

Channel coordination, pricing and replenishment policies
in three-echelon dual-channel supply chain*

by

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Abstract: This paper addresses pricing, replenishment policies, coordination, and issues of surplus profit division among the members of the dual-channel supply chain for a particular product whose unit cost decreases continuously over product's life time. In the dual-channel setting, manufacturer, distributor, and retailer with retail channel and the manufacturer's direct e-channel are considered. When manufacturer is a Stackelberg leader, the paper reveals that the channels are bound to compete with each other on price severely in the centralized as well as decentralized system. It is observed that more preference on the retail channel may lead to non coexistence of profitable retail-e-tail channel which results in a threshold for higher retail price than the online price. The mechanism of all unit quantity discount with agreement of franchise fee coordinates the channel and provides for the win-win outcome. Finally, Nash bargaining over product depicts particular profit division among the channel members. A numerical example is provided in order to test and justify the proposed model.

Keywords: three-echelon dual-channel; unit cost decrease; replenishment policy; quantity discount, franchise fee.

1. Introduction

The dazzling growth of e-commerce impels the manufacturers to introduce the direct online channels in order to secure sustainability in the highly competitive global market. Increasing competition for the customers of geographically

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diverse locations, reduced cost of searching, reduced time consumption for purchasing through e-channel propel a manufacturer to augment its market coverage and hence the business share realized through internet sales. As a result, for instance, in Czech Republic, 24% of the country's total trade turnover was generated via online channel in 2010. In 2012, global e-commerce sales reached \$1 trillion for the first time in history (www.wikipedia.org). The data for Western Europe show that investment in e-commerce reached 128 billion euros in 2013, 14.3% increase compared to the year 2012. It is forecasted that e-commerce spending in 2017 would reach 191 billion euros in Western Union, meaning the compound annual growth rate of about 11% (www.interretailer.com). E-commerce sales in US increased by 15.8% in 2013 in comparison with the year 2011. Thus, it is important for a manufacturer to restructure its traditional brick-and-mortar channel by engaging in direct sales through the internet channel, because the customers always prefer alternatives that are better suited for their needs (Takahashi, Hirotsu and Morikawa, 2011).

In high-tech industry, demand and unit cost of a particular product decrease due to the introduction of newer versions of the components. In such scenario, decision maker of a business sector faces a competitive situation regarding the determining of optimum selling price of a product. The studies of continuous decrement of unit cost in the supply chain are neglected, although existing literature in this direction offers some content considering single business entity. In the current competitive business environment, the manufacturers are being immensely interested on opening an internet channel and in the coexistence of brick-and-mortar and internet channels. Thus, in the current conditions of economy, it is essential to determine the pricing and replenishment policy of products (Cardenas-Barron, 2006; Sana, 2011, 2012a,b; Smith, Robles and Cardenas 2009; Panda, Saha and Basu 2008, 2013; Shah and Raykundaliya, 2010; Sarkar, Saren and Wee 2013; Pal, Sana and Chaudhuri 2012, 2014, 2015; Modak et al., 2014; Modak, Panda and Sana, 2015,2016) meant to achieve minimum cost or maximum profit of the business organization.

Although channel coordination using contract mechanisms in two-echelon supply chain has been explored extensively, models that would deal with resolving channel conflict in three-echelon supply chain are notably fewer. In practice, it is more difficult to resolve channel conflict in a three-tier supply chain by applying coordination contract than for the two-tier supply chain. When the number of echelons increases, the values of the self cost minimizing/profit maximizing objectives increase. As a result, dimensionality of the solution space increases and the channel coordination using contract becomes more complex. The problem further aggravates with the introduction of the direct channel of the manufacturer. Though the various aspects of dual channel supply chain have been discussed, there are no such research articles to date, which have addressed pricing and replenishment policies for the hi-tech products, whose unit costs decrease continuously in their short life span. Hi-tech products have high

online compatibility and tech savvy customers generally look at the specifications of the products through online channels and compare the retail prices with the prices of products in online manufacturer's sales channels. Thus, there is a need to determine pricing and replenishment policies for both of the channels. Also, it is essential to identify a coordination mechanism that would eliminate channel conflicts and to determine best channel performance and win-win division of profits among the channel members.

Besides a three-echelon brick-and-mortar channel, the paper assumes that the manufacturer operates e-tail as well as retail channel for a high-tech product whose unit costs decreases over time and which becomes obsolete after a finite time. For the Stackelberg leadership of the manufacturer, the present article explores the effects of customers' channel preference and of the number of replenishments on pricing and replenishment policies. Also, the paper analyzes how the mechanism of all unit quantity discount with agreement of franchise fee eliminates the channel conflict and Nash (1950) bargaining solution determines the win-win profits of the channel members.

2. Literature review

In the age of advanced and accelerating technology, continuous changes of components of the cost factor have become quite common in the literature concerning this subject domain. In this context, Erel (1992) proposed a model, involving the assumption of compound increment of the unit cost of the product in inflationary situation. Buzacott (1975) assumed compound increments of both unit cost and setup cost in inflationary situation. Teng and Yang (2004) and Teng, Ouyang and Chang (2005) developed inventory models under partial backlogging, where demand and cost fluctuate over time. They determine the optimal replenishment and purchasing policies so as to minimize the system running cost. They have claimed that their suggested policy fits today's high-tech market. The model of Khouja and Goyal (2006) may be considered to constitute a special case of that of Teng and Yang (2004) and Teng, Ouyang and Chang (2005) with constant demand and unit cost dependent holding cost. Khouja and Park (2003) presented a short review of the literature associating unit cost decrement with the existing industrial scenario. They developed an inventory model to determine the optimal operating policy in the case where the unit cost of the product decreases continuously by a constant percentage. Under the restriction of equal cycle lengths over finite time horizon, they derived an approximate closed form of the optimal cycle length to minimize the system operating cost. Panda (2011) determined the optimal pricing and replenishment policy for the decreasing demand with time and price sensitive market, where the unit cost of the product decreases linearly with time. Cardenas-Barron, Trevinio-Garza and Wee (2012) suggested a heuristic algorithm to solve the vendor inventory system management with multi-product and multi-constraint based EOQ model with backorders, considering two classical backorder costs:

linear and fixed. Panda et al. (2015) developed a model for the dual channel supply chain, where unit cost of the product decreases continuously with time. Sarkar and Majumder (2013) investigated an integrated vendor-buyer supply chain model to reduce total cost of the channel by considering the setup cost reduction of the vendor. Sarkar, Mandal and Sarkar (2014) investigated an economic manufacturing quantity (EMQ) model in an imperfect production process, in which demand depends on selling price and time. Sarkar et al. (2015) further extended the model and brought a reduction to the total cost by considering carbon emission during transportation for single-setup multi-delivery policy in supply chain management.

Considered besides the retail channel, an internet channel of the manufacturer has the potential of reducing retailer's dominance, addressing different customer segments, gaining higher profit margin etc. As a result, in the current global business scenario, dual channel supply chain acquires significant importance and becomes a hot topic of research and application in industrial problems. A variety of issues in dual channel supply chain have been addressed by the researchers. For example, Yan (2011) developed a dual channel supply chain model and analyzed the effect of differentiated branding. Sharma and Mehrotra (2007) claimed that the dual channel increases channel conflict though it has also the potential to increase the customer base. Cattani et al. (2006) determined the wholesale price of the manufacturer for the case when the channel members compete with each other. Chiang, Chhajed and Hess (2003) proposed a model that demonstrates that the dual channel supply chain can be used to control the retailer's price. Hua, Zhang and Xu (2011) analyzed the effect of delivery lead time on the pricing decisions in a dual channel supply chain. Dan, Xu and Liu (2012) determined the optimal retail service and prices in a dual channel supply chain. Chen, Zhang and Sun (2012) developed a dual channel supply chain model and proposed pricing strategies that maximize the decentralized dual channel performance. All the proposed models, mentioned above, have addressed pricing and replenishment policies, channel conflict and channel competition, mainly between the brick-and-mortar and internet channels, but have not focused on channel coordination.

Coordination among channel members is imperative for improving channel-wide performance, because it neutralizes the difference between the decentralized and centralized outcomes. The central objective of the coordination mechanism is the transfer of money from one channel member to another when they act coherently. The existing literature offers a rich content in this regard for two-echelon supply chains. Quantity discount (Panda, Modak and Pradhan, 2016), two-part tariff (Ingene and Parry, 1995), revenue sharing (Panda, 2014), mail-in-rebate (Saha et al., 2015), buyback (Ding and Chen, 2008), disposal cost sharing (Panda, Modak and Basu, 2014), profit sharing (Modak, Panda and Sana, 2015), etc., are the mechanisms used to resolve double marginalization in a two-tier supply chain. There are, however, some papers that have

focused on eliminating the channel conflict in dual channel supply chain. Chen, Zhang and Sun (2012) used wholesale price and manufacturer's direct channel price contract for channel coordination. They also suggested that two-part tariff and profit sharing in a range coordinate the channel, bringing, in effect, win-win situation among the members of the chain. Agarwal, Agarwal and Singh (2006) have shown that the sales effort resolves the channel conflict when the channels compete with each other. Cai (2010) proposed a hybrid revenue sharing and linear online retail prices relationship to resolve channel conflict and examined the influence of channel coordination on the supplier. Boyaci and Gallego (2004) demonstrated that revenue sharing, buy back, and wholesale price contract are unable to resolve the double marginalization problem. The authors quoted maintain that a penalty contract can coordinate a dual channel supply chain, though this is difficult to implement. However, all the models reported above, have examined the coordination issues in a dual channel supply chain of two echelons in the traditional brick-and-mortar channel.

The research reported in this paper differs from the prior work in many aspects, listed here, as follows. *Firstly*, the paper considers a retail-e-tail channel supply chain, where the retail channel has three echelons, which has not been addressed in the literature earlier. *Secondly*, the paper discusses the pricing and replenishment issues of a product, whose unit cost decreases continuously in its short life span. This is not highlighted earlier in the supply chain models. *Thirdly*, as the product has a short life time, the paper considers the system over a finite time horizon, consisting of multiple replenishment cycles rather than a single one. The earlier studies in this regard in inventory literature have considered pre-specified cycle length. Relaxing this assumption, the paper considers the number of replenishments - over the planning horizon as a decision variable. Obviously, the number of replenishments occurrences in centralized and decentralized decision making processes is different and double marginalization is considered. *Fourthly*, to resolve the channel conflict, the paper uses all unit quantity discount with agreement of franchise fee as the contract mechanism. It eliminates the channel conflict but is unable to determine the win-win profits for the channel members. *Fifthly*, the paper uses Nash bargaining product to divide the surplus profit among the channel members according to which the channel coordinated profits of the all members are win-win. *Sixthly*, the paper analyzes the effect of customer's channel preference on the channel competition for optimal prices.

