Control and Cybernetics

vol. **26** (1997) No. 3

Book review:
Modelling and control in solid mechanics

by

A.M. Khludnev, J. Sokolowski

1. Introduction to the book

The book consists of five chapters. The first one, of introductory role, provides the reader with the notions and fundamental theorems to be frequently referred to in the book. Thus the reader is acquainted with some fundamental theorems on solvability of variational inequalities, minimization of convex functionals, derivatives of convex functionals, also in the non-smooth case. Moreover, weak semicontinuity of functionals is discussed. This part of the Introduction ends with some elements of measure theory. In particular, the theorem on unique representation of a linear continuous functional, which identifies such functionals with measures, is reported. The second part of the Introduction provides a brief sketch of formulations of the equilibrium problems of linearly-elastic and anisotropic bodies as well as isotropic thin shallow shells. A thin plate is treated as a particular case of a thin shell. The contact problem of a thin plate resting on a rigid foundation or a rigid obstacle is outlined in the last section of the Introduction.

A very brief formulation of contact problems of elastic bodies is justified by an ample literature on the subject; it is sufficient to refer to classical monographs by Duvaut and Lions (1972), Necas and Hlavacek (1981), Panagiotopoulos (1985), and Hlavacek, Haslinger, Necas and Lovisek (1983).

2. Chapter 2

Chapter 2, entitled "Variational inequalities in contact problems of elasticity" concerns the following topics:

1. contact between an elastic and a rigid body;
2. contact between two elastic bodies;
3. contact between a shallow shell and a rigid body;
4. contact between two thin plates;
5. the Signorini problem for a shallow shell with moderately large deflections - Marguerre's model;

6. moderately large axial and transverse vibrations of beams and shallow shells with unilateral constraints.

The exposition of the main theorems of this chapter is rigorous. The reader is led directly to new original results concerning existence and regularity problems. Worth emphasising is the proper application of the measure theory, which makes it possible to take into account non-smooth and singular distribution of reaction forces in the contact zones. This part of the book develops the results of Duvaut and Lions (1972), Hlavacek et al. (1983) and Kinderlehrer and Stampacchia (1980).

Out of necessity, the physical meaning of the problems discussed has been dwarfed by mathematical technicalities. Thus, it is worth mentioning here that in all the problems analyzed the contact between two bodies has been viewed as frictionless. The reader interested in taking into account the friction effects is referred to Kalker (1986), Klarbring (1986), Telega (1988, 1990).

3. Chapter 3

The Chapter 3 entitled "Variational inequalities in plasticity" encompasses the problems of existence and uniqueness of solutions of bilateral and unilateral problems of statics and dynamics of three-dimensional bodies, plates, shells and beams within the framework of the Hencky plasticity theory. The results presented develop the results of Duvaut and Lions (1972) and Temam (1985). In an intelligible manner the authors acquaint the reader with the realm of BD spaces. These spaces arc suited to properties of the solutions of the problems of plasticity theory. To the displacement fields describing the plastic phenomena the strains are associated that assume bounded values. Hence the role of BD spaces or spaces of bounded deformations.

The elasto-plastic modelling of the three-dimensional bodies can be applied to modelling of the behaviour of plates, shells and beams. The Authors do not discuss this problem and start just from the lower dimensional formulations. In the case of shell modelling the shallow shell approximation is adopted.

The modern strong tools of analysis have made it possible for the Authors to solve the problems of existence and regularity of a series of complex engineering problems like the problem of a quasi static behaviour of a shallow elasto-plastic shell, Section 6 and the contact problem of a thin clasto-plastic plate, Section 7. The appropriate proofs are complete, the reader being not bothered by referring to the hardly available bibliographical items.

4. Chapter 4

Chapter 4 entitled "Optimal control problems" encompasses the following topics:

1. optimization of distribution of the loading; the merit function adopted is the sum of norms of reactions and loading or the sum of norms of deflection

and loading;
2. optimization of a shape of an obstacle; the functionals to be minimized characterize: a) the deflection and the shape of the obstacle, b) reactions and the shape of the obstacle, c) rotation of the edge and loading, d) rotation of the edge and the shape of the obstacle;
3. optimization of the shape of the obstacle in the bending problem of a clamped plate, fully plastified along its edge, which makes the plate simply supported; the merit function minimized characterizes deflections and the shape of the obstacle;
4. optimization of the shape of the obstacle in a beam compressed and bent; the non-penetration condition couples the axial displacement with the transverse one;
5. optimization of shapes of two punches that bend the plate and make it undergo moderately large deflections; the distribution of membrane forces is assumed as dependent of the norm of the deflection, which makes the problem involving only one scalar unknown; the merit function is minimized and describes deflection and shape of both punches; in particular, one can derive thercfrom a solution within the framework of the H.M. Berger theory, see (Berger (1955); the other special case is the membrane problem in which the bending stiffness is neglected;
6. finding the extreme shapes of the through-the-thickness cracks in plates with obstacles; considerations thereof concern: a) maximization of the norm of the deflection in the bent plate, where the shape of the obstacle and the shape of the crack are design variables, b) maximization of the norm of displacements in three directions in the problem in which the non-penetration condition couples the membrane and bending states, c) similar problems accounting for the elasto-plastic effects.

