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Comparison between GPC and adaptive GPC based on Takagi Sugeno multi-model for an Activated Sludge Reactor*

by

Lamia Matoug[1] **and Med Tarek Khadir**[2]

Badji-Mokhtar Annaba University, LabGED Laboratory, BP 12, 23200
Annaba, Algeria
[1]matoug@labged.net, [2]Khadir@labged.net

Abstract: This paper investigates the use of Adaptive Generalized Predictive Control (TS-AGPC) for an activated sludge reactor, based on a Takagi Sugeno (TS) model, and presents the comparison between the latter and Generalized Predictive Control using an overall TS model (TS-GPC). The reduced bio-reactor Activated Sludge ASM1 Model, which describes the biological degradation of an activated sludge reactor, is designed based on several simplifications, as a TS model, its structure being based on a set of linear submodels, covering the process input-output space, interpolated by a nonlinear weighting function μ. The adaptive GPC approach is obtained by switching between linear submodels of the TS formulation. This is performed by selecting, in turns, a portion of the weighting function μ. The winning model will then act as an internal model for the TS-AGPC control law formulation, whereas the complete TS model is used in the calculation of the TS-GPC control law. Finally, the performance under input and parametric disturbances as well as control variable constraints of the TS-AGPC controller are compared to those for a global TS-GPC controller and a benchmark PID in terms of error and response dynamics.

Keywords: adaptive generalized predictive control, generalized predictive control, Takagi Sugeno, activated sludge reactor, activated sludge model

1. Introduction

Since its first industrial applications, Model Predictive Control (MPC) (Froisy, 1994; Qin and Badgwell, 1996) has widely spread to a broad variety of application areas including chemicals, food processing, automotive branch, and aerospace applications, away from its first petrochemical nest (Qin and Badgwell, 2003).

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The concept of advanced controls can be referred to the work of Kalman, in the early 1960s (Kalman, 1960a, b), which was later on summarized as a problem of optimization, known as the Linear Quadratic Gaussian (LQG) controller. The LQG approach soon became a tool to solve control problems in a wide range of application areas. However, it did not have a great impact on control technology development in industry, because of the existence of heavy constraints, process nonlinearities, model uncertainty, etc. (Richalet et al., 1976, Garcia et al., 1989).

The first description of MPC applications was presented by Richalet in the form of Model Predictive Heuristic Control (MPHC) (Richalet et al., 1976, 1978). The solution software was referred to as IDCOM (IDentification and COMmand), representing the first generation of MPC technology.

Cutler and Ramaker presented an unconstrained multivariable control algorithm Dynamic Matrix Control (DMC) (Cutler and Ramaker, 1979). An application of a modified DMC, including nonlinearities and constraints, was presented by Prett and Gillette (1980).

Then, QDMC was developed in 1983, representing the DMC algorithm in terms of a Quadratic Program (QP) in which input and output constraints were included (Cutler, 1983; Garcia and Morshedi, 1986).

As MPC technology gained wider acceptance in industry for systems with more important complexities, a new generation of MPC technology has been developed, including IDCOM-M, HIECON, SMCA, SMOC, representing the third generation within this methodological domain of research endeavour.

Generalized Predictive Control (GPC) is one of the most popular implementations of MPC (Clarke et al., 1987) along with Predictive Functional Control (PFC) (Richalet, 1993), these approaches having been mainly designed for low complexity internal models, applied to petrochemical systems.

Later, Aspen Technology have developed DMC-plus and RMPCT as the fourth generation MPC technology.

Since then, MPC has established itself as a valuable and efficient form of advanced control in the industrial world, counting thousands of applications. In most cases, a linear model of the process is sufficient for ensuring good control as the robustness of MPC is sufficient for overcoming the process/model mismatches. However, when severe nonlinearities are in presence, Nonlinear Model Predictive Control (NMPC) may be a suitable option (Escano et al., 2009).

Wastewater treatment systems are, nowadays, a challenging control problem, needing advanced control to optimize water quality as well as to reduce costs, especially MPC (Caraman et al. 2007). The treatment is usually performed biologically, using activated sludge reactors. The reduced bio-reactor ASM1 model is able to model the biological processes, including carbon removal, nitrification and denitrification (Henze et al., 1987).

Usually, modelling of ASM1 systems involves the use of complex expressions, based on the knowledge of physical and chemical phenomena. To overcome this difficulty, a solution will be to use a Multi-Model Takagi-Sugeno (TS) approach, meaning to obtain of a set of linear models, combined with a nonlinear function μ. The nonlinearities (premise variables), with the function μ, are represented using a Quasi-LPV (Quasi-Linear Parameter Varying) approach (Huang and Jadbabaie, 1999).

The weight of the system or the model nonlinearity is therefore put in, or expressed through the weighting function μ, with the linear part being expressed by r linear models. Despite the clear resulting advantage, choosing the right premise variables, the adequate number of sub models,...etc., remains a challenge as regards assuring system observability and controllability (Matoug and Khadir, 2014, 2015; Nagy et al., 2010; Nagy, 2010).

The present work investigates the use of MIMO GPC for the control of an activated sludge reactor using as an internal model a TS multi-model, giving a multi-model MPC controller (TS-GPC). As an alternative to the global TS-GPC, an Adaptive Generalized Predictive Control (TS-AGPC) is also investigated, and is based on the same TS model, using, however, only one linear sub model at a time, by switching between sub models (TS-AGPC). The use of the two most significant internal models has, as well, been implemented and tested as TS-AGPC2.

The paper is organized as follows: Section 2 gives a comprehensive description of the activated sludge process and presents the mathematical model of the plant, stating the simplifications made.

In Section 3, the TS approach is presented. In Section 4, the steps taken in the MIMO multi-model GPC algorithm formulation are outlined and Section 5 gives the Adaptive MIMO GPC version. Section 6 presents the obtained results of the TS-GPC and TS-AGPC designs along with a comprehensive performance comparison with the benchmark PID design. Finally, the last section concludes with an extensive comparison between TS-GPC and TS-AGPC and a benchmark PID control in terms of performance and complexity.

2. Activated sludge bioreactor process

2.1. Working principle of the activated sludge treatment plant

Industrialization and urbanization have led to one of the most serious problems of our days: sewage, the polluted water that travels across our cities through underground sewers and ends up in our lakes and rivers, and eventually raises the concern of the states and governments, which subsequently appliy stringent conditions on industry and devices to overcome this problem.

The presence of nitrogen in the water and waste water was regulated by the Community law in the late 1970s. Nitrogen in municipal waste water is primarily treated biologically. The activated sludge is one of methods that are capable of carrying out this treatment.

In 1982, the International Association on Research and Control of Water Pollution (IAWPRC) has implemented the activated sludge model N°1. In 1995, model N°2, including nitrogen removal and biological phosphorus removal (ASM2) was published. In 1999, this process has been replaced by the ASM2d model, including combined denitrification.

In 1998, the working group, dealing with the problem, decided to develop a new process ASM3 (Jeppsson, 1996).

The treatment of organic materials and nitrogen is carried out in a single bio-reactor aerated intermittently (Fig. 1). In the presence of oxygen, autotrophic bacteria oxidize ammonia to nitrite nitrogen and nitrate (nitrification), which will be reduced to nitrogen gas by heterotrophic bacteria in the absence of oxygen (denitrification). The decrease of the carbonaceous filler produces biomass that has to be regularly removed from the processing systems. This is done in the clarifier and the respective output is largely recycled to the bio-reactor, so that only a small amount of sludge is extracted from the system.

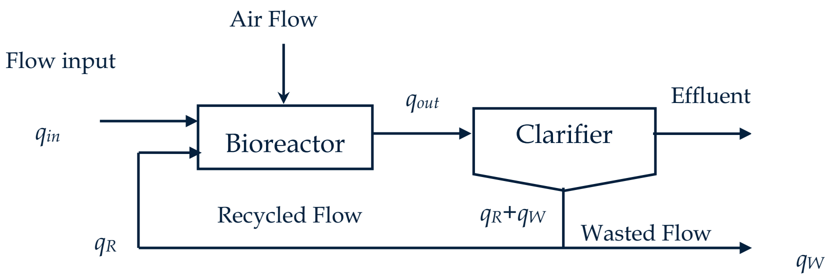


Figure 1: Diagram of activated sludge wastewater treatment process

2.2. Modelling of activated sludge waste water treatment plants

The ASM1 model (Henze et al., 1987) is able to accurately represent the behavior of the process when treating effluent loaded with nitrogen and carbon materials, the initial model being composed of thirteen variables.

In Nagy et al. (2010), a method for the scale model in the activated single aeration basin with sludge surface has been developed on the basis of a number of simplifications, listed below:

- The variable describing alkalinity is not included,
- The products of the biomass decomposition (dead) (X_P) are included together with the inert organic compounds and the particulate is given by the variable (X_I)
- Oxygen concentration is considered as state variable,
- The inert soluble and particulate organic matter (S_I) and (X_I) are neglected in the reduced model,
- Inhibition of the mechanism of hydrolysis and ammonification, and elimination of the two fractions of organic nitrogen, soluble S_{ND} and particulate X_{ND} are accounted for through, respectively,
- The growth of autotrophic active biomass (X_{BA}) is not taken into account,
- Nitrogen in the form of nitrate and nitrite, S_{NO}, is a small quantity and is thus eliminated

- The mechanism of formation of nitrogen in ammonia form, S_{NH}, is not considered to be critical for the control.