3. Notations

The following notations are used to develop the proposed model.

- L time horizon under consideration.
- T the cycle time during the planning horizon.
- n the total number of replenishments over $(0, L)$ (a decision variable).
- D_i^r the demand rate of the product in retail channel of i^{th} replenishment

- cycle.
- D_i^e the demand rate of the product in e-tail channel of i^{th} replenishment cycle.
- h_r the inventory holding cost per unit per unit time of the retailer.
- h_m the inventory holding cost per unit per unit time of the manufacturer.
- s_r the ordering or/and set-up cost of the retailer.
- s_d the ordering or/and set-up cost of the distributor.
- s_m the ordering or/and set-up cost of the manufacturer.
- $c(t)$ unit production cost of the manufacturer.
- w_i^m wholesale price of the manufacturer for the distributor.
- w_i^d wholesale price of the distributor for the retailer.
- p_i^r selling price per unit of the retailer.
- p_i^e selling price per unit of the manufacturer in e-tail channel.

4. Model formulation

4.1. The preliminaries

Besides the traditional brick-and-mortar channel of the manufacturer, distributor and retailer, the manufacturer sells the product through an internet channel. The channel operates for a single product over a finite time horizon L , in which n replenishments are made, each for the period of length T , such that $nT = L$. The unit cost of the product decreases continuously with respect to time at a constant rate and, at time t , it is equal $c(t) = \alpha - \beta t$, $t \in (0, \alpha/\beta)$. The parameter α is the introductory unit cost of the product and β is the time-related parameter. The price-demand relationship is deterministic and is known. Following Yue and Liu (2006), Kurata, Yao and Liu (2007), and Huang and Swaminathan (2009), we assume that the demand functions in two channels are linear, based on self-price and cross-price effects. The demand functions in the i^{th} replenishment cycle ($i = 1, 2, \dots, n$) in retail and e-tail channels are, respectively as follows

$$D_i^r = \theta a - b_1 p_i^r + r_1 (p_i^e - p_i^r)$$

and

$$D_i^e = (1 - \theta)a - b_2 p_i^e + r_2 (p_i^r - p_i^e).$$

Demand in the retail channel and in the direct channel depend on the e-tail price p_i^e and the retail price p_i^r . The parameter a represents the forecasted potential demand if the products were free of charge. The share of the demand that goes to the retail channel is θ , and the rest, $(1 - \theta)$, goes to the direct channel, when p_i^r and p_i^e are zeros. The term $(1 - \theta)$ captures the customers preference for the direct channel when the products are free of charge. The parameters $b_1 > 0$ and $b_2 > 0$ are the coefficients of self-price elasticity of D_i^r and D_i^e , $r_1 > 0$ and $r_2 > 0$ reflect the degree of competition between the two channels. To maintain the analytical tractability, following Yue and Liu (2006), and Hua, Zhang and Xu (2011), we assume that the price elasticity and the cross-price effects are

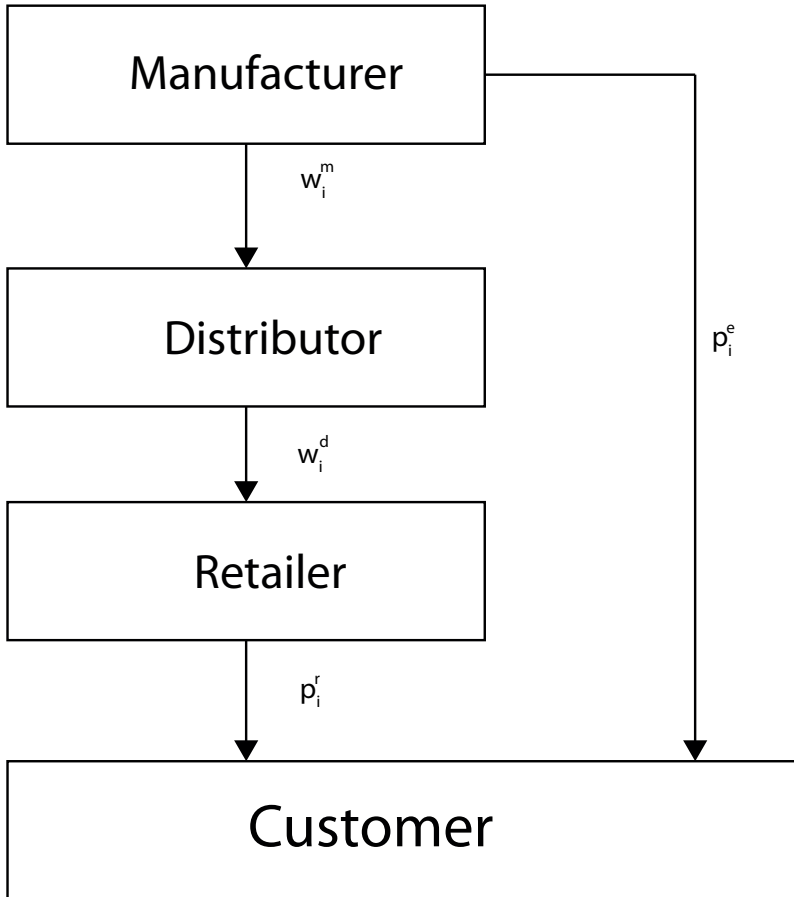


Figure 1. Structure of the proposed dual-channel supply chain

symmetric, i.e., $b_1 = b_2 = b$ and $r_1 = r_2 = r$. Thus, demands in the retail channel and e-tail channel take the forms as follows

$$D_i^r = \theta a - (b + r)p_i^r + rp_i^e, \quad (i = 1, 2, \dots, n) \tag{1}$$

and

$$D_i^e = (1 - \theta)a - (b + r)p_i^e + rp_i^r, \quad (i = 1, 2, \dots, n). \tag{2}$$

It is realistic to assume that the selling price in the online channel is higher than the manufacturer’s as well as distributor’s wholesale price in the i^{th} ($i = 1, 2, \dots, n$) replenishment cycle, i.e., $p_i^d > w_i^d > w_i^m, (i = 1, 2, \dots, n)$. Otherwise, the retailer would purchase the product through the online channel rather

than from the manufacturer. Under this model setting, we first find the optimal decisions in the decentralized and centralized decision making circumstances.

4.2. The decentralized dual-channel supply chain

In decentralized decision making, the manufacturer, the distributor and the retailer are interested in individual profit maximization. The retailer replenishes n^{ds} times each of length T^{ds} over the finite time horizon L such that $n^{ds}T^{ds} = L$, i.e., time for each replenishment is $T^{ds} = L/n^{ds}$. During the i^{th} ($i = 1, 2, \dots, n^{ds}$) replenishment, the profit functions of the retailer, the distributor and the manufacturer are, respectively, as follows

$$\pi_i^r(p_i^r) = D_i^r T^{ds} p_i^r - D_i^r T^{ds} w_i^d - s_r - \frac{D_i^r T^{ds} h_r T^{ds}}{2}, \tag{3}$$

$$\pi_i^d(w_i^d) = D_i^r T^{ds} w_i^d - D_i^r T^{ds} w_i^m - s_d \tag{4}$$

and

$$\begin{aligned} \pi_i^m(w_i^m, p_i^e) = \\ D_i^r T^{ds} w_i^m - D_i^r T^{ds} c[(i-1)T^{ds}] + D_i^e T^{ds} p_i^e - D_i^e T^{ds} \frac{h_m T^{ds}}{2} - \\ D_i^e T^{ds} c[(i-1)T] - s_m. \end{aligned} \tag{5}$$

Profit functions of the retailer, the distributor and the manufacturer over the planning horizon L are, respectively, given as

$$\pi^r(n^{ds}, p_i^r) = \frac{L}{n^{ds}} \sum_{i=1}^{n^{ds}} \left[D_i^r (p_i^r - w_i^d - \frac{h_r L}{2n^{ds}}) \right] - n^{ds} s_r, \tag{6}$$

$$\pi^d(w_i^d) = \frac{L}{n} \sum_{i=1}^{n^{ds}} [D_i^r (w_i^d - w_i^m)] - n^{ds} s_d \tag{7}$$

and

$$\begin{aligned} \pi^m(w_i^m, p_i^e) = \\ \frac{L}{n} \sum_{i=1}^n \left[D_i^r (w_i^m - c[(i-1)\frac{L}{n}]) + D_i^e (p_i^e - c[(i-1)\frac{L}{n}] - \frac{h_m L}{2n}) \right] - ns_m. \end{aligned} \tag{8}$$

Interactions among the channel members are considered in terms of a two-stage Stackelberg game, i.e., manufacturer is the decision maker and other members are the followers of him. The distributor follows manufacturer’s move and then reacts by playing the best move, consistent with the available information, whereas the retailer follows the distributor to make decision that will maximize

his profit. The objective of the leader is to design his own move in such a way as to maximize own profit after considering all rational moves of the followers who can devise their own moves (Basar and Olsder, 1999). In this game, the distributor maximizes own profit margin depending on manufacturer’s wholesale price and retailer maximizes own profit margin depending on distributor’s wholesale price. We use backward induction to find out the optimal decisions of the channel members in this manufacturer-led Stackelberg game. For given n^{ds} , the necessary condition $d\pi_i^r/dp_i^r = 0$ for the existence of the optimal solution in the i^{th} replenishment cycle yields

$$p_i^r = \frac{w_i^d}{2} + \frac{r}{2(b+r)}p_i^e + \frac{a\theta}{2(b+r)} + \frac{Lh_r}{4n^{ds}}, \quad (i = 1, 2, \dots, n^{ds}). \tag{9}$$

Here, $d^2\pi_i^r/d(p_i^r)^2 = -2L(b+r)/n^r < 0$, i.e., π_i^r is a concave function of p_i^r .

