\xi}\rangle dt\le c\quad\forall\bar{\xi},\ \|\bar{\xi}\|_{L^\infty(Q)}\le\mu$$

and, consequently,

$$\frac{1}{\delta}\|p(\sigma)\|_{L^1(Q)}\le c. \tag{30}$$

In the sequel this estimate is improved, namely, we show that ¼P(a-) is, in fact, bounded in £ 2 (0, T; L 1 (fl)).

We derive a priori estimate for the time derivatives V, er. It follows from (18), (19), (21) that we have the homogenous initial conditions for the derivatives,

vi (0) = 0, i = 1, 2, 3, irij(0) = 0, i,j = 1, 2, 3.

Differentiate with respect to t the equations (18), (19) and multiply by Vi , CTij - lij , respectively. Since the term

i(!p(a-(t)), ir(t))

is nonnegative for almost all t E (0, T) (see Lions, 1969, page 399) the above multiplication and integration over n results in the inequality

1 d
2 dt {allvll6 + llirll6} + all&ll6 - (aEij(v),j + irij ,j ,vi) -

-(Eij (v), irij - lij) - (i, v) (air+ er,1) + i(: t-p(a-),1).

Boundary conditions (20) can be taken into account in order to integrate by parts in the left-hand side of the latter inequality.

We integrate the last inequality with respect to t over the interval (0, t). This implies

$$\frac{1}{2}\{\alpha\|\dot{v}(t)\|_0^2 + \|\dot{\sigma}(t)\|_0^2\} + \alpha \sum_{i,j=1}^{3} \int_0^t \|\varepsilon_{ij}(\dot{v})\|_0^2 d\tau \leq$$

1
8(p(a-),l)16 - J rt Jo (p(a-),()dr+

+(ir,l)lt- Jo r(&l)dr+ !/2 Jo t lll(r)ll6dr.

By estimates (29), (30) and the condition l(O) = 0 the (a, 8)-uniform estimate follows

$$\|\dot{\sigma}\|_{L^2(Q)}^2 + \alpha\|\dot{v}\|_{L^2(Q)}^2 \leq c. \tag{31}$$

Let v = VN + PN be the decomposition of v into the sum of two orthogonal elements, VN E £ 2 (0, T; Hh(rl)), PN E £ 2 (0, T; R(fl)). We should note at this point that £ 2 (0, T; H1 (fl)) = £ 2 (0, T; R(rl)) EB£ 2 (0, T; Hh(rl)) provided that the scalar product (4) is used in H 1 (rl). For almost all t E (0, T), vN(t) = (vNl (t), vm (t), vN3(t)),

k vN(t) = 0, k (vNi(t)xj - VNj (t)xi) = 0, i,j = 1, 2, 3.

Hence, by (29), (31) it follows from (19) that $\varepsilon_{ij}(v_N), i,j = 1,2,3$, are bounded in $L^2(Q)$ uniformly in α for any fixed δ. This implies the estimate

$$\|v_N\|_{L^2(0,T;H^1_N(\Omega))} \leq c(\delta) \tag{32}$$

with the constant $c(\delta)$ depending, in general, on δ.

Now, observe that in view of the estimates (29), (31) we can use the Galerkin approximations of parabolic problems with monotone operators, Lions (1969), in order to show that for any fixed α, δ the solution to (18)-(21) exists in the sense of (22)-(25).

The estimates obtained allow us to pass to the limit as $\alpha \to 0$. Indeed, denote by $v^{\alpha\delta}, \sigma^{\alpha\delta}$ the solution to (22)-(25) and consider the decomposition $v^{\alpha\delta} = v_N + P_N$, $v_N \in L^2(0,T; H^1_N(\Omega))$, $P_N \in L^2(0,T; R(\Omega))$. Note that the solution $(v^{\alpha\delta}, \sigma^{\alpha\delta})$ satisfies the estimates (29), (31), (32). Hence, from the sequence $v^{\alpha\delta}, \sigma^{\alpha\delta}$ one can choose a subsequence (with the previous notation for the subsequence) such that for any fixed $\delta > 0$ and $\alpha \to 0$

$$
\begin{aligned}
&\alpha v^{\alpha\delta} \to 0 &&\text{weakly in} \quad L^2(0,T;H^1(\Omega)),\\
&v_N^{\alpha\delta} \to v^\delta &&\text{weakly in} \quad L^2(0,T;H^1_N(\Omega)),\\
&\alpha v^{\alpha\delta} \to 0 &&\text{weakly in} \quad L^2(Q),\\
&\sigma^{\alpha\delta} \to \sigma^\delta &&\text{weakly in} \quad L^2(Q),\\
&\dot{\sigma}^{\alpha\delta} \to \dot{\sigma}^\delta &&\text{weakly in} \quad L^2(Q).
\end{aligned}
$$