The treatment method with activated single aeration basin surface sludge in the aerobic phase consists in mixing used waters with a rich mixture of bacteria in order to degrade the organic matter. The ASM1 model will then only consider the organic substrate removal process, and will be reduced to the interaction of three state variables. The final mathematical representation is given by equations (1).

$$\begin{cases} \dot{X}_{BH} = \left[\hat{\mu_H}\dfrac{S_S}{K_S+S_S}\dfrac{S_O}{K_{OH}+S_O} - b_H\right]X_{BH} \\ \qquad -\dfrac{1+f_R}{(f_R+f_W)}(X_{BH}/V)q_W + \dfrac{q_{in}}{V}X_{BH}^{in}, \\ \dot{S}_S = \dfrac{q_{in}}{V}(S_S^{in}-S_S) + (1-f_P)b_H X_{BH} \\ \qquad -\dfrac{\hat{\mu_H}}{Y_H}\dfrac{S_S}{K_S+S_S}\dfrac{S_O}{K_{OH}+S_O}X_{BH}, \\ \dot{S}_O = -\dfrac{q_{in}}{V}S_O + Kq_a(S_O^{max}-S_O) \\ \qquad -\hat{\mu_H}\dfrac{1-Y_H}{Y_H}\dfrac{S_S}{K_S+S_S}\dfrac{S_O}{K_{OH}+S_O}X_{BH} \end{cases} \tag{1}$$

where X_{BH}, S_S, and S_O represent, respectively, the heterotrophic biomass concentration, substrate concentration and the dissolved oxygen concentration.

Usually, q_R and q_W, being, respectively, the input recycled flow and the wasted flow, represent the fractions of input flow q_{in}:

$$q_R = f_R q_{in}, 1 \leq f_R \leq 2$$

$$q_W = f_W q_{in}, 0 \leq f_W \leq 1.$$

The parameters used in this model are defined in Table 1 (Nagy, 2010).

In Table 1, Y_H is the conversion rate of substrate/ heterotrophic biomass, f_P is the fraction of inert DCO, $\hat{\mu}_H$ the maximum growth rate of heterotrophic biomass, b_H the mortality rate of heterotrophic biomass, K_S the coefficient of half saturation of rapidly biodegradable substrate, K_{OH} the coefficient of half saturation of oxygen for heterotrophic biomass, K the gain regulator of oxygen, V the volume setting of the reactor, S_O^{max} the concentration of oxygen saturation, and f_R, f_W the fractions of the recycled and wasted sludge, respectively.

3. Introduction to the fuzzy Takagi-Sugeno model

TS (Takagi-Sugeno) models represent a very interesting mathematical formulation of nonlinear systems. These can thereby be easily represented, regardless of their complexity, with a simple structure, based on a nonlinear combination

Table 1: Parameters of ASM 1 model

Notation	Value	Unite
Y_H	0.67	–
f_P	0.08	–
$\hat{\mu}_H$	4	$[1/24]h^{-1}$
b_H	0.3	$[1/24]h^{-1}$
K_S	10	g/m^3
K_{OH}	0.2	g/m^3
K	2.3	m^{-3}
V	6000	m^3
S_O^{max}	10	g/m^3
f_R	1.1	–
f_W	0.03	–

of a set of linear models (Murray-Smith and Johansen, 1997; Li et al., 2004; Smets et al., 2006). This simple structure with interesting properties, makes TS models easily exploitable from a mathematical point of view, allowing them to be used as internal models in linear MPC algorithms such as GPC, DMC and PFC, widely recognised for their proven efficiency in the industrial and academic world.

3.1. Representation of fuzzy Takagi Sugeno models

Fuzzy Takagi Sugeno models are represented by fuzzy rules such as "IF-THEN" (Takagi and Sugeno, 1985). The i^{th} fuzzy rule of continuous TS model is then written as:

if $z_1(t)$ is $F_1^i(z_1(t))$ and $\cdots$ $z_p(t)$ is $F_p^i(z_p(t))$,

$$\text{then} \begin{cases} x(t+1) = A_i x(t) + B_i u(t), \\ y(t) = C_i x(t) + D_i u(t) \quad i\text{=1,2,}\ldots\text{,}r. \end{cases} \tag{2}$$

where $F_j^i(z_j(t))$ for $j = 1, \ldots, p$ are fuzzy sets, p is the number of fuzzy rules, $z_j(t)$ are the premise variables that depend on the input and/or state of the system, $x(t) \in \Re^n, y(t) \in \Re^q, u(t) \in \Re^m$, respectively, represent the state vector, the output vector and the control vector. $A_i \in \Re^{n\times n}, B_i \in \Re^{n\times m}, C_i \in \Re^{q\times n}, D_i \in \Re^{q\times m}$ are matrices describing the system dynamics.

Each rule is assigned a weight noted $\mu_i(z(t))$, which depends on the degree of membership of the premise variables $z_j(t)$ in the fuzzy subsets $F_j^i(z_j(t))$ and the connector "and" connecting the premises selected such that:

$$\mu_i(z(t)) = \prod_{j=1}^{p} F_j^i(z_j(t)), \quad i{=}1{,}2{,}\ldots{,}r. \tag{3}$$

$F_j^i(z_j(t))$ represent the values of the membership function $z_j(t)$ with respect to the fuzzy set F_j^i. We then have the following properties:

$$\begin{cases} \sum_{i=1}^{r} \mu_i(z(t)) = 1, \\ \quad \mu_i(z(t)) \geq 0 \quad i{=}1{,}2{,}\ldots{,}r. \end{cases} \tag{4}$$

Finally, the defuzzification of the fuzzy model provides the state representation of a nonlinear model by interconnecting local time invariant models by nonlinear activation functions, obtaining :

$$\begin{cases} x(t+1) = \dfrac{\sum_{i=1}^{r} \mu_i(z(t))\{A_i x(t) + B_i u(t)\}}{\sum_{i=1}^{r} \mu_i(z(t))}, \\ y(t) = \dfrac{\sum_{i=1}^{r} \mu_i(z(t))\{C_i x(t) + D_i u(t)\}}{\sum_{i=1}^{r} \mu_i(z(t))} \end{cases}. \tag{5}$$

3.2. Construction of fuzzy Takagi-Sugeno models

For obtaining a TS model, three approaches are widely used in the literature. The first approach relies on identification techniques (Gasso, 2000). The second is based on the linearization of the nonlinear model around several operating points, and the third one is based directly on the analytical knowledge of the nonlinear model and is known as the nonlinear transformation sectors (Tanaka and Wang, 2001; Morere, 2001; Wang et al., 1996). The principle thereof is based on a polytopic convex nonlinear transformation of the dynamic system's nonlinear terms.

Unlike the first two approaches that give a definite approximation of the nonlinear model, the third method provides a representative and accurate TS model that is as close as possible to the nonlinear model. Note that the sector nonlinearity approach allows to associate an infinity of TS models with a nonlinear system, based on the division of nonlinearities achieved. A systematic approach to cutting nonlinear areas is based on the following Polytopic Convex Transformation (PCT) lemma (Tanaka and Wang, 2001; Morere, 2001):

Lemma 1 *Polytopic Convex Transformation (PCT) (Tanaka and Wang, 2001) Let $z_j((x(t), u(t))$ be a bounded continuous function on the domain $D \subset \Re^n \times \Re^m$, having values in $\Re$, with $x(t) \in \Re^n, u(t) \in \Re^m$.*

Then there exist two functions $(k = 1, 2)$

$$F_{j,k} : D \mapsto [0, 1]$$

$$(x(t), u(t)) \mapsto F_{j,k}(x(t), u(t))$$

$$\textit{with } F_{j,1}(x(t), u(t)) + F_{j,2}(x(t), u(t)) = 1 \textit{ such that}$$

$$z_j(x(t), u(t)) = F_{j,1}(x(t), u(t))z_{j,1} + F_{j,2}(x(t), u(t))z_{j,2}$$

for all $z_{j,1} \geq \underset{x,u\in D}{max}\{z_j(x,u)\}$ *and* $z_{j,2} \leq \underset{x,u\in D}{min}\{z_j(x,u)\}$.

The functions $F_{j,1}$ *and* $F_{j,2}$ *are defined by:*

$$F_{j,1}(x(t), u(t)) = \frac{z_j(x(t), u(t)) - z_{j,2}}{z_{j,1} - z_{j,2}}$$

$$F_{j,2}(x(t), u(t)) = \frac{z_{j,1} - z_j(x(t), u(t))}{z_{j,1} - z_{j,2}}$$

where

$z_{j,1} = \underset{x,u}{max}\{z_j(x,u)\}, z_{j,2} = \underset{x,u}{min}\{z_j(x,u)\}$.

3.3. Quasi-linear parameter variable form ”Quasi-LPV”

The first step is to convert the nonlinear model into the ”Quasi-Linear Variable Parameters” form, called ”Quasi-LPV” model, given by (6)

$$\begin{cases} \dot{x} = A(x,u)x + B(x,u)u, \\ y = C(x,u)x + D(x,u)u. \end{cases} \tag{6}$$

There are several possible choices for the Quasi-LPV form, depending on the choice of premise variables. The Quasi-LPV form, preferably contains a low number of premise variables, also depending on the lowest number of state variables. Reduction of the number of premise variables affects proportionally the number of sub-models, the overall model structure, as well as system feasibility.

Observability/controllability of the overall system must also be ensured for each submodel (Huang and Jadbabaie, 1999). In order to ensure the controllability/observability of the overall system, finding a solution for the LMIs using the Lyapunov method associated to each submodel is then necessary (Guerra et al., 2009).