In response to the retailer’s decision, for given n^{ds} and the manufacturer’s wholesale price and e-tail channel price pair (w_i^m, p_i^e) , distributor’s best wholesale price during the i^{th} ($i = 1, 2, \dots, n^{ds}$) replenishment, obtained by optimizing the respective profit function, can be obtained from $d\pi_i^d/dw_i^d = 0$ as follows

$$w_i^d = \frac{w_i^m}{2} + \frac{r}{2(b+r)}p_i^e + \frac{a\theta}{2(b+r)} - \frac{Lh_r}{4n^{ds}}, \quad (i = 1, 2, \dots, n^{ds}). \tag{10}$$

Also, we have

$$d^2\pi_i^d/d(w_i^d)^2 = -L(b+r)/n^{ds} < 0,$$

i.e. π_i^d is a concave function of w_i^d . Depending on the distributor’s reaction, the manufacturer will make the decision that maximizes his profit. The necessary conditions $\partial\pi_i^m/\partial p_i^e = 0$ and $\partial\pi_i^m/\partial w_i^m = 0$ for the maximization of the manufacturer’s profit yield

$$w_i^{m*} = \frac{a(r+\theta b)}{2b(b+2r)} - \frac{Lh_r}{4n^{ds}} + \frac{1}{2}c \left[(i-1)\frac{L}{n^{ds}} \right], \quad (i = 1, 2, \dots, n^{ds}) \tag{11}$$

and

$$p_i^{e*} = \frac{a[r+(1-\theta)b]}{2b(b+2r)} + \frac{Lh_m}{4n^{ds}} + \frac{1}{2}c \left[(i-1)\frac{L}{n^{ds}} \right], \quad (i = 1, 2, \dots, n^{ds}). \tag{12}$$

Again, observe that

$$\partial^2\pi_i^m/\partial(p_i^e)^2 = -[2(b+r)^2 - r^2]/(b+r) < 0; \partial^2\pi_i^m/\partial(w_i^m)^2 = -(b+r) < 0;$$

$$\partial^2\pi_i^m/\partial p_i^e \partial w_i^m = r = \partial^2\pi_i^m/\partial w_i^m \partial p_i^e$$

and

$$[\partial^2\pi_i^m/\partial(p_i^e)^2] \times [\partial^2\pi_i^m/\partial(w_i^m)^2] - (\partial^2\pi_i^m/\partial p_i^e \partial w_i^m)^2 = 2[(b+r)^2 - r^2] > 0.$$

Therefore, the manufacturer’s profit function is a concave function of

$$(w_i^m, p_i^e), (i = 1, 2, \dots, n^{ds}).$$

For given n^{ds} , using (9) and (10) in (11) and (12), the retailer’s optimal selling price and the distributor’s optimal wholesale price in the i^{th} ($i = 1, 2, \dots, n^{ds}$) replenishment cycle can be found as

$$w_i^{d*} = \frac{a}{2(b+2r)} \left[\frac{r}{b} + \frac{(3b+4r)\theta}{2(b+r)} \right] + \frac{L}{8n^{ds}} \left[\frac{rh_m}{b+r} - 3h_r \right] + \left(\frac{b+2r}{4(b+r)} \right) c \left[(i-1) \frac{L}{n^{ds}} \right] \tag{13}$$

and

$$p_i^{r*} = \frac{a}{2(b+2r)} \left[\frac{r}{b} + \frac{(7b+10r)\theta}{4(b+r)} \right] + \frac{L}{16n^{ds}} \left[h_r + \frac{3rh_m}{b+r} \right] + \left(\frac{b+4r}{8(b+r)} \right) c \left[(i-1) \frac{L}{n^{ds}} \right]. \tag{14}$$

Here, for any ($i = 1, 2, \dots, n^{ds}$),

$$w_{i-1}^{m*} - w_i^{m*} = \alpha L / 2n^{ds} > 0 \text{ and } p_{i-1}^{e*} - p_i^{e*} = \alpha L / 2n^{ds} > 0.$$

Similarly, for any ($i = 1, 2, \dots, n^{ds}$), it is easy to verify that

$$w_{i-1}^{d*} > w_i^{d*} \text{ and } p_{i-1}^{d*} > p_i^{d*}.$$

That is, the optimal wholesale prices and retail prices in a replenishment cycle are always higher than those of the immediate next replenishment cycle. The unit cost of the product decreases continuously with time and the product has limited lifetime. The wholesale prices of the product in the channel depend on the unit cost of the product. Also, the retail prices in the channel depend on the wholesale prices. So, as time progresses, the wholesale prices and hence the retail prices in the channel decrease. Thus, we have the following proposition.

PROPOSITION 1 *In a dual channel supply chain with continuously decreasing unit cost of the product, for the given number of replenishment cycles over the planning horizon, the optimal selling prices in the retail and e-tail channels as well as the wholesale prices of the manufacturer and the distributor are lower in every next replenishment cycle than in the preceding cycle.*

By substituting the optimal values of selling prices in (1) and (2), we can determine the demands of the product in retail and e-tail channels in i^{th} ($i = 1, 2, \dots, n^{ds}$) replenishment cycle as

$$D_i^{r*} = \frac{a\theta}{8} + \frac{[rh_m - (b+r)h_r]L}{16n^{ds}} - \left(\frac{b}{8} \right) c \left[(i-1) \frac{L}{n^{ds}} \right] \tag{15}$$

and

$$D_i^{e*} = \frac{a}{2} \left[1 - \frac{\theta(4b+r)}{4(b+r)} \right] - \frac{L}{16n^{ds}} \left[\frac{[4(b+r)^2 - 3r^2]h_m}{(b+r)} - rh_r \right] - \left(\frac{b(4b+7r)}{8(b+r)} \right) c \left[(i-1) \frac{L}{n^{ds}} \right]. \tag{16}$$

Thus, the optimal order quantity that the manufacturer faces in the i^{th} ($i = 1, 2, \dots, n^{ds}$) replenishment cycle is

$$Q_i^* = [D_i^{r*} + D_i^{e*}] \frac{L}{n^{ds}} = \frac{L}{n^{ds}} \left(\frac{a}{2} \left(1 - \frac{3b\theta}{4(b+r)} \right) - \frac{Lb}{16n^{ds}} \left(\frac{(4b+7r)h_m}{(b+r)} + h_r \right) - \left(\frac{b(5b+8r)}{8(b+r)} \right) c \left((i-1) \frac{L}{n^{ds}} \right) \right). \tag{17}$$

The profit functions of the retailer, distributor and the manufacturer in the i^{th} ($i = 1, 2, \dots, n$) replenishment cycle are, respectively, given by the following formulae:

$$\pi_i^{r*} = \frac{L}{n^{ds}(b+r)} \left[\frac{a\theta}{8} + \frac{(rh_m - (b+r)h_r)L}{16n^{ds}} - \frac{b}{8} c \left((i-1) \frac{L}{n^{ds}} \right) \right]^2 - s_r, \tag{18}$$

$$\pi_i^{d*} = \frac{L}{2n^{ds}(b+r)} \left[\frac{a\theta}{4} + \frac{(rh_m - (b+r)h_r)L}{8n^{ds}} - \frac{b}{4} c \left((i-1) \frac{L}{n^{ds}} \right) \right]^2 - s_d \tag{19}$$

and

$$\begin{aligned} \pi_i^{m*} = & \frac{a^2L(4(b+r)^2 - 8b(b+r)\theta + b(5b+2r)\theta^2)}{16bn^{ds}(b+r)(b+2r)} \\ & - \frac{aL^2}{16(n^{ds})^2} \left[\frac{\theta(b+r)h_r + h_m(4(b+r) - (4b+r)\theta)}{(b+r)} \right] \\ & - \left[\frac{L[2a(4(b+r) - 3b\theta) - b(L/n^{ds})(b(4h_m + h_r) + (7h_m + h_r)r)]}{16n^{ds}(b+r)} \right] c \left[(i-1) \frac{L}{n^{ds}} \right] \\ & - s_m + - \frac{(b^2(4h_m^2 + h_r^2) + 2b(4h_m^2 - h_mh_r + h_r^2)r + (h_m - h_r)^2r^2)L^3}{64(n^{ds})^3(b+r)} \\ & + \left[\frac{bL(5b+8r)}{16n^{ds}(b+r)} \right] c^2 \left[(i-1) \frac{L}{n^{ds}} \right]. \end{aligned} \tag{20}$$

Consequently, the total optimal order quantity over the sales season L in the decentralized scenario can be derived from equation (17) to have the following

form

$$\begin{aligned}
 Q^* &= \sum_{i=1}^{n^{ds}} Q_i^{ds} = \frac{aL}{2} \left[1 - \frac{3b\theta}{4(b+r)} \right] - \frac{L^2b}{16n^{ds}} \left[\frac{(2b+3r)h_m}{(b+r)} + h_r \right] \\
 &\quad - L \left(\frac{b(5b+8r)}{8(b+r)} \right) \left[\alpha - \frac{1}{2}\beta L \left(1 - \frac{1}{n^{ds}} \right) \right]. \tag{21}
 \end{aligned}$$

Total profit of the channel members over the planning horizon L can be found as follows:

$$\begin{aligned}
 \pi^{m*} &= \sum_{i=1}^{n^{ds}} \pi_i^{m/ds} = \frac{a^2L(4(b+r)^2 - 8b(b+r)\theta + b(5b+2r)\theta^2)}{16b(b+r)(b+2r)} \\
 &\quad - \frac{aL^2}{16n^{ds}} \left[\frac{\theta(b+r)h_r + h_m(4(b+r) - (4b+r)\theta)}{(b+r)} \right] \\
 &\quad - \left[\frac{L[2a(4(b+r) - 3b\theta) - b(L/n^{ds})(b(4h_m+h_r) + (7h_m+h_r)r)]}{16(b+r)} \right] \\
 &\quad \left[\alpha - \frac{1}{2}\beta L \left(1 - \frac{1}{n^{ds}} \right) \right] - n^{ds} s_m \\
 &\quad + \frac{(b^2(4h_m^2+h_r^2) + 2b(4h_m^2-h_mh_r+h_r^2)r + (h_m-h_r)^2r^2)L^3}{64(n^{ds})^2(b+r)} \\
 &\quad + \left[\frac{bL(5b+8r)}{16(b+r)} \right] \left(\alpha^2 - \alpha\beta L \left(1 - \frac{1}{n^{ds}} \right) + \frac{\beta^2L^2}{6} \left(1 - \frac{1}{n^{ds}} \right) \left(2 - \frac{1}{n^{ds}} \right) \right), \tag{22}
 \end{aligned}$$