Passing to the limit in (18), (19) as $\alpha \to 0$ we dotain that

$$(\sigma^\delta_{ij}, \varepsilon_{ij}(v)) = (f, v) \quad \forall v \in L^2(0,T;H^1(\Omega)), \tag{33}$$

$$\dot{\sigma}^\delta_{ij} - \varepsilon_{ij}(v^\delta) + \frac{1}{\delta} p(\sigma^\delta)_{ij} = 0, \quad i,j = 1,2,3. \tag{34}$$

A justification of the convergence $p(\sigma^{\alpha\delta}) \to p(\sigma^\delta)$ can be done by the monotonicity arguments. We omit the details.

Now let us show that $\frac{1}{\delta} p(\sigma^\delta)$ are bounded in $L^2(0,T; L^1(\Omega))$ uniformly in δ. It follows from (33) that for a.e. $t \in (0,T)$,

$$-(\sigma^\delta_{ij}(t), \varepsilon_{ij}(v)) = (f(t), v) \quad \forall v \in H^1(\Omega).$$

Hence, for almost all $t \in (0,T)$

$$-(\sigma^\delta_{ij}(t), \varepsilon_{ij}(v^\delta(t))) = (f(t), v^\delta(t)). \tag{35}$$

Multiply (34) by $\sigma^\delta_{ij} - \xi_{ij}$ and integrate over Ω. This yields

$$\langle \varepsilon_{ij}(v^\delta(t)), \sigma^\delta_{ij}(t) - \xi_{ij}(t) \rangle + \frac{1}{\delta} \langle p(\sigma^\delta(t)), \xi(t) - \sigma^\delta(t) \rangle = \tag{36}$$

$$= \langle \dot\sigma^\delta(t), \sigma^\delta(t) - \xi(t)\rangle.$$

By (29) and (31) the right-hand side of (36) is bounded in $L^2(0,T)$ uniformly in 8. Summing up (35) and (36) we obtain that

$$\frac{1}{\delta}\langle p(\sigma^\delta(t)), \sigma^\delta(t) - \xi(t)\rangle \quad \text{are bounded in} \quad L^2(0,T) \tag{37}$$

uniformly in 8. Introduce the convex functional on the space L^2 (D),

$$F(\sigma) = \|\sigma - \pi\sigma\|_0^2, \quad \sigma = \{\sigma_{ij}\}, i,j = 1,2,3.$$

The Gateaux derivative of F is given by $F'(u) = 2p(u)$. Let us take a function a = {a_{i1}} E $L^{00}(Q)$. Then by the conditions imposed on t it follows that t(t) + *a(t)* belongs to the set *K*, *tt* E (0, *T*), provided that the norm llall£oo(Q) is sufficiently small. By the convexity of F we have

$$i(p(u^6(t)),a(t)) \text{ :S } i(p(u^6(t)),u^6(t) - t(t)) + \tag{38}$$

$$+\frac{1}{28}F(t(t) + a(t)) - \frac{1}{28}F(u^6(t)).$$

The second term of the right-hand side of (38) equals to zero by the inclusion t(t) + *a(t)* E K and consequently, in view of (37),

$$i(p(u^6(t)),a(t)) \quad \text{are bounded in} \quad L^2(0,T).$$

Since a is an arbitrary element of the space L^{00} (Q) with a small norm we infer the desired estimate,