3.4. Application to ASM1 model

The activated sludge process, shown in Fig. 1, is represented by the system of differential equations (1) that contains four nonlinearities. The selected premise variables that ensure the satisfaction of the Quasi-LPV form selection criteria, previously cited, are shown in formulae (7):

$$\begin{cases} z_1(S_S, S_O) = \dfrac{S_S}{K_S + S_S}\dfrac{S_O}{K_{OH} + S_O} \\ z_2(q_{in}, V) = \dfrac{q_{in}}{V}, \\ z_3(q_a) = q_a, \\ z_4(X_{BH}) = X_{BH}. \end{cases} \tag{7}$$

The number of sub-models is given by 2^p with p being the number of premise variables, i.e, $2^4 = 16$ sub-models, which are presented by pairs $(A_i, B_i)(i = 1, \ldots, 16)$, as this is shown below

$$\begin{aligned}
&A_1 = A(z_{1,1}, z_{2,1}, z_{3,1}) \;\; B_1 = B(z_{2,1}, z_{4,1})\\
&A_2 = A(z_{1,1}, z_{2,1}, z_{3,1}) \;\; B_2 = B(z_{2,1}, z_{4,2})\\
&A_3 = A(z_{1,1}, z_{2,1}, z_{3,2}) \;\; B_3 = B(z_{2,1}, z_{4,1})\\
&A_4 = A(z_{1,1}, z_{2,1}, z_{3,2}) \;\; B_4 = B(z_{2,1}, z_{4,2})\\
&A_5 = A(z_{1,1}, z_{2,2}, z_{3,1}) \;\; B_5 = B(z_{2,2}, z_{4,1})\\
&A_6 = A(z_{1,1}, z_{2,2}, z_{3,1}) \;\; B_6 = B(z_{2,2}, z_{4,2})\\
&A_7 = A(z_{1,1}, z_{2,2}, z_{3,2}) \;\; B_7 = B(z_{2,2}, z_{4,1})\\
&A_8 = A(z_{1,1}, z_{2,2}, z_{3,2}) \;\; B_8 = B(z_{2,2}, z_{4,2})\\
&A_9 = A(z_{1,2}, z_{2,1}, z_{3,1}) \;\; B_9 = B(z_{2,1}, z_{4,1})\\
&A_{10} = A(z_{1,2}, z_{2,1}, z_{3,1}) \;\; B_{10} = B(z_{2,1}, z_{4,2})\\
&A_{11} = A(z_{1,2}, z_{2,1}, z_{3,2}) \;\; B_{11} = B(z_{2,1}, z_{4,1})\\
&A_{12} = A(z_{1,2}, z_{2,1}, z_{3,2}) \;\; B_{12} = B(z_{2,1}, z_{4,2})\\
&A_{13} = A(z_{1,2}, z_{2,2}, z_{3,1}) \;\; B_{13} = B(z_{2,2}, z_{4,1})\\
&A_{14} = A(z_{1,2}, z_{2,2}, z_{3,1}) \;\; B_{14} = B(z_{2,2}, z_{4,2})\\
&A_{15} = A(z_{1,2}, z_{2,2}, z_{3,2}) \;\; B_{15} = B(z_{2,2}, z_{4,1})\\
&A_{16} = A(z_{1,2}, z_{2,2}, z_{3,2}) \;\; B_{16} = B(z_{2,2}, z_{4,2}).
\end{aligned}$$

Finally, the nonlinear system (1) will be the sum of 16 linear models interpolated by the nonlinear functions, given as a final result in (8):

$$x(t+1) = \frac{\sum_{i=1}^{16} \mu_i(z(t))\{A_i x(t) + B_i u(t)\}}{\sum_{i=1}^{16} \mu_i(z(t))}, \tag{8}$$

where $\mu_i(z(t))$ (Fig. 4) are given through the formula (3) as the following expressions:

$$
\begin{aligned}
\mu_1(z(t)) &= F_{1,1}(z_1(t))F_{2,1}(z_2(t))F_{3,1}(z_3(t))F_{4,1}(z_4(t))\\
\mu_2(z(t)) &= F_{1,1}(z_1(t))F_{2,1}(z_2(t))F_{3,1}(z_3(t))F_{4,2}(z_4(t))\\
\mu_3(z(t)) &= F_{1,1}(z_1(t))F_{2,1}(z_2(t))F_{3,2}(z_3(t))F_{4,1}(z_4(t))\\
\mu_4(z(t)) &= F_{1,1}(z_1(t))F_{2,1}(z_2(t))F_{3,2}(z_3(t))F_{4,2}(z_4(t))\\
\mu_5(z(t)) &= F_{1,1}(z_1(t))F_{2,2}(z_2(t))F_{3,1}(z_3(t))F_{4,1}(z_4(t))\\
\mu_6(z(t)) &= F_{1,1}(z_1(t))F_{2,2}(z_2(t))F_{3,1}(z_3(t))F_{4,2}(z_4(t))\\
\mu_7(z(t)) &= F_{1,1}(z_1(t))F_{2,2}(z_2(t))F_{3,2}(z_3(t))F_{4,1}(z_4(t))\\
\mu_8(z(t)) &= F_{1,1}(z_1(t))F_{2,2}(z_2(t))F_{3,2}(z_3(t))F_{4,2}(z_4(t))\\
\mu_9(z(t)) &= F_{1,2}(z_1(t))F_{2,1}(z_2(t))F_{3,1}(z_3(t))F_{4,1}(z_4(t))\\
\mu_{10}(z(t)) &= F_{1,2}(z_1(t))F_{2,1}(z_2(t))F_{3,1}(z_3(t))F_{4,2}(z_4(t))\\
\mu_{11}(z(t)) &= F_{1,2}(z_1(t))F_{2,1}(z_2(t))F_{3,2}(z_3(t))F_{4,1}(z_4(t))\\
\mu_{12}(z(t)) &= F_{1,2}(z_1(t))F_{2,1}(z_2(t))F_{3,2}(z_3(t))F_{4,2}(z_4(t))\\
\mu_{13}(z(t)) &= F_{1,2}(z_1(t))F_{2,2}(z_2(t))F_{3,1}(z_3(t))F_{4,1}(z_4(t))\\
\mu_{14}(z(t)) &= F_{1,2}(z_1(t))F_{2,2}(z_2(t))F_{3,1}(z_3(t))F_{4,2}(z_4(t))\\
\mu_{15}(z(t)) &= F_{1,2}(z_1(t))F_{2,2}(z_2(t))F_{3,2}(z_3(t))F_{4,1}(z_4(t))\\
\mu_{16}(z(t)) &= F_{1,2}(z_1(t))F_{2,2}(z_2(t))F_{3,2}(z_3(t))F_{4,2}(z_4(t)).
\end{aligned}
$$

Here, the state vector is: $x = [X_{BH}, S_S, S_O]^T$, and the input vector is $u = \left[q_W, q_a, X_{BH}^{in}, S_S^{in}\right]^T$, respectively, the wasted flow (the control variable), the air flow input, heterotrophic bacteria and carbon substrate inputs being treated as non controllable input disturbances.

The choice of the input vector for the model is heavily influenced by real conditions, challenging the chosen MPC control strategies, these real conditions including constraints, disturbances as well as realistic control variables.

3.5. Model evaluation

The evolution of the nonlinear system outputs, given by the differential equations (1) and the TS model outputs, obtained through application of the formulation (5), using the inputs shown in Fig. 2, can be seen in Fig. 3.

For space and clarity reasons, only eight of the sixteen weighting functions of the multiple model are presented in Fig. 4.

The air flow input q_a influences directly the dissolved oxygen concentration S_O, an increase of the air flow produces an increase of the oxygen concentration while its drop produces the decrease of the oxygen concentration.

The soluble carbon substrate and the heterotrophic biomass concentration (respectively S_S and X_{BH}) are influenced by their corresponding input concentrations (S_S^{in} and X_{BH}^{in}).

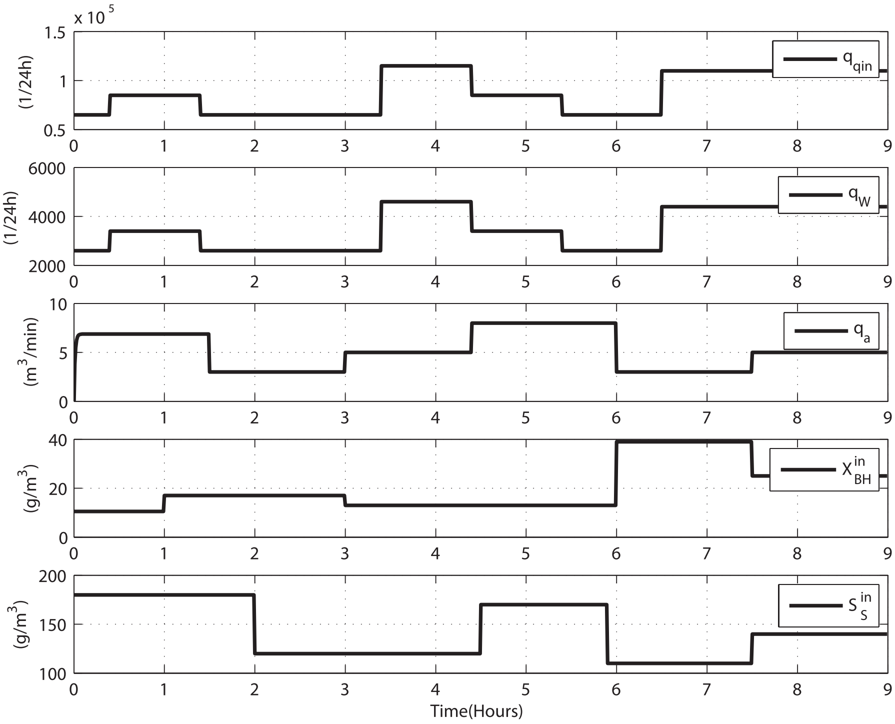


Figure 2: System inputs

The q_{in} input flow influences the three concentrations: X_{BH}, S_S, and S_O. An increase of the input flow q_{in} produces an increase of the substrate concentration S_S and the heterotrophic biomass concentration X_{BH}, as well as a decrease of the dissolved oxygen concentration S_O.