$$\begin{aligned}
 \pi^{d*} &= \sum_{i=1}^{n^{ds}} \pi_i^{d/ds} \\
 &= \frac{L}{32(b+r)} \left[a^2\theta^2 + \frac{(rh_m - (b+r)h_r)^2L^2}{4n^{ds2}} + \frac{a\theta(rh_m - (b+r)h_r)L}{n^{ds}} \right] \\
 &\quad - \frac{bL}{32(b+r)} \left(2a\theta - \frac{(rh_m - (b+r)h_r)L}{n^{ds}} \right) \left(\alpha - \frac{\beta L}{2} \left(1 - \frac{1}{n^{ds}} \right) \right) \\
 &\quad + \frac{b^2L}{32(b+r)} \left(\alpha^2 - \alpha\beta L \left(1 - \frac{1}{n^{ds}} \right) + \frac{\beta^2L^2}{6} \left(1 - \frac{1}{n^{ds}} \right) \left(2 - \frac{1}{n^{ds}} \right) \right) - n^{ds} s_d \tag{23}
 \end{aligned}$$

and

$$\begin{aligned}
 \pi^{r*} &= \sum_{i=1}^{n^{ds}} \pi_i^{r/ds} \\
 &= \frac{L}{64(b+r)} \left[a^2\theta^2 + \frac{(rh_m - (b+r)h_r)^2L^2}{4n^{ds^2}} + \frac{a\theta(rh_m - (b+r)h_r)L}{n^{ds}} \right] \\
 &\quad - \frac{bL}{64(b+r)} \left(2a\theta - \frac{(rh_m - (b+r)h_r)L}{n^{ds}} \right) \left(\alpha - \frac{\beta L}{2} \left(1 - \frac{1}{n^{ds}} \right) \right) \\
 &\quad + \frac{b^2L}{64(b+r)} \left(\alpha^2 - \alpha\beta L \left(1 - \frac{1}{n^{ds}} \right) + \frac{\beta^2L^2}{6} \left(1 - \frac{1}{n^{ds}} \right) \left(2 - \frac{1}{n^{ds}} \right) \right) - n^{ds}s_r.
 \end{aligned} \tag{24}$$

4.3. The centralized policy

The traditional centralized policy views the system as a single entity, in which there is one central planner, who makes all decisions, meant to maximize the profit of the entire system. The centralized policy determines the suitable selling prices of the product for both the retail and the direct channel, as well as production cycle such that the total profit of the chain is maximized. The relevant costs, considered in this policy, are similar to those taken into account in the decentralized replenishment policy. Suppose T^c and n^c denote the length of the replenishment cycle and the number of replenishment cycles in the time horizon L , respectively, in the centralized policy. Then, the profit function of the integrated channel is

$$\begin{aligned}
 \pi_i^c &= D_i^{rc}T^c p_i^{rc} - D_i^{rc}T^c \frac{h_r T^c}{2} - D_i^{rc}T^c c[(i-1)T^c] \\
 &\quad + D_i^{ec}T^c p_i^{ec} - D_i^{ec}T^c \frac{h_m T^c}{2} - D_i^{ec}T^c c[(i-1)T^c] - s_r - s_d - s_m.
 \end{aligned} \tag{25}$$

The necessary conditions for the existence of the optimal solution, i.e.,

$$\partial \pi_i^c / \partial p_i^{rc} = 0$$

and

$$\partial \pi_i^c / \partial p_i^{ec} = 0$$

provide the basis for the calculation of the optimal selling prices for the retail and e-tail channels in the i^{th} replenishment cycle, which are expressed as follows:

$$p_i^{rc*} = \frac{a(r+b\theta)}{2b(b+2r)} + \frac{h_r L}{4n^c} + \frac{1}{2}c \left[(i-1) \frac{L}{n^c} \right], \quad (i = 1, 2, \dots, n^c) \tag{26}$$

and

$$p_i^{ec*} = \frac{a[r+b(1-\theta)]}{2b(b+2r)} + \frac{h_m L}{4n^c} + \frac{1}{2}c \left[(i-1) \frac{L}{n^c} \right], \quad (i = 1, 2, \dots, n^c). \tag{27}$$

Moreover, for given n^c ,

$$\begin{aligned} \partial^2 \pi_i^c(p_i^{rc}, p_i^{ec}) / \partial (p_i^{rc})^2 &= -2(b+r) < 0, \\ \partial^2 \pi_i^c(p_i^{rc}, p_i^{ec}) / \partial (p_i^{ec})^2 &= -2(b+r) < 0, \\ \partial^2 \pi_i^c(p_i^{rc}, p_i^{ec}) / \partial p_i^{ec} \partial p_i^{rc} &= 2r = \partial^2 \pi_i^c(p_i^{rc}, p_i^{ec}) / \partial p_i^{ec} \partial p_i^{rc} \end{aligned}$$

and

$$\begin{aligned} \partial^2 \pi_i^c(p_i^{rc}, p_i^{ec}) / \partial p_i^{rc} \times \partial^2 \pi_i^c(p_i^{rc}, p_i^{ec}) / \partial p_i^{ec} - (\partial^2 \pi_i^c(p_i^{rc}, p_i^{ec}) / \partial p_i^{ec} \partial p_i^{rc})^2 \\ = 4[(b+r)^2 - r^2] > 0, \end{aligned}$$

i.e., π_i^c is a concave function of p_i^{rc} and p_i^{ec} .

Optimal demands for the product in the retail and direct channels in the i^{th} ($i = 1, 2, \dots, n^c$) replenishment cycle are

$$D_i^{rc*} = \frac{1}{2} \left[a\theta - \frac{(b+r)h_r L}{2n^c} + \frac{rh_m L}{2n^c} - bc \left[(i-1) \frac{L}{n^c} \right] \right] \tag{28}$$

and

$$D_i^{ec*} = \frac{1}{2} \left[a(1-\theta) - \frac{(b+r)h_m L}{2n^c} + \frac{rh_r L}{2n^c} - bc \left[(i-1) \frac{L}{n^c} \right] \right]. \tag{29}$$

Optimal centralized system profit in the i^{th} ($i = 1, 2, \dots, n^c$) replenishment cycle is

$$\begin{aligned} \pi_i^{c*} &= \frac{L}{2n^c} \\ &\left(\frac{[r+b-2b\theta(1-\theta)]a^2}{2b(b+2r)} - \frac{a\theta h_r L}{2n^c} - \frac{a(1-\theta)h_m L}{2n^c} \right. \\ &\quad \left. + \frac{[(b+r)(h_r^2 + h_m^2) - 2rh_r h_m]L^2}{8n^{c2}} \right) \\ &- \frac{L}{2n^c} \left[\left(a - \frac{b(h_r + h_m)L}{2n^c} \right) c \left[(i-1) \frac{L}{n^c} \right] - bc \left[(i-1) \frac{L}{n^c} \right]^2 \right] - s_m - s_d - s_r. \end{aligned} \tag{30}$$

Total optimal order quantity and total profit in the centralized scenario over the sales season L are

$$\begin{aligned} Q^{c*} &= \\ \sum_{i=1}^{n^c} Q_i^c &= \sum_{i=1}^{n^c} (D_i^{rc*} + D_i^{ec*}) \frac{L}{n^c} = \frac{L}{2} \left[a - \frac{bL(h_m + h_r)}{2n^c} - 2b \left(\alpha - \frac{\beta L}{2} \left(1 - \frac{1}{n^c} \right) \right) \right]. \end{aligned} \tag{31}$$

and

$$\begin{aligned}
 \pi^{c*} = & \frac{L}{2} \left(\frac{[r + b - 2b\theta(1 - \theta)]a^2}{2b(b + 2r)} - \frac{a\theta h_r L}{2n^c} - \frac{a(1 - \theta)h_m L}{2n^c} + \right. \\
 & \left. \frac{[(b + r)(h_r^2 + h_m^2) - 2rh_r h_m]L^2}{8n^{c2}} \right) \\
 & - \frac{L}{2} \left(a - \frac{b(h_r + h_m)L}{2n^c} \right) \left(\alpha - \frac{\beta L}{2} \left(1 - \frac{1}{n^c} \right) \right) \\
 & + \frac{bL}{2} \left[\alpha^2 - \alpha\beta L \left(1 - \frac{1}{n^c} \right) + \frac{\beta^2 L^2}{6} \left(1 - \frac{1}{n^c} \right) \left(2 - \frac{1}{n^c} \right) \right] \\
 & - n^c s_m - n^c s_d - n^c s_r. \tag{32}
 \end{aligned}$$

4.4. Impact of product compatibility

Product compatibility is becoming increasingly important due to a high intensity of network effect in the industry. When the product closely matches the individual’s needs, wants, beliefs, values, and consumption patterns, it can be considered as being highly compatible with this consumer. The percentage of the primary demand a that goes to the e-tail channel in the model here analysed is $(1 - \theta)$. When the value of θ is lower, the product’s compatibility with the e-tail channel is bigger and more consumers would purchase the product from the e-tail channel. Computer-related products, books, information items, magazines, and digital products have more compatibility with the internet channel than the products like water, rice, gasoline, or, say, milk. Notice that

$$\begin{aligned}
 \partial p_i^{e*} / \partial \theta &= -a / (2(b + 2r)) < 0; \\
 \partial p_i^{r*} / \partial \theta &= a(7b + 10r) / (8(b + r)(b + 2r)) > 0; \\
 \partial w_i^{d*} / \partial \theta &= a(3b + 4r) / (4(b + r)(b + 2r)) > 0; \\
 \partial w_i^{m*} / \partial \theta &= a / (2(b + 2r)) > 0; \\
 \partial D_i^{e*} / \partial \theta &= -a(4b + r) / (8(b + 2r)) < 0; \\
 \partial D_i^{r*} / \partial \theta &= a / 8 > 0
 \end{aligned}$$

and

$$\partial Q_i^* / \partial \theta = -3abL / (8(b + 2r)n^{ds}) < 0.$$

On the other hand, the expressions for the impact of product compatibility on the centralized selling prices and order quantity are as follows:

$$\begin{aligned}
 \frac{\partial p_i^{ec*}}{\partial \theta} &= -\frac{a}{2(b + 2r)} < 0; \\
 \frac{\partial p_i^{rc*}}{\partial \theta} &= \frac{a}{2(b + 2r)} > 0;
 \end{aligned}$$

$$\frac{\partial D_i^{ec*}}{\partial \theta} = -\frac{a}{2} < 0;$$

$$\frac{\partial D_i^{rc*}}{\partial \theta} = \frac{a}{2} > 0$$

and

$$\frac{\partial Q_i^*}{\partial \theta} = 0$$

From the above results we have the following proposition.