$$ip(u^6(t)) \quad \text{are bounded in} \quad L^2(0,T;L^1(D)).$$

Hence, from (34) it follows that

$$\text{llci1}(v^6)\text{IIL2(DT;L1(11)) :Sc}, \quad i,j = 1,2,3. \tag{39}$$

The inequality (39) and the inclusion v^6 E L^2(0,T; H);,(D)) yield the estimate

$$\text{llv}^6\text{IIL2(o,T;LDN(n)) : S c},$$

uniformly in 8. Moreover, the space L^1(D) is continuously imbedded in M^1(D), consequently

$$\text{llv}^6\text{IIL2(0,T;BDN(O)) :Sc}. \tag{40}$$

On the other hand the inequality

$$\|\sigma^\delta\|_{L^2(Q)} + \|\dot\sigma^\delta\|_{L^2(Q)} \le c \tag{41}$$

is satisfied uniformly in 8 Recall that *BD(O)* c $L^3 1^2$ (0,) in the three-dimensional space. Moreover, we have the following estimate

$$\|\sigma^{\delta}(T)\|_0 \leq c\{\|\sigma^{\delta}\|_{L^2(Q)} + \|\dot{\sigma}^{\delta}\|_{L^2(Q)}\} \qquad (42)$$

with the constant c independent of functions o'^6, 17^6.

By (40), (41), (42) we can choose a subsequence (with the previous notation for the subsequence) such that as 8 -+ 0,

o'^6 - +O weakly in $L^2(Q)$,

17^6 -+ 17 weakly in $L^2(Q)$,

v^6 - +v weakly in $L^2(0, T; £^3 1^2(0,))$,

$Ei_j(v^6)$ - +$Ei_j(v)$ (*)-weakly in $L^2(0,T; M^1(0))$, i,j = 1, 2, 3,

o''^6 *(T)* -+ *O'(T)* weakly in $L^2(0,)$.

As a result, passing to the limit as 8-+ 0 in (33), we obtain

$(Oi_j, Ei_j(v)) = (f,v)$ Vv E $L^2(0,T;H^1(0))$. (43)

The equations (34) imply

$(17^6_j, ai_j$ - $O^6_j)$ - $(Ei_j(v^6), aij$ - $O^6_j)$ 2: 0 Va E $L^2(0,T; Vo(O))$, (44)

a(t) E K, t E (0, *T*).

The identity (43) ensures the fulfilment of the equations

$-O't_{,j}$ $=Ji$, i = 1,2,3, (45)

in the sense of distributions.

Furthermore, by (33)

$(si_j(v^6), Ot) = -(O't_j, vf)$

and, moreover,

$$(\varepsilon_{ij}(v^{\delta}), \bar{\sigma}_{ij}) = -(v_i^{\delta}, \bar{\sigma}_{ij,j}) \quad \forall \bar{\sigma} \in L^2(0,T; V_0(\Omega)).$$

Hence, inequality (44) can be rewritten in the following form

$(17^6_j, ai_j$ - $O'^6_j)$ + $(vf, ai_{j,j}$ - $O^6_j)$ 2 0 Va E $L^2(0,T; Vo(O))$, (46)

a(t) E K, t E (0, T).

By (45) the functions $O^6_{j,j}$ can be replaced by - Ji and from (46) it follows that

$$(\dot{\sigma}_{ij}^{\delta}, \bar{\sigma}_{ij}) + (v_i^{\delta}, \bar{\sigma}_{ij,j} + f_i) \geq \frac{1}{2}\|\sigma^{\delta}(T)\|_0^2 \,. \qquad (47)$$

Passing to the lower limit in both sides of (47) and replacing Ji by -aij,j we obtain (16). The inclusion *a(t)* E K, *t* E (0, *T),* can be verified by the standard arguments. Since v^8 E L^2 *(0, T;H1 (D)),* the convergence of v^8 to v implies the inclusionv E L^2 *(0, T;BDN(D))*. Thepropertya(t) E V_0 *(D)*, *t* E *(0,T),* actually follows from (43) taking into account the equations

-aij,j = Ji, i = 1, 2, 3,

and the Green formula (6). Therefore, Theorem 2.1 is proved. ■

3. Domain with a nonsmooth boundary (body with a crack)

In this section we prove the existence theorem for elastoplastic problem in the case of the domain with nonsmooth boundary. In applications, nonsmooth boundaries occur due to the presence of cracks in the body. In the framework of elasticity the properties of the solutions in domains with cracks are analysed in many papers, and we refer the reader to e.g. Oleinik, Kondratiev, Kopacek (1981, 1982), Grisvard (1992), Duduchava, Wendland (1995), Nazarov, Plamenevskii (1991), Morozov (1984), Khludnev (1995, 1996a,b), Nicaise (1992) for the related results.