As expected, it can be clearly seen that the obtained TS model, based on a polytopic convex nonlinear transformation (see Section 3.3) called "Quasi-LPV" model (Matoug and Khadir, 2012), follows scrupulously the chosen ASM1 model.

The evaluation gives the MSE (Mean Square Error) equal to 0.1614. The TS multi-model formulation can therefore be used in a control based strategy also because of subsystem linearity.

4. Multi-model predictive control based on TS model

A TS-GPC formulation may be obtained based on the original GPC algorithm (Clarke et al., 1987) using as an internal prediction model the TS formulation, obtained in Section 3.4. In what follows, the original MIMO GPC algorithm is described and applied to each sub-model as the first step. The global control law

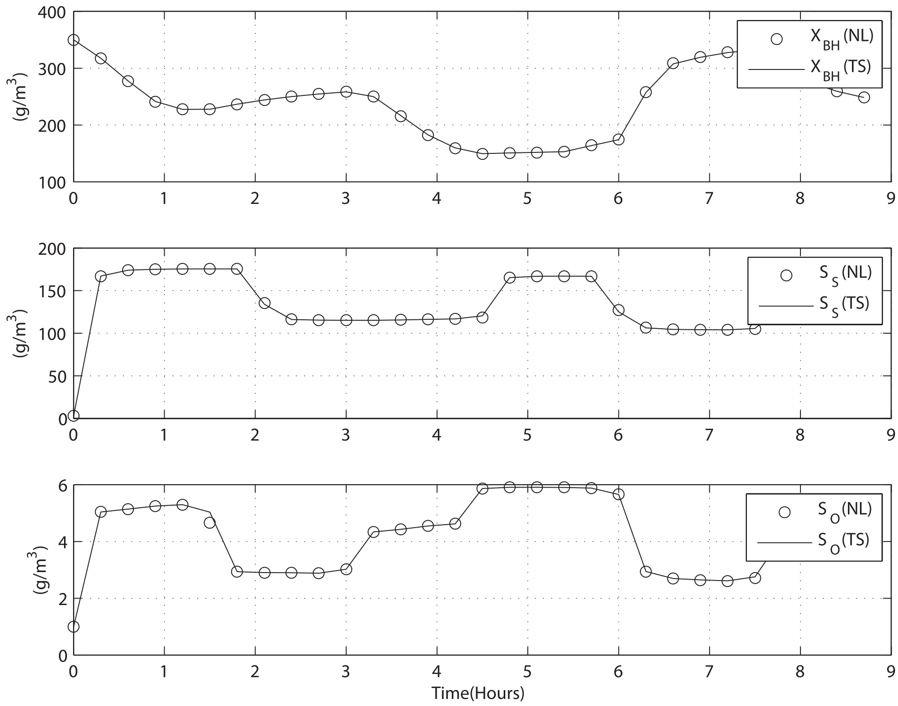


Figure 3: Outputs of the nonlinear system and multi-model

modifications needed to use the complete TS model (a nonlinear aggregation of the 16 submodels) are presented as the second step.

4.1. Local MIMO GPC

In this section, the procedure to obtain the GPC control law formulation is presented for the MIMO case. System dynamics is formulated using state space description as:

$$\begin{cases} x(k+1) = A_i x(k) + B_i u(k) \\ \quad\;\; y(k) = C_i x(k) + D_i u(k) \end{cases} \tag{9}$$

where x is the n-dimensional state vector, A_i, B_i, C_i, D_i are matrices of dimensions $(n \times n)$, $(n \times m)$, $(r \times n)$, and $(s \times r)$, respectively. The MIMO GPC

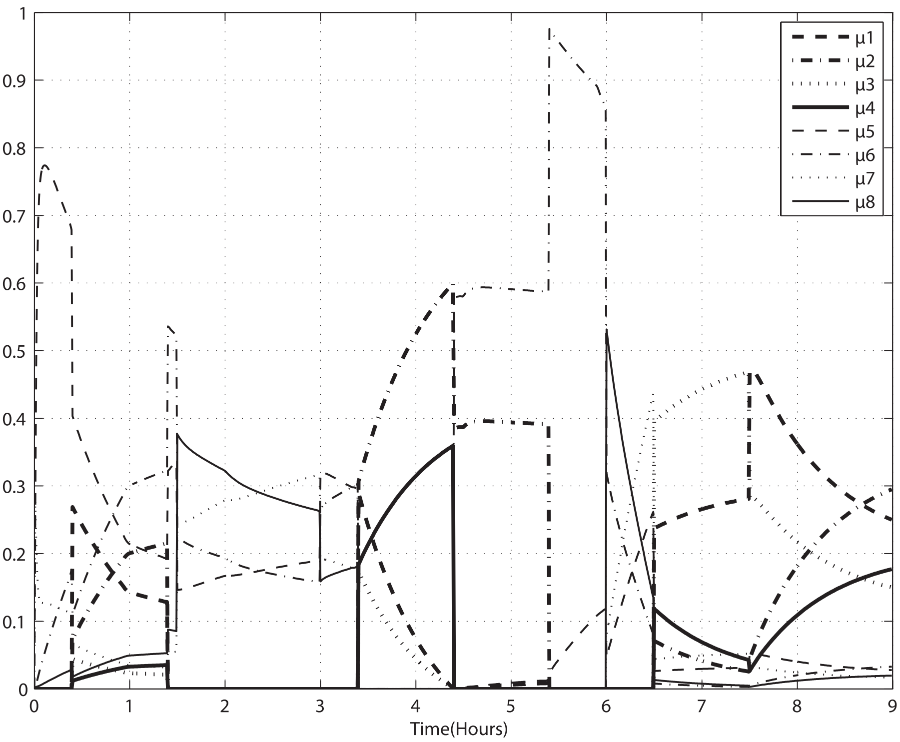


Figure 4: Nonlinear weighting function

control law is calculated to minimize the following cost function:

$$J(N_1, N_2, N_c, \lambda) = \sum_{j=N_1}^{N_2} (y(k+j) - w(k+j))^T (y(k+j) - w(k+j)) + \sum_{j=1}^{N_c} \lambda(\Delta u(k+j-1))^T \Delta u(k+j-1)), \tag{10}$$

where y is the r-vector of predicted outputs, w is the r-vector of the setpoint and Δu is the s-vector of input increments. The following section develops the calculation steps of the MPC optimal control law minimizing the above cost function (10).

4.1.1. Predicted outputs:

Consider a MIMO system (9). The outputs $y(k+j)(j = N1, \ldots, N2)$ based on the system information available up to the sampling time k using a recursion procedure from all the subsystems can be computed and are given by:

$$y_i(k+j) = CA^j x(k) + \sum_{p=0}^{j-1} CA_i^{j-p-1} B_i \Delta u(k+p). \tag{11}$$

In the matrix form we get:

$$y_i(k+j) = [CA_i^{j-1}B_i \quad CA_i^{j-2}B_i \cdots CA_iB_i \quad CB_i]$$
$$\begin{pmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \vdots \\ \Delta u(k+j-2) \\ \Delta u(k+j-1) \end{pmatrix} + CA_i^j x(k) \tag{12}$$

where

$$y = \begin{bmatrix} y_1(k+N_1) \\ \vdots \\ y_r(k+N_1) \\ \vdots \\ \\ \vdots \\ y_1(k+N_2) \\ \vdots \\ y_r(k+N_2) \end{bmatrix}_{(N_2-N_1+1)r\times 1} \tag{13}$$

$$\Delta u_i = \begin{bmatrix} \Delta u_1(k) \\ \vdots \\ \Delta u_s(k) \\ \Delta u_1(k+1) \\ \vdots \\ \Delta u_s(k+1) \\ \vdots \\ \\ \vdots \\ \Delta u_1(k+Nu-1) \\ \vdots \\ \Delta u_s(k+Nu-1) \end{bmatrix}_{(N_c s\times 1} . \tag{14}$$

The predicted outputs are given in a more compact form as follows:

$$y_i = G_i \Delta u + f_i. \tag{15}$$

Matrices G and f are calculated using the Diophantine-based technique (Clarke et al., 1987), obtaining: to fit the control law requirements:

$$G_i = \begin{bmatrix} CA_i^{N_1-1}B_i & CA_i^{N_1-2}B_i & \cdots 0 \\ \vdots & \vdots & \ddots \; \vdots \\ CA_i^{N_2-1}B_i & CA_i^{N_2-2}B_i & \cdots 0 \end{bmatrix}_{(N_c s \times 1} \tag{16}$$

$$f = \begin{bmatrix} CA^{N_1} \\ \vdots \\ CA^{N_2} \end{bmatrix} x(k). \tag{17}$$

4.1.2. MIMO GPC control law formulation

The cost function (10) can be written in a more compact form, by substituting the predicted outputs by the respective expression:

$$\begin{aligned} J(N_1, N_2, N_c, \lambda) &= (y-w)^T(y-w) + \lambda \Delta u^T \lambda \Delta u \\ &= (G\Delta u + w - w)^T (G\Delta u + w - w)^T \\ &+ \lambda \Delta u^T \lambda \Delta u. \end{aligned} \tag{18}$$

Simplifying the equation above one gets:

$$J = \tfrac{1}{2}\Delta u^T H \Delta u + 2[(f-w)^T G] + f_0 \tag{19}$$

where $H = 2[G^T G + \lambda I]$, $f_0 = (y-w)^T(y-w)$.