PROPOSITION 2 (a) In the decentralized channel, for given n^{ds} , in the i^{th} ($i = 1, 2, \dots, n^{ds}$) replenishment cycle (i) the selling price in e-tail channel decreases, whereas the wholesale prices of the distributor and manufacturer and the selling price of retailer increase with decreasing product compatibility; (ii) the demand for the product in the e-tail channel decreases, the demand of the product in the retail channel increases, but overall order quantity decreases with decreasing product compatibility.

(b) In the centralized channel, for given n^c , in the i^{th} , ($i = 1, 2, \dots, n^c$) replenishment cycle (i) the selling price of the product in e-tail channel decreases, whereas the selling price of retailer increases with decreasing product compatibility; (ii) the demand for the product in e-tail channel decreases, while demand for the product in retail channel increases with decreasing product compatibility. But, there is no effect of product compatibility on the overall order quantity.

Proposition 2 indicates that, as the customers' preference for the retail channel increases, the manufacturer reduces online selling price, but the retailer increases its price. In order to gain a portion of the retailer's profit, the manufacturer increases its wholesale price and, as the mediator, the distributor also increases its wholesale price.

Now, the optimal pricing strategy of the decentralized channel is acceptable to the channel members only when $p_i^{r*} > w_i^{d*}$ and $p_i^{e*} > w_i^{d*}$ for $i = 1, 2, \dots, n^{ds}$. We have $p_i^{r*} > w_i^{d*}$ if

$$\theta > \frac{b}{a}c \left[(i - 1) \frac{L}{n^{ds}} \right] - \frac{L(rh_m + 7(b + r)h_r)}{2an^{ds}}. \tag{33}$$

Also, $p_i^{e*} > w_i^{d*}$ if

$$\theta < \frac{2(b + r)}{5b + 6r} + \frac{(b + 2r)[3(b + r)h_r + (2b + r)h_m]L}{2a(5b + 6r)n^{ds}} + \frac{b(b + 2r)}{a(5b + 6r)}c \left[(i - 1) \frac{L}{n^{ds}} \right]. \tag{34}$$

The right hand sides of (33) and (34) depend on n^{ds} and i . Since the system operates over the planning horizon L , during which n^{ds} replenishment cycles

take place, the maximum value of the right hand side of (34) occurs at $i = 1$, i.e.,

$$\frac{b\alpha}{a} - \frac{L(rh_m + 3(b+r)h_r)}{2an^r} = \theta^{min}, \tag{35}$$

The minimum value of the right hand side of (37) occurs at $i = n^{ds}$, i.e.,

$$\begin{aligned} \frac{2(b+r)}{5b+6r} + \frac{(b+2r)[3(b+r)h_r + (2b+r)h_m]L}{2a(5b+6r)n^{ds}} + \frac{b(b+2r)}{a(5b+6r)} \left[\alpha - \frac{\beta L(n^{ds}-1)}{n^{ds}} \right] \\ = \theta^{max}, \end{aligned} \tag{36}$$

and we have the following Lemma:

LEMMA 1 *For given n^{ds} , the manufacturer will participate in the dual-channel for a product, whose unit cost decreases continuously over L if $\theta \in (\theta^{min}, \theta^{max})$.*

Lemma 1 indicates that the customer’s channel preference is one of the determining factors for operating an online channel besides the traditional retail channel. When $\theta < \theta^{min}$, the retailer cannot do business because its selling price is less than the distributor’s wholesale price. On the other hand, for $\theta > \theta^{max}$, the manufacturer cannot set the optimal selling price as the online price. We consider the maximum of the lower threshold and the minimum of the upper threshold of product’s compatibility with the retail channel. As there are multiple replenishment cycles over L , the retailer’s optimal selling prices are profitable in these cycles only when the lower limit of θ is the maximum among all of these values. Moreover, the online selling price is higher than the distributor’s wholesale price in these cycles only when the upper limit of θ is minimum among all of these values. However, Lemma 1 does not ensure that the retailer and the distributor will participate in the profit making retail-e-tail channel for a product that experiences continuous unit cost decrease over L because of its setup cost. In the i^{th} ($i = 1, 2, \dots, n^{ds}$) replenishment cycle, the distributor will participate in the dual-channel only when its profit is positive, i.e., $\pi_i^{d*} > 0$, that is, if

$$\theta > \frac{1}{a} \left[4\sqrt{2}\sqrt{\frac{n^{ds}(b+r)s_d}{L}} + \frac{L[(b+r)h_r - rh_m]}{2n^{ds}} + bc[(i-1)\frac{L}{n^{ds}}] \right] = \theta_i^d,$$

$$i = 1, 2, \dots, n^{ds}.$$

Yet, the value of θ_i^d depends on n^{ds} and it attains the maximum value when $i = 1$, i.e.,

$$\underline{\theta}^d = \frac{1}{a} \left[4\sqrt{2}\sqrt{\frac{n^{ds}(b+r)s_d}{L}} + \frac{L[(b+r)h_r - rh_m]}{2n^{ds}} + b\alpha \right]. \tag{37}$$

Similarly, the retailer will participate in the dual-channel only when its profit is positive, i.e., $\pi_i^{r*} > 0$, that is, if

$$\theta > \frac{1}{a} \left[8\sqrt{\frac{n^{ds}(b+r)s_r}{L}} + \frac{L[(b+r)h_r - rh_m]}{2n^{ds}} + bc[(i-1)\frac{L}{n^{ds}}] \right] = \theta_i^r,$$

$$i = 1, 2, \dots, n^{ds}.$$

The value of θ_i^d depends on n^{ds} , and it attains the maximum when $i = 1$. That is,

$$\underline{\theta}^r = \frac{1}{a} \left[8\sqrt{\frac{n^{ds}(b+r)s_r}{L}} + \frac{L[(b+r)h_r - rh_m]}{2n^{ds}} + b\alpha \right]. \tag{38}$$

Thus, the retail channel members obtain a positive profit in the i^{th} ($i = 1, 2, \dots, n^{ds}$) replenishment cycle if $\theta > \max\{\underline{\theta}^r, \underline{\theta}^d\} = \underline{\theta}$. In eqs. (37) and (38), the differences, related to $\underline{\theta}^d$ and $\underline{\theta}^r$ depend on the setup costs. If $s_d > s_r$ then $\underline{\theta}^d > \underline{\theta}^r$, otherwise the reverse relations occur. On the other hand, the manufacturer will operate the online channel as long as the demand in the online channel is positive, i.e., $D_i^{e*} > 0$. In distinction from the other channel members' case, for positive profit of the manufacturer, we consider the positive online demand, because the setup cost for operating the online channel is included in the system from which the manufacturer supplies the product. It is assumed that the setup cost of the manufacturer consists of the setup cost for manufacturing the product and the setup cost for operating the online channel, because we concentrate only on the overall profitability (profit from the retail channel and profit from the online channel) of the manufacturer. By simplifying the above inequality, we obtain

$$\theta < \frac{4(b+r)}{4b+r} - \frac{[(4(b+r)^2 - 3r^2)h_m - r(b+r)h_r]L}{2an^{ds}(4b+r)} - \frac{b(4b+7r)}{(4b+r)a} c \left[(i-1)\frac{L}{n^{ds}} \right]$$

$$= \theta_i^m, (i = 1, 2, \dots, n^{ds}). \tag{39}$$

The term θ_i^m also depends on n^{ds} and it attains the minimum value when $i = 1$, i.e.,

$$\underline{\theta}^m = \frac{4(b+r)}{4b+r} - \frac{[(4(b+r)^2 - 3r^2)h_m - r(b+r)h_r]L}{2an^{ds}(4b+r)} - \frac{b(4b+7r)\alpha}{(4b+r)a}. \tag{40}$$

Equations (37), (38) and (39) suggest that when the customers' preference for the retail channel lies between $\max\{\underline{\theta}^r, \underline{\theta}^d\}$ and $\underline{\theta}^m$, the manufacturer can successfully operate a profitable dual-channel. Thus, we have the following lemma:

LEMMA 2 *Over the planning horizon L , for given n^{ds} , the dual channel is profitable for all the channel members when the customers' retail channel preference $\theta \in (\underline{\theta}, \underline{\theta}^m)$.*

Therefore, we obtain the following proposition from Lemmas 1 and 2.

PROPOSITION 3 *Over the planning horizon L , for given n^{ds} , the retail-e-tail channel is a profitable for all the channel members with successful implementation of pricing strategy when the customers' retail channel preference $\theta \in (\max\{\underline{\theta}, \theta^{min}\}, \min\{\theta^{max}, \overline{\theta^m}\})$.*

Now, comparing the selling prices of the product in the retail channel and in the internet channel in the i^{th} ($i = 1, 2, \dots, n^r$) replenishment cycle, we have $p_i^{r*} \geq p_i^{e*}$ if

$$\begin{aligned} \theta &\geq \\ \frac{2(b+r)}{(11b+14r)} + \frac{(b+2r)((h_m-h_r)r+b(4h_m-h_r))L}{a(22b+28r)n^{ds}} + \frac{(b+2r)3bc\left[(i-1)\frac{L}{n^{ds}}\right]}{a(11b+14r)} \\ &= \theta_i^{rd}. \end{aligned}$$

So, as far as the competition between the retail channel and online channel is concerned, we have the following proposition.

PROPOSITION 4 *For given n^{ds} , in the i^{th} ($i = 1, 2, \dots, n^r$) replenishment cycle, the optimal retail price of the decentralized system is higher than the online selling price if $\theta \geq \theta_i^{rd}$, while the reverse takes place for $\theta \leq \theta_i^{rd}$.*

Proposition 4 suggests that there exists severe price competition between the retail and internet channels mainly due to customers' channel preference. When the customers' retail channel preference is higher than a threshold, the retailer sets the price higher than the online price. This happens, because the retailer capitalizes on the customers' retail channel loyalty. Also, from Proposition 2, it can be concluded that the manufacturer and the distributor set higher wholesale prices when θ is higher, because both want to take away some profit from the retailer that is generated by setting higher selling price due to taking the advantage of higher value of θ . A reverse trend may be observed when θ is lower than the threshold. Thus, it is very important for a decision maker to take into account the effect of customers' channel preference while making any pricing decision.