Again, let D C IR3 be a bounded domain with the smooth boundary 8D = r and c C D be a smooth orientable two-dimensional surface with a regular boundary. We assume that c can be extended in such a way that the domain D is divided into two parts with Lipschitz boundaries. The surface c is described parametrically

$$Xi = Xi(Y1, Y2), \quad i = 1, 2, 3, \tag{48}$$

where (Y1, Y2) E w, w C IR^2 is a bounded domain with smooth boundary *ry;* w = w U ry. Assume that for any point (y_1, y_2) E w the rank of the Jacobi matrix [1:;] equals 2 and the map (48) is one-to-one. In this case one can choose a unit normal vector to the surface c,

$$V = \frac{kxk}{\left|\frac{oy1}{OX} \times \frac{oy2}{OX}\right|} \quad oy1 \quad oy2$$

Denote D_e = D \ c, Q_c = D_e x (0, *T), T* > 0.

The equilibrium problem for elastoplastic body occupying the domain D_e can be formulated as follows.

Find functions v = $(v1, v_2, v_3)$, a = {aij}, T/ij,i,j = 1,2,3, defined in Q_c and satisfying the following equations and inequalities

$$-aij,j = Ji, \quad i = 1, 2, 3, \tag{49}$$

$$Eij (V) = Cijkli,kl + T/ij, \quad i, j = 1, 2, 3, \tag{50}$$

$$\Phi(\sigma) \leq 0, \quad \eta_{ij}(\bar{\sigma}_{ij} - \sigma_{ij}) \leq 0 \;\; \forall \bar{\sigma}, \; \Phi(\bar{\sigma}) \leq 0, \tag{51}$$

$$\sigma_{ij} n_j = 0, \;\; i = 1, 2, 3, \quad \text{on } \Gamma \times (0, T), \tag{52}$$

$$\sigma_{ij} \nu_j = 0, \;\; i = 1, 2, 3, \quad \text{on } \Sigma_c^{\pm} \times (0, T), \tag{53}$$

$$\sigma = 0, \quad t = 0. \tag{54}$$

The same notations are used as in sections 1,2. As we see, in this case the boundary of the domain Ω_c consists of the parts Γ, Σ_c^+, Σ_c^-, where $\Sigma_c^{\pm}$ correspond to the positive and negative directions of the normal ν, respectively.

Introduce the space

$$H^1(\Omega_c) = \{v = (v_1, v_2, v_3) \mid v_i \in L^2(\Omega_c), i = 1,2,3;$$

$$v_{i,j} \in L^2(\Omega_c), \; i, j = 1, 2, 3\}.$$

Note that functions from the space $H^1(\Omega_c)$ have, in general, different boundary values on $\Sigma_c^{\pm}$. This holds for functions from functional spaces considered below, provided that the boundary conditions are well defined even in a weak sense.

In this section we shall consider functions satisfying the following relations

$$\int_{\Omega_c} v = 0, \quad \int_{\Omega_c} (v_i x_j - v_j x_i) = 0, \;\; i, j = 1, 2, 3, \;\; v = (v_1, v_2, v_3). \tag{55}$$

Let

$$LD(\Omega_c) = \{v = (v_1, v_2, v_3) \mid v_i \in L^1(\Omega_c), i = 1, 2, 3, \;\; \varepsilon_{ij}(v) \in L^1(\Omega_c),$$

$$i,j = 1,2,3\}.$$

The subspaces $H_N(\Omega_c)$, $LD_N(\Omega_c)$ consist ofall function from $H^1(\Omega_c)$ and $LD(\Omega_c)$ respectively, satisfying (55). In the subspaces $H_N(\Omega_c)$ and $LD_N(\Omega_c)$ the following norms are introduced,

$$|v|_{H_N(\Omega_c)} = \sum_{i,j=1}^{3} \|\varepsilon_{ij}(v)\|_{0,c}, \quad |v|_{LD_N(\Omega_c)} = \sum_{i,j=1}^{3} \|\varepsilon_{ij}(v)\|_{L^1(\Omega_c)}$$