All the problems mentioned above arc solved completely and convincingly.

5. Chapter 5

The last, fifth, chapter entitled "Sensitivity analysis" is aimed at solving of the following optimization problems:
1. find the distribution of thickness of a thin plate of variable thickness, which minimizes the maximal deflection of the plate with an obstacle;
2. find the shape of an obstacle to minimize the maximal deflection of a clamped plate of a constant thickness;
3. solve the previous problem if the plate is simply supported;
4. recover a domain, known partly by a given observation, where the unknown variable solves the Dirichlet problem;
5. domain optimization problem for parabolic equations.

The last sections of the book concern the sensitivity problems of thin shells within the frameworks of the Koiter and Mushtari-Vlasov shell models. This former model satisfies the rigid body test: equating the deformations to zero implies rotation by an infinitesimal angle and a translation of the whole shell as a rigid body. This model is fully correct. By using the notions of the material, shape, displacement and Euler derivatives of functionals, introduced in Sections 1 and 2, the Authors have performed a sensitivity analysis of solutions to the static problems, with respect to variation of the shell domain. In particular, the method of finding the second derivative of the cost functional is put forward. The lecture of this chapter should be preceded by reading the book of Sokolowski and Zolesio (1992). The analytical methods presented in this chapter complement the independently developed regularization techniques based on homogenization (admitting composite domains), see Bendsoe (1995). One can cherish the hope that the techniques of sensitivity analysis will make it possible to figure out an abundant number of analytical solutions found recently by using the regularization techniques based on admitting the composite regions. In particular the present reviewer sees now no bridging relationship between the results of problem (1) mentioned above for Chapter 5 and the solution technique sketched in Sec. 6.3 of the paper by Lurie and Cherkaev (1986).

6. Final remarks

The book reviewed concerns the brand-new problems of vital interest. A wide range of the subject matter covered by this comprehensive book as well as the rigorous style it has been written in places this monograph among the up-to-date and important texts 9n optimization and control problems of solid mechanics. The book is strongly recommended to specialists in optimum design. On the other hand, the book may tum out too difficult, from the point of view of mathematical tools used, to be recommended for engineers.

Tomasz Lewinski

References

BENDS0E, M.P. (1995) *Optimization of structural topology, shape and material.* Springer-Verlag, Berlin.

BERGER, H.M. (1955) *J. Appl. Mech,* 22, 465-472.

DUVAUT, G. and LIONS, J.L. (1972) *Les inequations en mecanique et physique.* Dunod, Paris.

HLAVACEK, I., HASLINGER, J., NECAS, J. *and* LOVISEK, J. (1983) *Riesenie variacnych nerovnosti v mechanike (Solving variational inequalities of* mechanics). SNTL-Praha.

KALKER, J.J. (1986) Mathematical models of friction for contact problems in elasticity. *Wear,* 113, 61-77.

KINDERLEHRER, D. and STAMPACCHIA, G. (1980) *An Introdv,ction to Variational Inequalities and their Applications.* Academic Press, New York.

KLARBRING, A. (1986) A mathematical programming approach to three-dimensional contact problems with friction. *Comp. Meth. Appl. Mech.*

Engng., 58, 175-200.

LURIE, A.K. and CHERKAEV, A.V. (1986) Effective characteristics of composities and optimum structural design (in Russian). *Adv. Mech.*, 9, 3-81.

NECAS J. and HLAVACEK I. (1981) *Mathematical Theory of Elastic and Elastoplastic Bodies. An IntrodV,ction.* Elsevier, Amsterdam.

PANAGIOTOPOULOS P.D. (1985) *IneqV,ality Problems in Mechanics and Applications. Convex and Nonconvex Energy FV,nctions.* Birkhauser Verlag, Basel-Boston.

SOKOLOWSKI, J. and ZOLESIO, J.P. (1992) *Introdv,ction to Shape Optimization. Shape Sensitivity Analysis.* Springer-Verlag, New York.

TELEGA, J.J. (1988) Topics on unilateral contact problems of elasticity and inelasticity. In: *Nonsmooth Mechanics and Applications,* J.J. Moreau, P.D. Panagiotopoulos, eds. Springer-Verlag, Wien-New York, 341-462.

TELEGA, J.J. (1990) Variational methods and convex analysis in contact problems and homogenization (in Polish). *IFTR Reports,* 38/90, 209.

TEMAM, R. (1985) *Mathematical Problems in Plasticity.* Gauthier-Villars, Paris, Trans-Inter-Scientia, Tonbridge.

A.M. Khludnev, J. Sokolowski: *Modelling and Control in Solid Mechanics.* Birkhauser Verlag AG, Basel, 1997. ISBN 3-7643-5238-8. Price: DM 178.-