The optimal solution is obtained upon setting:

$$\frac{\partial J}{\partial \Delta u} = 0$$

$$\begin{aligned} \Delta u &= -2H^{-1}G^T(f-w), \\ &= [G^T G + \lambda I]^{-1} G^T (w - f). \end{aligned} \tag{20}$$

Finally, the actual control signal sent to the process is constituted by the first s elements of the optimal solution obtained above.

The MIMO GPC control law developed in (20) is based on the system having input Δu. The model has to be transformed accordingly. Once this is done, GPC control equation can be derived and used to compute the output. The control variable u is then computed as: $u(k+1) = u(k) + \Delta u(k+1)$ and only then, constraints, limiting u at every sample time instant, are applied.

4.2. The multi-model generalized predictive control

In this section, a parallel control configuration (shown in Fig. 5) is used to implement the multi-model predictive controller. In Section 4.1, the local stable generalized controller (controller i) has been designed for each model (A_i, B_i), and so, the global controller can be implemented by weighted integration of all the local controllers. That is, for the given regression vector $x(k)$, the global control law is given by

$$\Delta u(k) = \sum_{i=1}^{r} \mu_i \Delta u_i / \sum_{i=1}^{r} \mu_i. \quad (21)$$

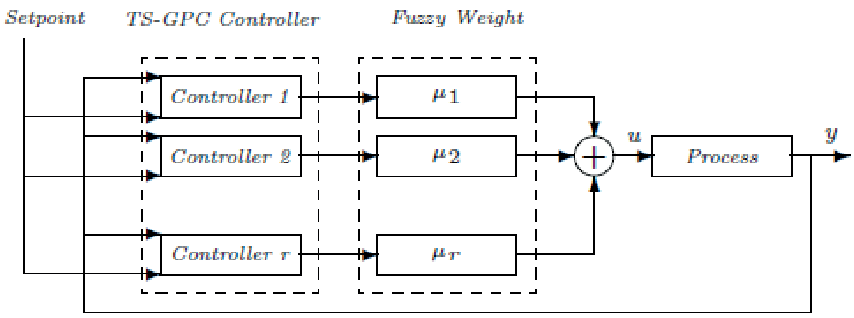


Figure 5: Weighted integration of local controller

5. Adaptive TS-GPC

The adaptive TS-AGPC controller uses an internal model, resulting from an adaptive switching among local TS models. Therefore, a unique i^{th} local model is used to describe the global system influencing the resulting control input.

Fig. 6 shows the switching scheme used in the TS-AGPC approach, based on the biggest value of the weighting functions μ_i. At an instant k, the weighting functions μ_i, associated with the 16 submodels are calculated and the biggest value of μ_i is found. Consequently, this μ_i is the weight of the selected model i and, therefore, we use the latter to predict the process output and calculate the optimal control increment $\Delta u(k)$.

Unlike the TS-GPC control approach that uses a combination of all 16 sub control increments wherein the output of each submodel will be calculated through the appropriate control increment, the TS-AGPC control uses at each time k of a control increment i, a unique selected submodel in order to calculate the associated control law.

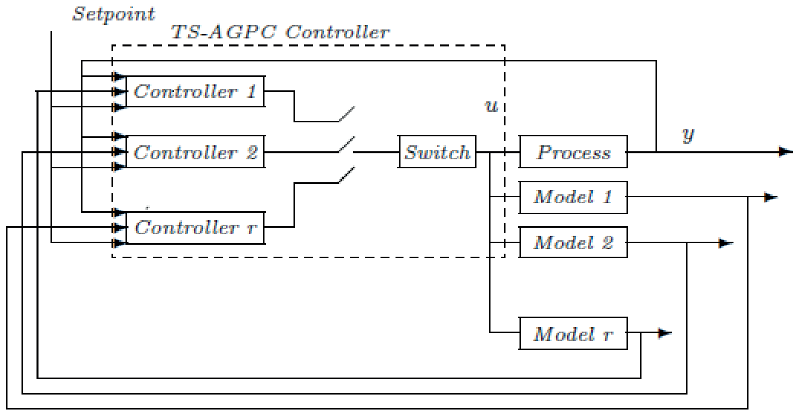


Figure 6: Switching of local controller in the TS-AGPC approach

6. Simulation results

A set of simulations have been performed in order to test the developed controllers, under the following assumptions and parameters:

1. 9 hours of simulation time, corresponding to 900 samples, obtained with the sampling interval of 36 s.
2. To ensure flexible control, N_c is equal to 1, in the case where there is no unstable pole in the open-loop system.
3. The prediction horizon $N_2 = 50$ (for the maximum efficiency, N_2 is equal to the response time of the system).
4. Input constraints are imposed on the waste water flow q_W.
5. Disturbances appear through air flow input q_a, heterotrophic bacteria X_{BH}^{in} and carbon substrate input S_S^{in}.
6. Uncertainties concern the parameter of the mortality rate of heterotrophic biomass b_h.

Fig. 7 represents the fuzzy weight $\mu_i(z(t))$, given by equation (3), when the above listed assumptions are applied.

According to Fig. 7, the introduced input and parametric disturbances have seriously impaired the weighting functions μ_i as well as the performance of the overall model. The explanation is given by the fact that the premise variables, which depend on q_a, X_{BH}^{in}, and S_S^{in}, are in this case heavily affected by disturbances in the air flow q_a, heterotrophic bacteria X_{BH}^{in} and carbon substrate S_S^{in} (see Fig. 7).

From Fig. 7 one can clearly see that at the sampling times 3h50mn and 4h20mn, μ_2 and μ_4 overlap. The same observations can be made for the samples 7h50mn, 8h00mn and 8h20mn, where μ_1 and μ_3 overlap. Overlapping in μ_i is the evidence for the increased importance of associated submodels, for instance,

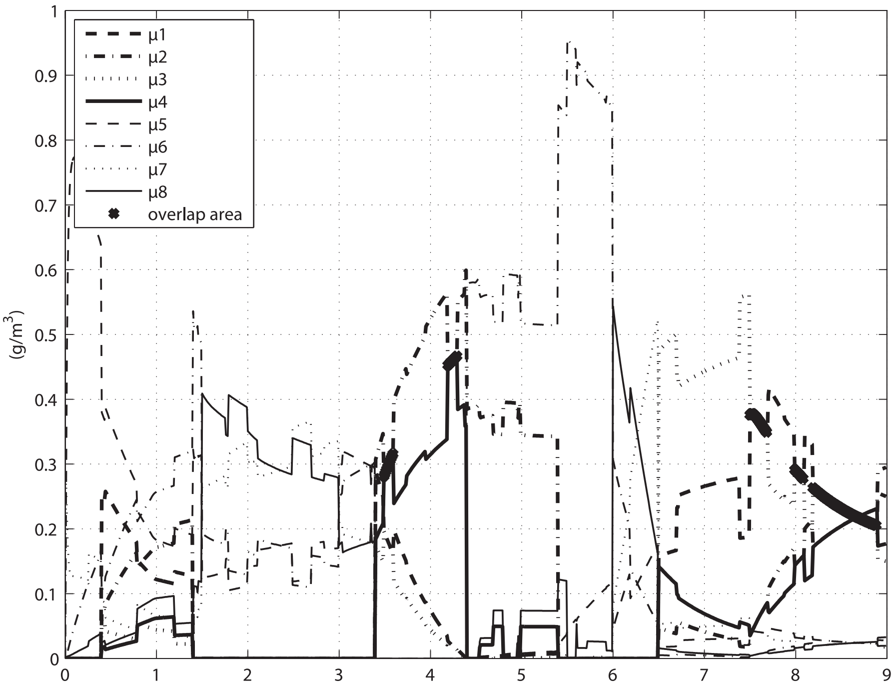
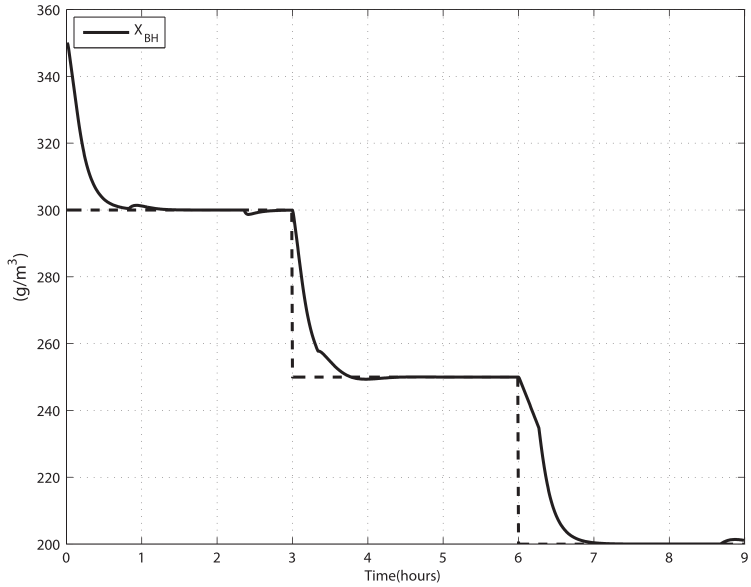


Figure 7: Nonlinear weighting function with disturbances

models 2 and 4 at time 3h50mn, and models 1 and 3 at time 7h50mn.