which are equivalent to the standard ones. Here $\|\cdot\|_{0,c}$ stands for the norm in $L^2(\Omega_c)$. The proof of the norm equivalency uses the compactness of imbeddings $H^1(\Omega_c) \subset L^2(\Omega_c)$, $LD(\Omega_c) \subset L^1(\Omega_c)$ which are obtained under the conditions imposed on Σ_c and Γ. Indeed, let us prove that there exists a positive constant c such that

$$|v|_{LD(\Omega_c)} \geq c\|v\|_{LD(\Omega_c)} \quad \forall v \in LD(\Omega_c), \tag{56}$$

where

$$|v|_{LD(\Omega_c)} = \left|\int_{\Omega_c} v\right| + \sum_{i,j=1}^{3} \left|\int_{\Omega_c} (v_i x_j - v_j x_i)\right| + \sum_{i,j=1}^{3} \|\varepsilon_{ij}(v)\|_{L^1(\Omega_c)}.$$

Assume that the inequality (56) is false. In this case there exists a sequence v^k,

$$\|v^k\|_{LD(\Omega_c)} = 1, \tag{57}$$

$$|v^k|_{LD(\Omega_c)} \le \frac{1}{k}, \tag{58}$$

According to the assumptions on E_e we can divide the domain D_e into two subdomains D_1 and D_2 with Lipschitz boundaries. Since the imbeddings $LD(D_i) \subset L^1(D_i)$, $i = 1, 2$, are compact, it follows that the imbedding $LD(D_e) \subset L^1(D_e)$ is also compact. Consequently, by (57) we can suppose that

$$v^k \to v \quad \text{strongly in} \quad L^1(D_e),$$

and hence, by (58)

$$v^k \to v \quad \text{strongly in} \quad LD(D_e). \tag{59}$$

In view of (58), we have $\varepsilon_{ij}(v) = 0$ in D_e, ie. $v_i(x) = c_i + b_{ij} x_j$, where $c_i, b_{ij} \in \mathbb{R}$, $b_{ij} = -b_{ji}$, $i, j = 1, 2, 3$. Moreover, the inequality (58) implies that v satisfies (55), whence $v = 0$ which contradicts (57), (59). Therefore, (56) is proved.

Consider the following spaces,

$$BDN(D_e) = \{v \in BD(D_e) \mid v \text{ satisfies } (55)\},$$

$$U_a(D_e) = \{a = \{a_{ij}\} \mid a_{ij} \in H^1(D_e), i,j = 1,2,3; \; a_{ij,j} \in L^3(D_e), i = 1,2,3;$$

$$a_{ij} n_j = 0, i = 1, 2, 3, \text{ on } \Gamma; \; a_{ij} \nu_j = 0, i = 1, 2, 3, \text{ on } \Sigma \}.$$

Again, to simplify the formulae we assume $C_{ijkl} = \delta_{ik}\delta_{jl}$. Recall that the set (12) contains zero as its interior point. The set K is introduced in the same way as in section 2,

$$K = \{a = \{a_{ij}\} \mid a_{ij} \in L^2(D_e), i,j = 1,2,3, \; \Phi(a(x)) \le 0 \quad \text{a.e. in} \quad D_e\}.$$

The scalar products in $L^2(D_e)$ and $L^2(Q_e)$ are denoted by $(\cdot, \cdot)_e$, $(\cdot, \cdot)_e$, respectively. The space of rigid displacements on D_e is denoted by $R(D_e)$.

We assume that there exists a function $f = \{f_{ij}\}, i, j = 1, 2, 3$, $f \in C^2(Q_c)$, $f(0) = \dot{f}(0) = 0$, satisfying the equation

$$(f_0, \varepsilon_{ij}(v))_e = (f, v)_e \quad \forall v \in L^2(0,T; H^1(D_e)) \tag{60}$$

and such that $(1 + \kappa)f(t) \in K \cap U_0(D_e)$, $\kappa \in \mathbb{R}$, $\kappa > 0$, $t \in [0,T]$. Now we are in a position to prove the theorem of existence of solutions to (49)-(54).