Now, in the centralized decision making context, the manufacturer would be interested in opening the online channel only when the online channel demand in the i^{th} ($i = 1, 2, \dots, n^c$) replenishment cycle is positive, i.e., $D_i^{ec*} > 0$, which, after simplification, yields

$$\theta < 1 - \frac{[(b+r)h_m - rh_r]L}{2an^c} - \frac{b}{a}c \left[(i-1)\frac{L}{n^c} \right] = \overline{\theta}_i^c, i = 1, 2, \dots, n^c$$

where $\overline{\theta}_i^c$ depends on n^c and it attains the minimum value for $i = 1$, i.e.,

$$\overline{\theta}^c = 1 - \frac{[(b+r)h_m - rh_r]L}{2an^c} - \frac{b\alpha}{a}. \tag{41}$$

On the other hand, the manufacturer will operate the retail channel only when the demand for the product in the retail channel in the i^{th} , ($i = 1, 2, \dots, n^c$), replenishment cycle is positive, i.e., $D_i^{rc*} > 0$, which after simplification, yields

$$\theta > \frac{[(b+r)h_r - rh_m]L}{2an^c} + \frac{b}{a}c \left[(i-1)\frac{L}{n^c} \right] = \underline{\theta}_i^c, i = 1, 2, \dots, n^c.$$

Here, $\underline{\theta}_i^c$ depends on n^c and it attains the maximum value for $i = 1$, i.e.,

$$\underline{\theta}^c = \frac{[(b+r)h_r - rh_m]L}{2an^c} + \frac{b\alpha}{a}. \tag{42}$$

Although in the centralized channel, the members co-operate and take decision jointly, product compatibility has an impact on the operating retail-e-tail channel and thus we have the following proposition.

PROPOSITION 5 *In the centralized channel, for given n^c , the decision maker will operate a retail-e-tail channel if $\theta \in (\underline{\theta}^c, \theta^c)$.*

Now, upon comparing the selling prices of the product in the centralized retail channel and the direct channel in i^{th} , ($i = 1, 2, \dots, n^c$) replenishment cycle, we get $p_i^{rc*} \geq p_i^{ec*}$ if

$$\theta \geq \frac{1}{4} \left(2 + \frac{(h_m - h_r)(b + 2r)L}{an^c} \right) = \theta_1^c.$$

Thus, we have the following proposition.

PROPOSITION 6 *For given n^c , in the i^{th} ($i = 1, 2, \dots, n^c$) replenishment cycle, the optimal retail price of the centralized system is higher than online selling price if $\theta \geq \theta_1^c$, while the reverse takes place for $\theta \leq \theta_1^c$.*

Although in the centralized channel, the channel members cooperate, still the compatibility of the product with the channel has a significant impact on pricing. In fact, based on the customers' channel preference, the prices are set in integrated channel.

4.5. Impact of the number of replenishment cycles

In the previous section, we have analyzed the effect of product compatibility on the optimal decision for a given number of replenishment cycles. Now, we investigate the effect of the number of replenishment cycles on the optimal decision. Observe that

$$\frac{d}{dn^{ds}} \left(\frac{1}{n^{ds}} \sum_{i=1}^{n^{ds}} w_i^{m*} \right) = \frac{L}{4n^{ds2}}(h_r - \beta) < 0 \text{ if } h_r < \beta,$$

$$\frac{d}{dn^{ds}} \left(\frac{1}{n^{ds}} \sum_{i=1}^{n^{ds}} w_i^{d*} \right) = \frac{L}{8(b+r)n^{ds^2}} [3(b+r)h_r - rh_m - \beta(b+2r)] < 0 \text{ if}$$

$$h_r < \frac{rh_m + \beta(b+2r)}{3(b+r)},$$

$$\frac{d}{dn^{ds}} \left(\frac{1}{n^{ds}} \sum_{i=1}^{n^{ds}} p_i^{r*} \right) = - \left[\frac{L}{16(b+r)n^{ds^2}} [(b+r)h_r + 3rh_m - \beta(b+4r)] \right] < 0$$

and

$$\frac{d}{dn^{ds}} \left(\frac{1}{n^{ds}} \sum_{i=1}^{n^{ds}} p_i^{e*} \right) = - \frac{L(h_m + \beta)}{4n^{ds^2}} < 0.$$

Here, the average wholesale prices of the manufacturer and the distributor, and average retail price and online price over the planning horizon L decrease as the retailer’s number of replenishments increases. The selling prices in the channels depend on the wholesale price of the distributor. The wholesale price of the distributor depends on the manufacturer. Finally, the wholesale price of the manufacturer depends on the unit cost of the product. The unit cost of the product decreases continuously. For higher number of replenishment cycles, the cycle length is shorter and the distributor receives the product at a lower wholesale price and hence the retailer buys the product at a lower wholesale price. As a result, the retail price decreases. On the other hand,

$$\frac{dQ^*}{dn^{ds}} = \frac{L^2b}{16n^{ds^2}} \left[h_r + \left(\frac{2b+3r}{b+r} \right) h_m + \beta \left(\frac{5b+8r}{b+r} \right) \right] > 0.$$

The above inequality implies that the optimal order quantity of the channel increases as the number of replenishments increases. As the selling prices in the channel decrease for increasing values of n^{ds} , the customers buy more, and hence Q^* increases over L . Now, as far as the optimal number of replenishments in the decentralized scenario over the time horizon L is concerned, we have the following proposition.

PROPOSITION 7 *Over the selling season L , retailer’s profit attains the maximum for the number of replenishments n_r^{ds} , where n_r^{ds} is given by*

$$n_r^{ds} = \begin{cases} [n_0] & \text{if } \pi^{r*}([n_0]) > \pi^{r*}([n_0] + 1) \\ [n_0] + 1 & \text{otherwise} \end{cases}$$

and

$$n_0 = (-d + \sqrt{d^2 + b^3})^{\frac{1}{3}} + (-d - \sqrt{d^2 + b^3})^{\frac{1}{3}} \tag{43}$$

where $b = L^2(A - b\beta)[2a\theta - b(2\alpha - \beta L)]/384(b+r)s_r$
 and $d = L^3[3A(A - 2b\beta) + 2b^2\beta^2]/768(b+r)s_r$.

PROOF: See Appendix.

Proposition 7 suggests that, based on the distributor’s wholesale price and the manufacturer’s online selling price, the retailer chooses n_r^{ds} as the number of replenishments over L in order to maximize its profit. As the manufacturer is dependent on the retail channel, based on the retailer’s replenishment policy, the manufacturer will determine the online pricing schedule.

On the other hand, in the centralized system, the decision maker takes n_0^{c*} as the number of replenishments over L , this number maximizing the total channel profit. In this context, n_0^{c*} can be determined by using the following proposition.

PROPOSITION 8 *Over the selling season L , the number of replenishments for which system’s profit is maximum is given by*

$$n_0^{c*} = \begin{cases} [n_0^c] & \text{if } \pi^{c*}([n_0^c]) > \pi^{c*}([n_0^c] + 1) \\ [n_0^c] + 1 & \text{otherwise} \end{cases}$$

where

$$n_0^c = (-d_c + \sqrt{d_c^2 + b_c^2})^{\frac{1}{3}} + (-d_c - \sqrt{d_c^2 + b_c^2})^{\frac{1}{3}} \tag{44}$$

and $b_c = -[2aL^2(\beta + \theta h_r + (1 - \theta)h_m) - bL^2(2\alpha - \beta L)(2\beta + h_r + h_m)]/24(s_r + s_d + s_m)$,
 $d_c = L^3[3((b + r)(h_r^2 + h_m^2) - 2rh_r h_m) + 4b\beta^2 + 3b\beta(h_r + h_m)]/48(s_r + s_d + s_m)$
 and $[n_0^c]$ denotes the largest integer not greater than n_0^c .

PROOF: The proof follows the same reasoning as for Proposition 7 and so we omit it here.

5. Channel coordination using all unit quantity discount contract

5.1. The coordination mechanism

It is well established in the supply chain literature that coordination among the channel members is imperative to optimize system performance. Thus, a key issue in supply chain management is to develop a mechanism that can align channel members’ objectives and coordinate their activities so as to obtain the centralized channel profit. A variety of coordination mechanisms exists in the supply chain literature, meant to cope with this situation. We assume here that the manufacturer and the distributor provide all unit quantity discount to their immediate next downstream channel member. That means that the manufacturer supplies the product at a wholesale price $\phi_i^m w_i^{m*}$ ($0 \leq \phi_i^m \leq 1$) to the distributor in the i^{th} ($i = 1, 2, \dots, n^c$) replenishment cycle. The distributor supplies the product to the retailer at a wholesale price $\phi_i^d w_i^{d*}$ ($0 \leq \phi_i^d \leq 1$) in the i^{th} replenishment cycle. The online selling price of the manufacturer depends on its own wholesale price, which is claimed from the distributor when supplying the product, as the manufacturer provides a discount $R_i^m \geq 0$ on its

decentralized online selling price. Under this mechanism, the profit functions of the retailer, the distributor and the manufacturer during the i^{th} ($i = 1, 2, \dots, n^c$) replenishment cycle are, respectively, expressed as

$$\pi_i^{rco} = D_i^r T^c p_i^{rco} - D_i^r T^c \phi_i^d w_i^{d*} - s_r - D_i^r T^c \frac{h_r T^c}{2}, \quad (i = 1, 2, \dots, n^c), \quad (45)$$

$$\pi_i^{dco} = D_i^r T^c \phi_i^d w_i^d - D_i^r T^c \phi_i^m w_i^{m*} - s_d, \quad (i = 1, 2, \dots, n^c) \quad (46)$$

and

$$\begin{aligned} \pi_i^{mco} = & D_i^r T^c \phi_i^m w_i^{m*} - D_i^r T^c [(i - 1)T^c] + D_i^e T^c (p_i^{eco} - R_i^m) - \\ & D_i^e T^c \frac{h_m T^c}{2} - D_i^e T^c c [(i - 1)T^c] - s_m, \quad (i = 1, 2, \dots, n^c). \end{aligned} \quad (47)$$

The necessary condition $d\pi_i^{rco}/dp_i^{rco} = 0$ for the existence of the optimal solution π_i^{rco} yields

$$p_i^{rco} = \frac{2(p_i^{eco} - R_i^m)r + 2a\theta + (b + r)(h_r T^c + 2w_i^{d*} \phi_i^d)}{4(b + r)}. \quad (48)$$