From Fig. 8, it can be seen that the TS-AGPC control, in the absence of disturbances, gives excellent results, slightly better than TS-GPC control; however, the effects of the sudden changes in the submodel selection around 0h83mn, as we switch from submodel 5 to submodel 6, and around 3h36mn, as we switch from submodel 5 to submodel 2, can nevertheless be noticed. This phenomenon is, logically, not apparent when using TS-GPC, as the controller uses the complete TS model with the combination of the 16 submodel outputs (see Fig. 9).

In the presence of disturbances and constraints, the TS-AGPC shows weaknesses in different scenarios (see Fig. 10), as it uses only one model at a time for control law calculation, based on the maximum values of the weighting functions μ_i. It will, then, select one submodel at a time, neglecting the output of the second submodel of importance in the case of the overlapping in μ_i. This will induce a loss of valuable information and process model mismatch, as the global model is the sum of all sub-models, where each submodel represents better the global system in a specific region of input/output space. The rest of the models represent additional information, necessary to better render the behavior of the global nonlinear model. These pieces of information are lost during the switching process.



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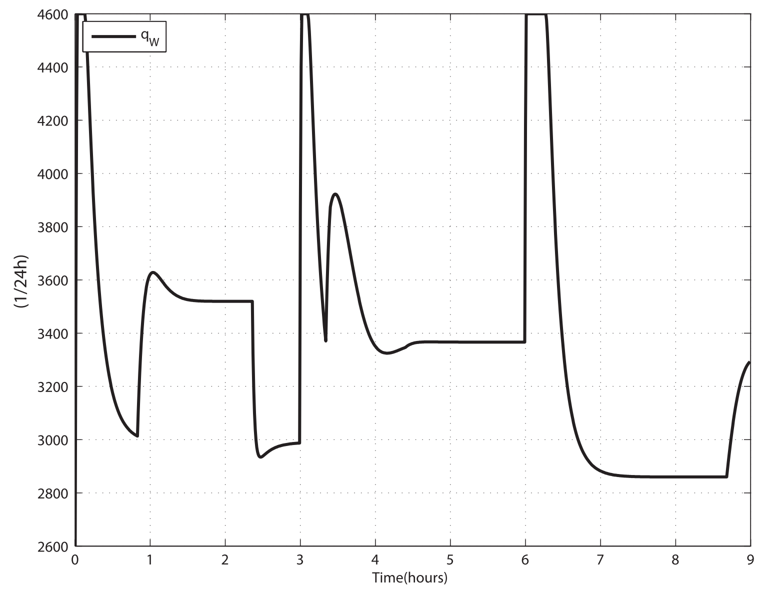
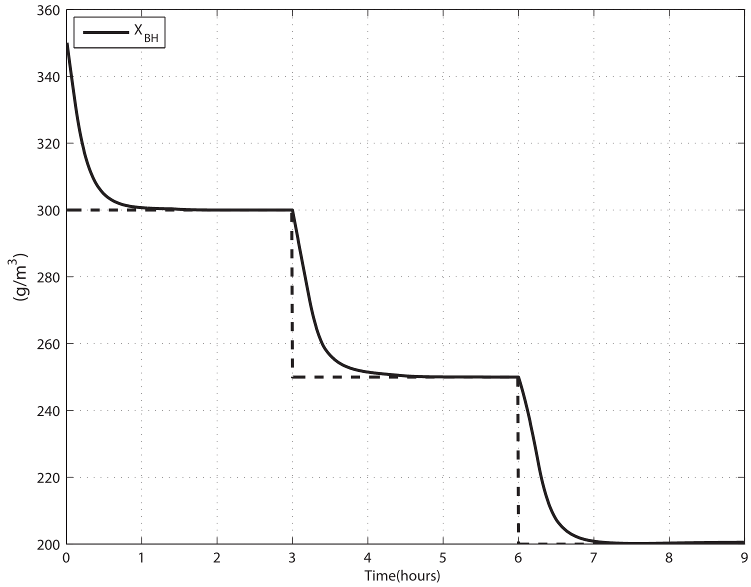


(b)

Figure 8: TS-AGPC without disturbances: (a) X_{BH}; (b) q_w



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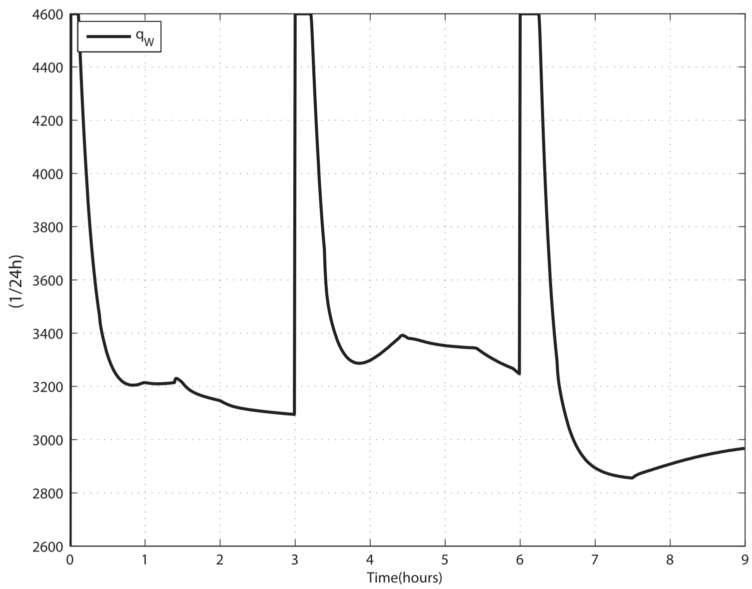
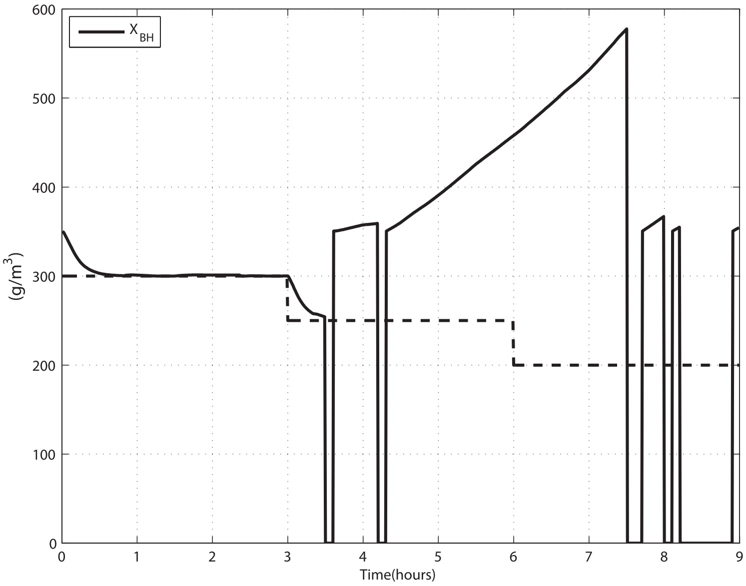


(b)

Figure 9: TS-GPC without disturbances: (a) X_{BH}; (b) q_w



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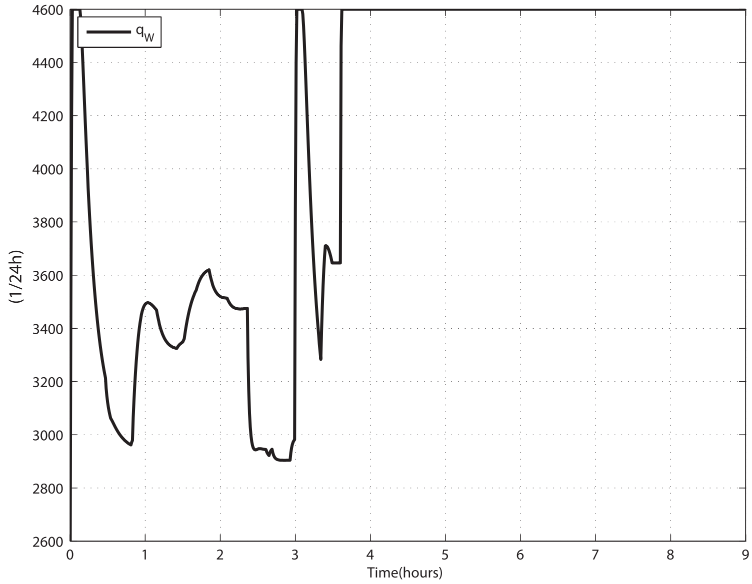


(b)

Figure 10: TS-AGPC with disturbances: (a) X_{BH}; (b) q_w

To overcome this problem, we propose to use more than one submodel at a time, i.e., for instance, choosing both submodels 1 and 3 when μ_1 and μ_3 overlap, as well as both sub models 2 and 4 when μ_2 and μ_4 overlap. The performance of the approach, which uses the combination of the two winning models (TS-AGPC2) during switching, is shown in Fig. 11.

The controller can be implemented by weighted integration of the two winning local controllers during switching and is then given by

$$\Delta u(k) = \sum_{i=2,1}^{4,3} \mu_i \Delta u_i / \sum_{i=2,1}^{4,3} \mu_i. \tag{25}$$

Opting for a combination of sub models as a multi-model control strategy gives rise to the following question: why not then use the complete TS model, leading to the TS-GPC approach, instead?