THEOREM 3.1 *Assume that* $f \in L^3(Q_e), j \in L^2(Q_e), f(0) = 0$, $(f(t), p)_e = 0$ $\forall p \in R(D_e), t \in [0, T]$, *and the above assumption on* f *hold. Then there exist functions* $v = (v_1, v_2, v_3)$, $a = \{a_{ij}\}$ *such that*

$$v \in L^2(0,T; BDN(D_e)), \; a, \dot{a} \in L^2(Q_e), \; a(t) \in K, \; t \in (0, T),$$

$$(\sigma_{ij}, \varepsilon_{ij}(\bar{v}))_c = (f, \bar{v})_c \quad \forall \bar{v} \in L^2(0,T; H^1(\Omega_c)), \tag{61}$$

$$(\dot{\sigma}_{ij}, \bar{\sigma}_{ij} - \sigma_{ij})_c + (v_i, \bar{\sigma}_{ij,j} - \sigma_{ij,j})_c \geq 0 \quad \forall \bar{\sigma} \in L^2(0,T; U_0(\Omega_c)), \tag{62}$$

iJ(t) E K *ae. in* (0, *T),*

a-= 0, $t = 0.$ (63)

Proof. The general scheme of the proof coincides with that used for Theorem 1 and our attention now focusses on details related to the nonsmoothness of the boundary.

Let p be the penalty operator related to the set *K,* p(a-) = a- - no-, where 1r is the orthogonal projection operator of the space $[L^2(D_c)]^9$ onto the set *K.*

Consider two positive parameters *a,* 8 and auxiliary boundary value problem in O_c of the following form. Find v = (v1,v2,v3) and a-= {o-ij}, such that

avi -0'.Eij(v),j -O"ij,j = Ji ,i = 1,2,3, (64)

CYj + OOj - Gj (V) + $\frac{1}{5P(}$a-)ij = 0, i, j = 1, 2, 3, (65)

O"ijnj + ac:ij (v)nj = 0, i = 1, 2, 3, on r x (0, *T),* (66)

O"ijnj + O'.Eij(v)vj = 0, i = 1, 2, 3, on :Ez x (0, *T),* (67)

v = 0, o- = 0, $t = 0.$ (68)

We first obtain a priori estimates for solutions to (64)-(68). Multiply (64), (65) by Vi, Oj - ij and integrate over n. In the same way as for (29) we derive

$$\sup_{0\leq t\leq T} \|\sigma(t)\|_{0,c}^2 + \alpha\|v\|_{L^2(Q_c)}^2 + \alpha \sum_{i,j=1}^{3} \int_0^T \|\varepsilon_{ij}(v)\|_{0,c}^2 d\tau \leq c \tag{69}$$

with a constant c uniform in a, 8, a :S a0.

From (64), (65), (68) we have

vi(O) = 0, i = 1,2,3, aij(O) = 0, *i,j* = 1,2,3.

Hence, the differentiation of (64), (65) with respect to t and multiplication by Vi, aij - ij result in the estimate

$$\|\dot{\sigma}\|_{L^2(Q_c)}^2 + \alpha\|\dot{v}\|_{L^2(Q_c)}^2 \leq c. \tag{70}$$

Moreover, if v = VN + PN, VN E L^2(0,T;H'Jv(Dc)), PN E L^2(0,T;R(Dc)), then (65) provides

llvNIIL2(0,T;H}; (Dc)) :S *c(8)* (71)

where the constant *c(8)* depends, in general, on *8*

The estimates (69)-(70) allow us to prove the solvability of (64)-(68) for the fixed parameters α, δ in the following sense

$$v^\alpha \in L^2(0,T;H^1(\Omega_c)),\quad \dot v^\alpha, \sigma^\alpha, \dot\sigma^\alpha \in L^2(Q_c),$$

$$\alpha(\dot v^\alpha,\bar v)_c + \alpha(\varepsilon_{ij}(v^\alpha),\varepsilon_{ij}(\bar v))_c + (\sigma^\alpha_{ij},\varepsilon_{ij}(\bar v))_c = (f,\bar v)_c \tag{72}$$

$$\forall \bar v \in L^2(0,T;H^1(\Omega_c)), \tag{73}$$

$$\dot\sigma^\alpha_{ij} + \alpha\sigma^\alpha_{ij} - \varepsilon_{ij}(v^\alpha) + \frac{1}{\delta}p(\sigma^\alpha)_{ij} = 0,\quad i,j=1,2,3, \tag{74}$$

$$v^\alpha = 0,\quad \sigma^\alpha = 0,\quad t=0. \tag{75}$$

The solution of the above problem is denoted by v^α, σ^α, and the following step is a passage to the limit as $\alpha \to 0$. Note that boundary conditions (66), (67) are included in the identity (73).