The channel will be effectively coordinated only when $p_i^{rco} = p_i^{rc*}$. Thus, the retailer will agree to sell the product at the centralized retail price if it gets the wholesale price discount $\phi_i^d = \phi_i^{d1}$ in the i^{th} replenishment cycle, where

$$\phi_i^{d1} = \frac{N(\phi_i^{d1})}{D(\phi_i^{d1})}, \quad (49)$$

$$N(\phi_i^{d1}) = 8(b^3c + b^2(3c - p_i^e)r + ar^2 + br(a + 2(c - (p_i^e - R_i^m)r - a\theta))),$$

and

$$\begin{aligned} D(\phi_i^{d1}) = & 4ar^2 + b^3(2c - 3h_r T^c) + b^2(8cr + (h_m - 9h_r)rT^c + 6a\theta) \\ & + 2br(r(4c + h_m T^c - 3h_r T^c) + a(2 + 4\theta)). \end{aligned}$$

In the second stage, in response to the retailer's decision, the distributor optimizes its profit function. The optimal value of ϕ_i^d , which optimizes the distributor's profit function, is denoted by ϕ_i^{d2} , and is expressed as follows

$$\phi_i^{d2} = \frac{N(\phi_i^{d2})}{D(\phi_i^{d2})} \quad (50)$$

where:

$$N(\phi_i^{d2}) = (b + r)(2ar + b(2a\theta + (b + 2r)(2c - h_r T^c + 4w_i^{m*} \phi_i^m))),$$

and

$$\begin{aligned} D(\phi_i^{d2}) = & 4ar^2 + b^3(2c - 3h_r T^c) + b^2(8cr + (h_m - 9h_r)rT^c + 6a\theta) \\ & + 2br(r(4c + h_m T^c - 3h_r T^c) + a(2 + 4\theta)). \end{aligned}$$

The channel will be effectively coordinated if ϕ_i^{d1} coincides with ϕ_i^{d2} . Now, from ϕ_i^{d1} and ϕ_i^{d2} , we get the value of ϕ_i^m , which is given as follows

$$\phi_i^m = \frac{N(\phi_i^m)}{D(\phi_i^m)}, \tag{51}$$

where:

$$N(\phi_i^m) = 6ar^2 + b^3(6c + h_r T^c) + 2br(r(6c - 8(p_i^e - R_i^m) + h_r T^c) + a(3 - 5\theta)) + b^2(18cr - 8(p_i^e - R_i^m)r + 3h_r r T^c - 2a\theta)$$

and

$$D(\phi_i^m) = (b + r)(2ar + b(b + 2r)(2c - h_r T^c) + 2ab\theta).$$

Finally, in the last stage of the channel coordination mechanism, the manufacturer will maximize its profit function depending on the distributor's reaction. Since the manufacturer provides discount on optimal decentralized wholesale price, the necessary condition $d\pi_i^{mco}/dp_i^{eco} = 0$ for the existence of the optimal solution yields

$$(p_i^{eco} - R_i^m) = \frac{N(p_i^{eco} - R_i^m)}{D(p_i^{eco} - R_i^m)}, \tag{52}$$

where:

$$N(p_i^{eco} - R_i^m) = (b + r) (2a(b^2 + 2br + 3r^2) + b(b + 2r)(2bc + 6cr + bh_m T^c + h_m r T^c + 2h_r r T^c) - 2ab(b^2 + 4br + 7r^2)\theta),$$

and

$$D(p_i^{eco} - R_i^m) = 4b(b + 2r)(b^2 + 2br + 3r^2).$$

Notice from (12) and (27) that $p_i^{e*} = p_i^{ec*}$ if $T^c = T^{ds}$, i.e., $n^c = n^{ds}$. The channel will be effectively coordinated in the i^{th} replenishment cycle if $(p_i^e - R_i^m) = p_i^{ec*}$, i.e., $R_i^m = p_i^{eco*} - p_i^{ec*}$, which, after simplification, yields

$$R_i^m = -\frac{r[2a\theta - ((b + r)h_r - rh_m)T^c - 2bc]}{2(b^2 + 2br + 3r^2)}, i = 1, 2, \dots, n^c. \tag{53}$$

Thus, in the i^{th} ($i = 1, 2, \dots, n^c$) replenishment cycle, the optimal profits of the channel members under this contract can be determined to be equal

$$\pi_i^{rco*} = \frac{T^c[2a\theta - ((b + r)h_r - rh_m)T^c - 2bc]^2}{(16(b + r))} - s_r, \tag{54}$$

$$\pi_i^{dco*} = \frac{T^c[2a\theta - ((b + r)h_r - rh_m)T^c - 2bc]^2}{(16(b + r))} - s_d \tag{55}$$

and

$$\begin{aligned}
 \pi_i^{mco*} = & \frac{4ab(b+2r)(-(b+r)(2c+h_mT^c) + (4bc+b(h_m+h_r)T^c + (-h_m+h_r)rT^c)\theta)T^c}{16b(b+r)(b+2r)} \\
 & + \frac{T^c(-(h_m-h_r)^2r^2T^{c2} + b^2(h_m-h_r)T^c(4c+(h_m+h_r)T^c))}{16(b+r)} \\
 & + \frac{T^c(2br(4c^2+2c(3h_m-h_r)T^c + (h_m^2+h_mh_r-h_r^2)T^{c2}))}{16(b+r)} \\
 & + \frac{4a^2((b+r)^2-2b(b+r)\theta-2br\theta^2)T^c}{16b(b+r)(b+2r)} - s_m. \tag{56}
 \end{aligned}$$

Notice that $\pi_i^{rco*} + \pi_i^{dco*} + \pi_i^{mco*} = \pi_i^{c*}$ for any $i = 1, 2, \dots, n^c$. This means that the channel is coordinated in the i^{th} replenishment cycle. Obviously, the generality of i implies that all unit quantity discount coordinates the channel over the planning horizon L and we have the following proposition.

PROPOSITION 9 *The all unit quantity discount with the set of contracts $(\phi_i^d, \phi_i^m, R_i^m)$ coordinates the three level dual-channel supply chain.*

The above analysis shows that the mechanism, based on all unit quantity discount can effectively coordinate the supply chain, while allowing the manufacturer to earn a positive profit. Due to coordination, the profits of the retailer and the distributor increase significantly, compared with the corresponding decentralized profits, but the mechanism fails to provide any additional benefit to the manufacturer. Now, we shall discuss the implementation of the contract with a complementary agreement between the manufacturer and the distributor and between the distributor and the retailer that can not only coordinate the dual-channel supply chain but also ensure a win-win strategy for all the members of the channel.

5.2. Franchise fees for win-win outcome

Although the all unit quantity discount effectively coordinates the channel, it does not ensure the win-win outcomes for all the channel members. For successful implementation of the contract, suppose the manufacturer charges a franchise fee F_m to the distributor over the planning horizon L , besides the offer (R_i^m, ϕ_i^m) in the i^{th} replenishment cycle for all i ($i = 1, 2, \dots, n^c$). In this case, the distributor charges a franchise fee F_d to the retailer over the planning horizon L besides the offer ϕ_i^d in the i^{th} replenishment cycle. As long as a franchise fee F_d satisfies the relation $\sum_{i=1}^{n^c} \pi_i^{rco*} - F_d \geq \pi^{r*}$, the retailer will accept the (ϕ_i^d, F_d) contract that yields

$$F_d \leq \sum_{i=1}^{n^c} \pi_i^{rco*} - \pi^{r*} = \overline{F_d}. \tag{57}$$

On the other hand, the minimum value of the franchise fee, charged by the manufacturer to the distributor, is given by

$$F_m \geq \pi^{m*} - \sum_{i=1}^{n^c} \pi_i^{mco*} = \underline{F_m}. \tag{58}$$

The profit of the distributor over the planning horizon L is greater than or equal to its decentralized profit if

$$F_m - F_d \leq \sum_{i=1}^{n^c} \pi_i^{dco*} - \pi^{d*}. \tag{59}$$

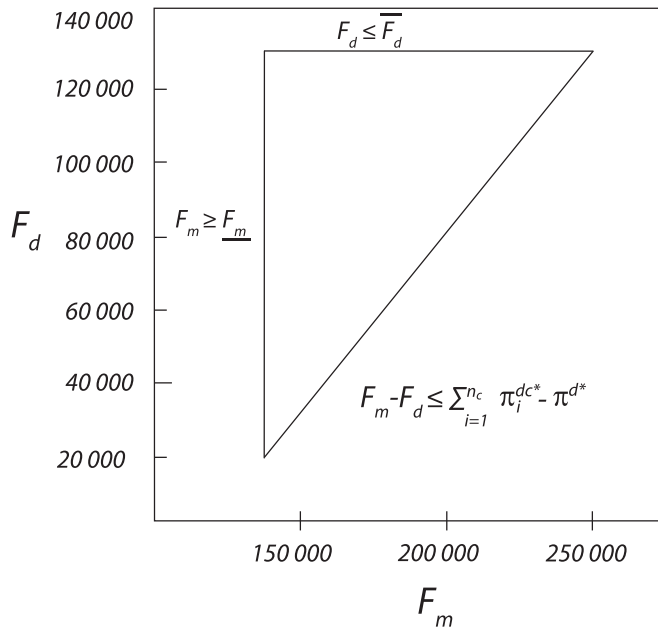


Figure 2. The win-win region of the franchise fees

Interestingly, in the traditional brick-and-mortar channel, the distributor plays the central role, because it actually maintains the lot streaming between the manufacturer and the retailer. Thus, there may arise two questions: (i) What is the minimum of franchise fee ($\underline{F_d}$) that the distributor will accept from the retailer? (ii) What is the maximum of franchise fee ($\overline{F_m}$) that the distributor can pay to the manufacturer? From equation (58), it is clear that $\underline{F_d}$ and $\overline{F_m}$ depend on the distributor's previous decision, i.e., $\underline{F_d}$ depends on the value of the franchise fee that the distributor pays to manufacturer and $\overline{F_m}$ depends

on the value of the franchise fee that the distributor accepts from the retailer. In this context, Fig. 2 shows the win-win region of the franchise fees under the conditions of all unit quantity discount. Thus, we have the following next proposition.