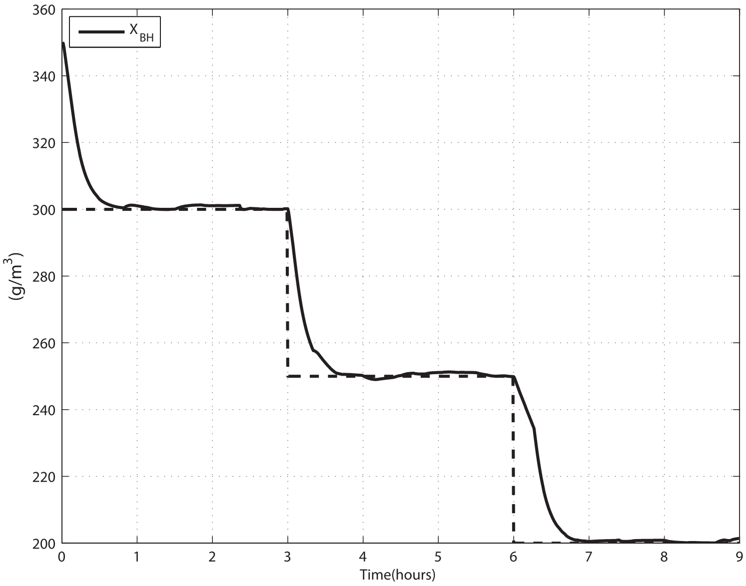
No complementary efforts from the designed controllers ought to be undertaken, even in the presence of disturbance, when the complete TS model is used as an internal model in the TS-GPC approach, as all submodels are used in the calculation of the control increment (see Fig. 12). This gives the TS-GPC the power to follow any reference trajectory and reject any kind of reasonable disturbances.

To show the robustness-related superiority of MPC strategies over classical control, represented by a PID (tuned in the best manners, yielding $K_p = 170; K_i = 20; K_d = 90$), the corresponding tests have been performed. It can be clearly seen that the PID, no matter how optimally tuned, cannot stand up to the performance of MPC in the absence of disturbances (Fig. 13), whereas in the presence of disturbances and constraints it fails to provide acceptable reference tracking (Fig. 14). It can be clearly seen from Fig. 14 that the control variable (q_w) quickly reaches the constraints and the controller is not able to adjust afterwards.

On the contrary, the MPC strategies succeeded in providing accurate control without disturbances (Figs. 8 and 9) for both strategies, TS-GPC and TS-AGPC, respectively. However, in the presence of severe constraints and disturbances, TS-AGPC suffers from performance losses when model switching occurs (Fig. 10), while TS-GPC maintains practically the same level of performance (Fig. 12).

Indeed, on the one hand, the output errors, given in Table 2, show that TS-GPC produces a sensibly lower tracking error than the one obtained with TS-AGPC. On the other hand, the input constraints are scrupulously respected by TS-GPC, while in the case of TS-AGPC control with disturbances, constraints are clearly violated, leading to a faulty following of the set reference.

The use of the combination of two dominant sub-models in the switching process of the TS-AGPC2 approach, improves the performance of TS-AGPC, and the results outperform those of the TS-GPC in the presence of disturbances.



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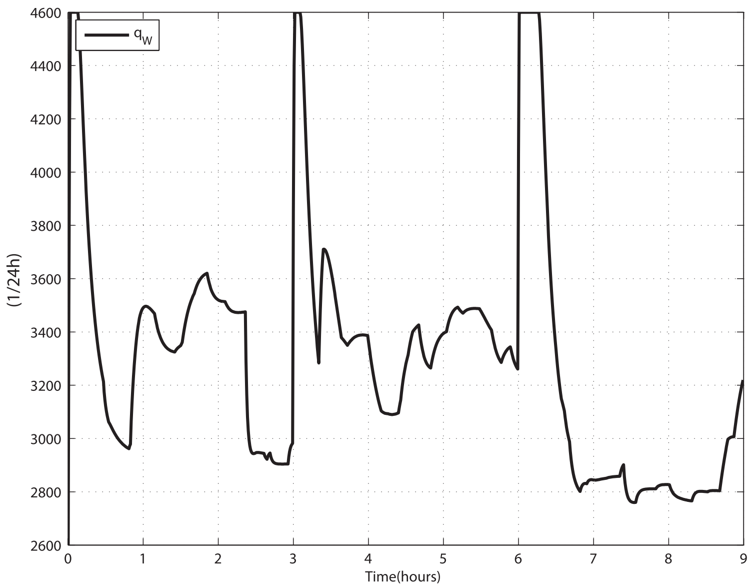
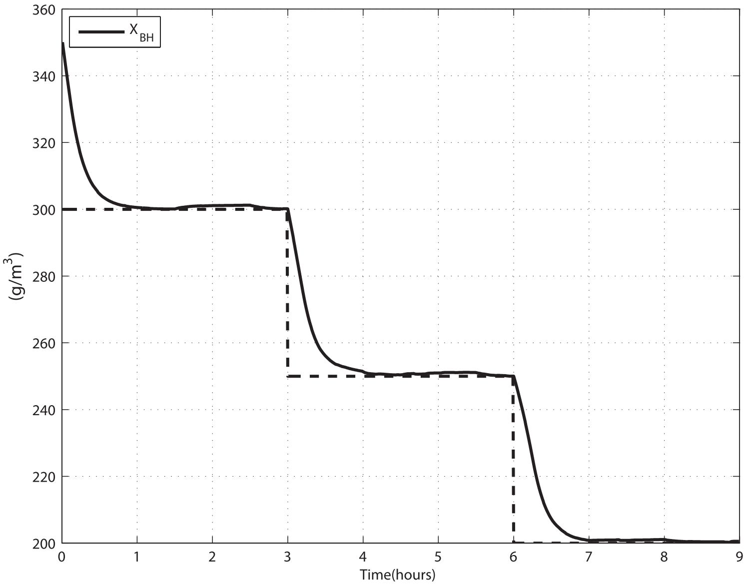


(b)

Figure 11: TS-AGPC2 with disturbances: (a) X_{BH}; (b) q_w



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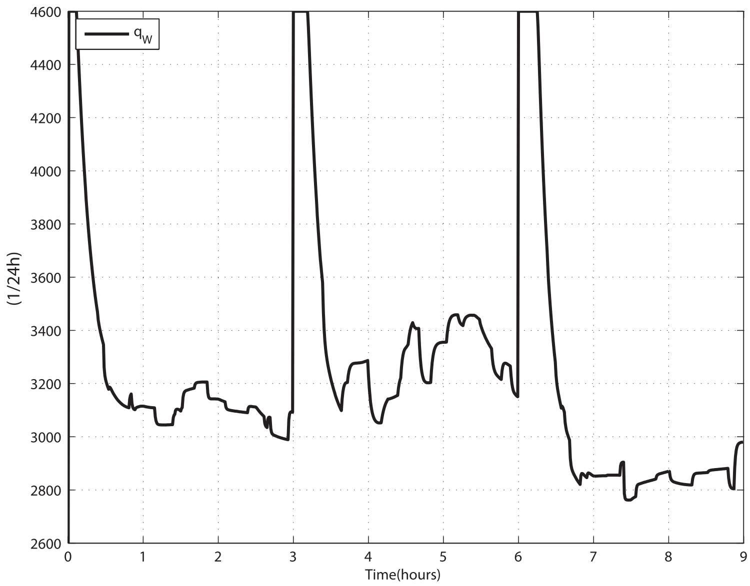
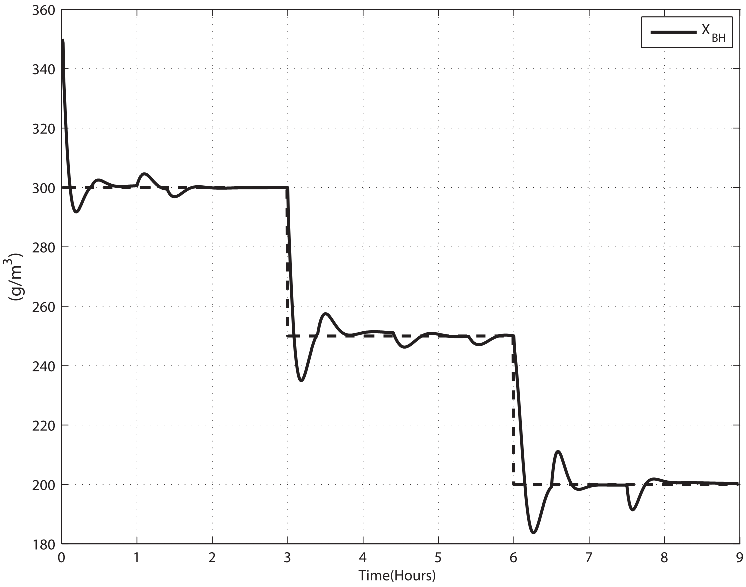


(b)