In view of the estimates (69)-(71) for any fixed $\delta > 0$ one can choose a subsequence, v^α, σ^α, such that as $\alpha \to 0$

$\alpha v^\alpha \to 0$ weakly in $L^2(0,T;H^1(\Omega_c))$,

$v^\alpha \to v^\delta$ weakly in $L^2(0,T;H^1(\Omega_c))$,

$\alpha \dot v^\alpha \to 0$ weakly in $L^2(Q_c)$,

$\sigma^\alpha, \dot\sigma^\alpha \to \sigma^\delta, \dot\sigma^\delta$ weakly in $L^2(Q_c)$.

After the passage to the limit as $\alpha \to 0$ we obtain

$$(\sigma^\delta_{ij},\varepsilon_{ij}(\bar v))_c = (f,\bar v)_c \quad \forall \bar v \in L^2(0,T;H^1(\Omega_c)), \tag{76}$$

$$\dot\sigma^\delta_{ij} - \varepsilon_{ij}(v^\delta) + \frac{1}{\delta}p(\sigma^\delta)_{ij} = 0,\quad i,j = 1,2,3. \tag{77}$$

Analogously to (40) the following estimate holds

$$\|v^\delta\|_{L^2(0,T;BD(\Omega_c))} \le C \tag{78}$$

being uniform in δ. Consequently, without any loss of generality we can assume that there exists a subsequence still denoted by v^δ, σ^δ such that as $\delta \to 0$

$\sigma^\delta, \dot\sigma^\delta \to \sigma, \dot\sigma$ weakly in $L^2(Q_c)$,

$v^\delta \to v$ weakly in $L^2(0,T;L^{3/2}(\Omega_c))$,

$\varepsilon_{ij}(v^\delta) \to \varepsilon_{ij}(v)$ (*)-weakly in $L^2(0,T;M^1(\Omega_c))$, $i,j = 1,2,3$,

$\sigma^\delta(T) \to \sigma(T)$ weakly in $L^2(\Omega_c)$.

From (76) it follows that

(ai$_j$ › G$_j$ (v))$_c$ = (J, v)$_c$ 'vv E £^{2}(0, T; H^1(D$_c$)), (79)

hence, the equations

-(]'i$_j$ ›$_j$ = Ji, i = 1, 2, 3,

are satisfied in Q$_c$ in the sense of distributions. Also, (76) implies that

- a f$_j$ ›$_j$ = Ji, i = 1, 2, 3. (80)

Whence, by (76), (80) we have

(ci$_j$ (v^6)› a f$_j$)c = - (a f$_j$ ›$_j$ " vf)c = (Ji› vf)c.

Moreover,

(ci$_j$ (v^6)› 8\$_j$)c = - (v f, <Ij ›$_j$)c

for all iJ E £^{2}(0, T; UO(D$_c$)). As a result, it follows from (77) for any iJ E £^{2}(0, T; Uo(D.$_c$)), a-(t) E K a. e. in (0, T), that

$$(\dot{\sigma}_{ij}^{\delta}, \bar{\sigma}_{ij} - \sigma_{ij}^{\delta})_c + (v_i^{\delta}, \bar{\sigma}_{ij,j} - \sigma_{ij,j}^{\delta})_c \geq 0. \tag{81}$$

The passage to the limit as 8 -+ 0 can be performed in (81) in the same way as in (46). Therefore, we arrive at (62). The property a(t) E K, t E (0, T), is obtained in the standard way. Boundary conditions (52) are fulfilled in the sense of the space H - ^{1}1^2 (r), and the conditions (53) are satisfied by identity (61) in the weak sense. This completes the proof of Theorem 3.1.

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