Figure 12: TS-GPC with disturbances: (a) X_{BH}; (b) q_w



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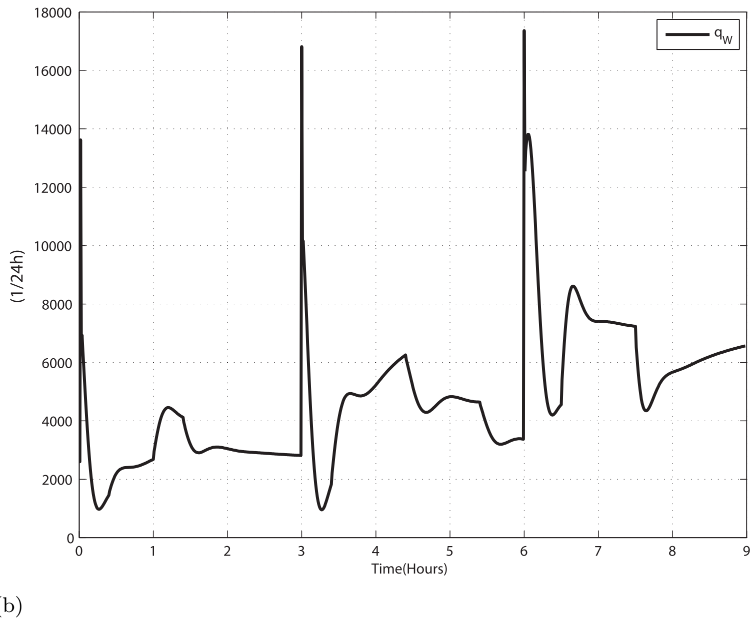
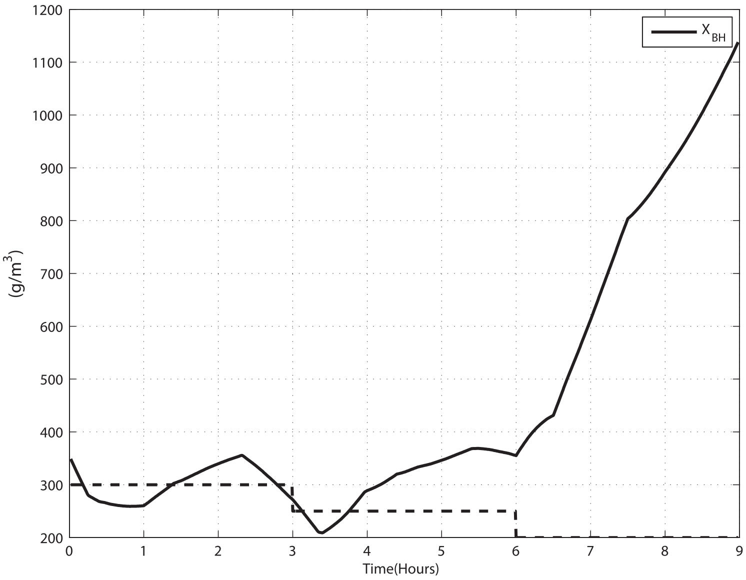


(b)

Figure 13: PID without disturbances: (a) X_{BH}; (b) q_w



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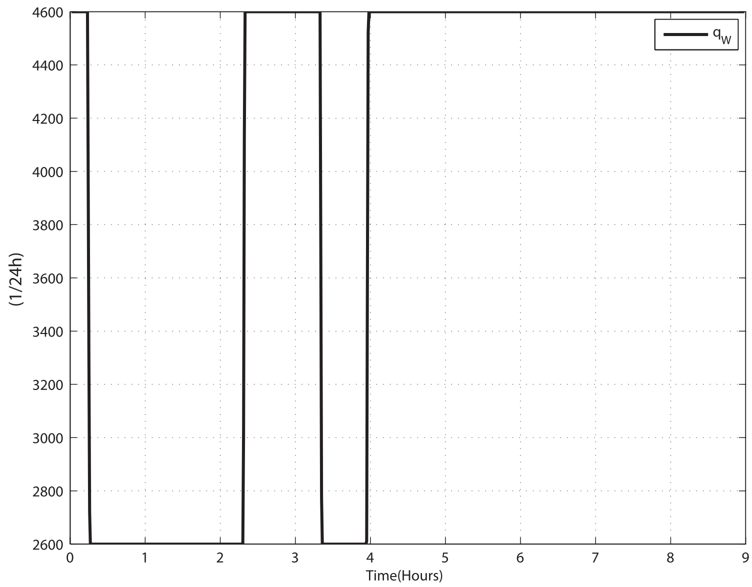

(b)

Figure 14: PID with disturbances: (a) X_{BH}; (b) q_w

Table 2: TS-GPC, TS-AGPC, TS-AGPC2, and PID control performances

TS-GPC	TS-AGPC	TS-AGPC2	PID
MAE with parametric and input disturbances			
4.8307	127.5360	4.6243	2.8917
MAE without parametric and input disturbances			
4.5967	4.3898	4.3898	214.1286

Figures 15 and 16 illustrate the performance of the different controllers, derived in this paper, without and with disturbances, respectively. The figures relate faithfully the results obtained and displayed in Table 2, and are shown here for a better visual comparison in terms of MAE, overshooting, constraint violation, etc. However, the presented PID performance (Fig. 15) is the one without constraints, as when constraints considered, the PID performance drops drastically.

7. Conclusions

The TS multi-model approach permitted to accurately model the ASM1 sludge reactor process, giving a model construction, which can be used in MPC based control strategy. MPC in general and GPC in particular, based on an accurate linear model, are well able to obtain high control performances in case of setpoint changes and parametric disturbances and constraints. The designed TS-GPC controller has been shown to outperform, for the here considered system, the TS-AGPC and the benchmark PID, especially when taking into account input constraints. However, the performance of TS-AGPC or TS-AGPC2 may be preferred over that of TS-GPC, if severe disturbances are not considered, due to simpler control law formulations.

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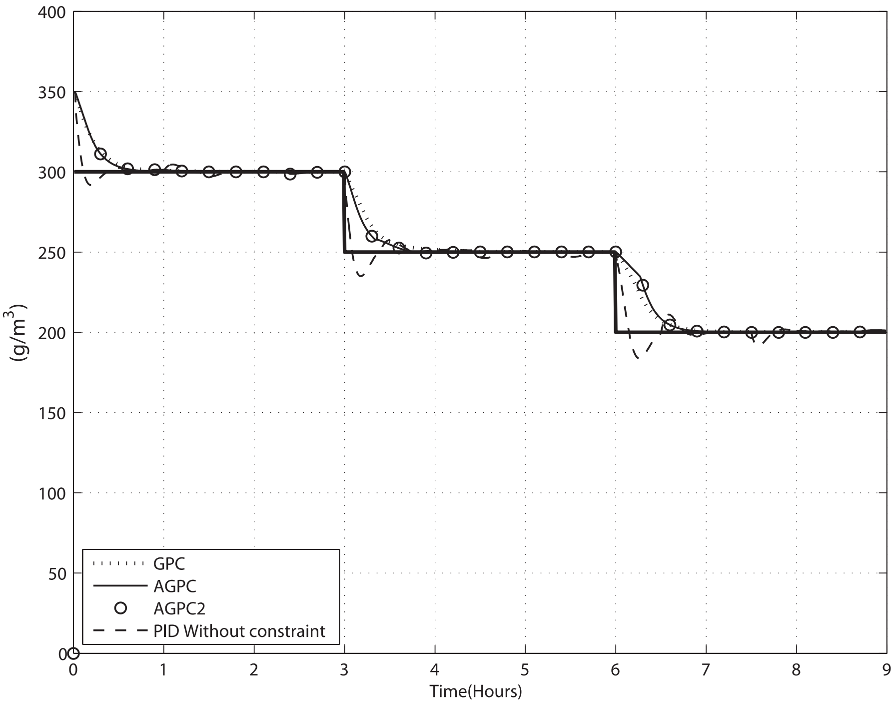

Figure 15: Comparison between control strategies without disturbances

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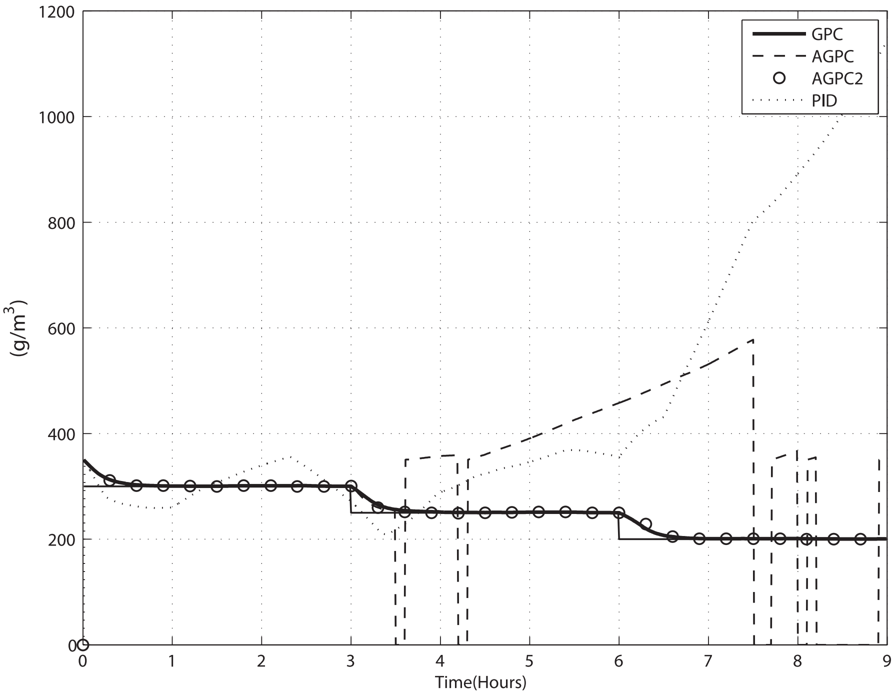

Figure 16: Comparison between control strategies with disturbances

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