PROPOSITION 10 *The all unit quantity discount with agreement of franchise fees coordinates the three-level dual-channel supply chain and provides the win-win opportunity for the channel members for the set of franchise fees (F_d, F_m) if they satisfy the inequalities (i) $F_d \leq \overline{F_d}$, (ii) $F_m \geq \underline{F_m}$ and (iii) $F_m - F_d \leq \sum_{i=1}^{n^c} \pi_i^{dco*} - \pi^{d*}$*

The relations (56), (57) and (58) suggest that a higher F_m provides a benefit to the manufacturer, whereas a lower F_d benefits the retailer. The profit of the distributor depends on its negotiations with its upstream as well as downstream channel members. The values of F_m and F_d depend heavily on the bargaining powers of the manufacturer and the retailer in the supply chain. In the next subsection, we discuss the outcomes of the bargaining.

5.3. Determination of franchise fees through bargaining

Bargaining refers to situations, where two or more players, who have the opportunity to collaborate, can take advantage from the mutual benefit in more than one way. To determine the exact value of the franchise fees and the profits of respective channel members, we use the generalized asymmetric Nash bargaining solution (Nash, 1950). Nash proposed a basic framework for constructing negotiation model among players. Suppose the manufacturer, distributor and retailer have bargaining powers: $\theta_1 \in (0, 1)$, $\theta_2 \in (0, 1)$ and $(1 - \theta_1 - \theta_2) \in (0, 1)$, respectively. Let Δ_m , Δ_d and Δ_r , denote the surplus profit share of the manufacturer, the distributor and the retailer, respectively. The functional forms of Δ_m , Δ_d and Δ_r are as follows:

$$\Delta_m(F_m) = \sum_{i=1}^{n^c} [\pi_i^{mco*}] + F_m - \pi^{m*} = X_m + F_m,$$

$$\Delta_d(F_m, F_d) = \sum_{i=1}^{n^c} [\pi_i^{dco*}] - F_m + F_d - \pi^{d*} = X_d - (F_m - F_d)$$

and

$$\Delta_r(F_d) = \sum_{i=1}^{n^c} [\pi_i^{rco*}] - F_d - \pi^{r*} = X_r - F_d$$

where

$$X_m = \sum_{i=1}^{n^c} [\pi_i^{mco*}] - \pi^{m*}; X_d = \sum_{i=1}^{n^c} [\pi_i^{dco*}] - \pi^{d*}$$

and

$$X_r = \sum_{i=1}^{n^c} [\pi_i^{rco*}] - \pi^{r*}.$$

The total surplus profit, generated through cooperation, is equal to $\Delta_m + \Delta_d + \Delta_r = X_m + X_d + X_r$.

According to the generalized asymmetric Nash bargaining model, we have to maximize the following function:

$$Max_{F_m, F_d} \Delta(F_m, F_d) = \Delta_m^{\theta_1} \Delta_d^{\theta_2} \Delta_r^{(1-\theta_1-\theta_2)}. \quad (60)$$

The optimal solution of the above Nash bargaining product can be obtained by solving the equations for $\partial \log \Delta / \partial F_m = 0$, and $\partial \log \Delta / \partial F_d = 0$. After simplification, the optimal values of $F_m = F_m^b$ and $F_d = F_d^b$ are found as follows:

$$F_m^b = \theta_1(X_d + X_r) - (1 - \theta_1)X_m \quad (61)$$

and

$$F_d^b = (\theta_1 + \theta_2)X_r - (1 - \theta_1 - \theta_2)(X_m + X_d). \quad (62)$$

Using the bargaining solution for the franchise fees, given in (60) and (61), the bargaining profits of the manufacturer, distributor and retailer are obtained, respectively, as

$$\pi_b^{m*} = \pi^{m*} + \theta_1(X_m + X_d + X_r), \quad (63)$$

$$\pi_b^{d*} = \pi^{d*} + \theta_2(X_m + X_d + X_r) \quad (64)$$

and

$$\pi_b^{r*} = \pi^{r*} + (1 - \theta_1 - \theta_2)(X_m + X_d + X_r). \quad (65)$$

Note that, in particular, if all channel members have equal bargaining powers, i.e., $\theta_1 = \theta_2 = 1 - \theta_1 - \theta_2 = 1/3$, then each gets equal share $((X_m + X_d + X_r)/3)$ of the total surplus. Thus, the all unit quantity discount with agreement of franchise fees coordinates effectively the channel and leads to win-win profits for the channel members. The bargaining outcomes that depend on the negotiation powers of the channel members exactly specify the division of the surplus profit, which is generated through channel coordination and hence, the profits of the channel members after coordination are higher.

6. A numerical example

Suppose that, in a high-tech industry, the planning horizon or sales season of a product is $L = 6$ months (180 days). At the beginning of the sales season, the initial unit cost of the product is $\alpha = \$ 200$ and the cost of the product decreases at a rate $\beta = \$0.25$ per day. Holding costs of the retailer and the

Table 1. Profit and order quantity for changing order cycles in decentralized and centralized scenarios

n	Decentralized channel					Centralized channel	
	Q^*	π^{m*}	π^{d*}	π^{r*}	Channel profit	Q^{c*}	Profit
1	4207	120541	2552	1276	124369	5355	144852
2	6381	285283	7570	3785	296637	8190	348700
3	7105	345570	8544	4272	358386	9135	423833
4	7467	371096	8150	4075	383321	9607	455997
5	7685	381334	7152	3576	392062	9891	469246
6	7830	383543	5836	2918	392297	10080	472558
7	7933	381024	4332	2166	387522	10215	470016

manufacturer are $h_r = \$0.4$ and $h_m = \$0.5$ per unit per day, respectively. The set up costs of the retailer, the distributor, and the manufacturer are $s_m = \$15000$, $s_d = \$2000$, $s_r = \$1000$ per cycle, respectively. Other parameter values are $b = 0.5$, $r = 0.2$ and $\theta = 0.45$. The optimal values in the decentralized and centralized settings are provided in Table 1.

As can be inferred from Propositions 5 and 6, $n^{ds} = 3.07$ and $n^c = 6.05$. These two values are approximated to the nearest integers 3 and 6, respectively. Based on these parameters, optimal values are presented in Table 1, with $n^c > n^{ds}$, which is quite reasonable, because the effect of unit decrement can be fully utilized only when the replenishment cycle length is shorter. Observe that the retailer’s profit in the centralized channel is higher than the total profit of the decentralized channel. Thus, the channel is not coordinated.

In Table 2, optimal selling prices in centralized and decentralized channels are provided. So, in Table 2, it is observed that the optimal selling prices in the retail as well as in online channels decrease as the number of replenishment cycles increases in both decentralized and centralized channels. Also, same results may be observed for the wholesale prices of the manufacturer and the distributor.

Customers’ preference for the retail channel intensifies the channel competition. Under the present model setting, Figs. 3 and 4 represent behavior of prices with respect to customers preference for the retail channel in decentralized and centralized channels, respectively. Notice that above the thresholds of $\theta = 0.432$

Table 2. Optimal prices and profits in different replenishment cycles in decentralized and centralized scenarios

n	w_i^{m*}	Decentralized channel			Centralized channel	
		w_i^{d*}	p_i^{r*}	p_i^{e*}	p_i^{rc*}	p_i^{ec*}
1	126.4	134.9	175.2	178.0	162.4	178.0
2	135.4	146.8	170.5	166.8	153.4	166.8
	124.2	139.6	165.3	155.5	142.2	155.5
3	138.4	150.8	169.0	163.1	150.4	163.1
	130.9	146.0	165.5	155.5	141.9	155.5
	123.4	141.2	162.0	148.1	135.4	148.1
4	139.9	152.8	168.2	161.2	148.9	161.2
	134.3	149.2	165.6	155.5	143.3	155.5
	128.7	145.5	162.9	149.9	137.7	149.9
	123.1	141.9	160.3	144.3	132.1	144.3
5	140.8	154.0	167.7	160.0	148.0	160.0
	136.3	151.1	165.6	155.5	143.5	155.5
	131.8	148.2	163.6	151.0	139.0	151.0
	127.3	145.3	161.5	146.5	134.5	146.5
	122.8	142.4	159.4	142.0	130.0	142.0
6	141.4	154.8	167.4	159.3	147.4	159.3
	137.7	152.4	165.7	155.6	143.7	155.5
	133.9	149.9	163.9	151.8	139.9	151.8
	130.2	147.5	162.2	148.0	136.2	148.0
	126.4	145.1	160.5	144.3	132.4	144.3
	122.7	142.7	158.7	140.5	128.7	140.5

Table 3. Optimal profits under all unit quantity discount (AQD) and bargaining solutions under AQD with franchise fee (FF)

n^c	i	π_i^m	π_i^d	π_i^r	$\pi_i^I = \pi_i^m + \pi_i^d + \pi_i^r$	
	1	21852.6	12906.7	13906.7	48666	
	2	26469.1	16054.7	17054.7	59578.5	
	3	31326.7	19504	20504	71334.7	
AQD	6	4	36425.4	23254.7	24254.7	83964.8
	5	41765.1	27306.7	28306.7	97378.5	
	6	47345.9	31660	32660	111666	
Total profit		205184.8	130686.8	136686.8	472558.4	
Bargaining total profit	6	402656.2	42796.1	27106.1	472558.4	

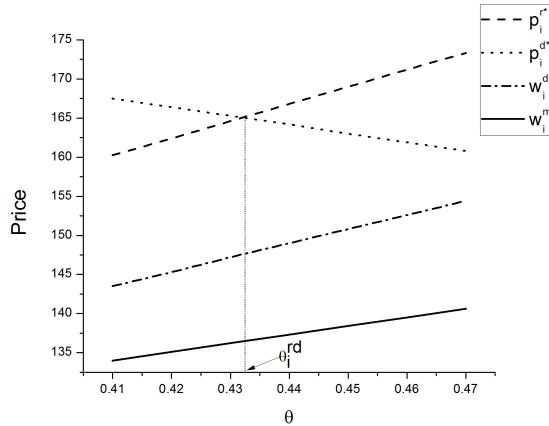


Figure 3. Nature of decentralized prices with respect to product compatibility

and $\theta = 0.507$, the optimal retail price is higher than the optimal online price in decentralized and centralized channels, respectively. For any $\theta \in (0.41, 0.83)$ and $\theta \in (0.277, 0.709)$, the manufacturer can operate profitably decentralized and centralized channels, respectively, for continuously decreasing unit cost of the product over the planning horizon. As indicated, the wholesale prices of the manufacturer and the distributor and the selling prices in retail and online channels decrease in every subsequent replenishment cycle.

Table 3 contains the outcomes of the proposed coordination contract and bargaining assuming that the bargaining powers of the manufacturer, the distributor and the retailer are 0.5, 0.3 and 0.2, respectively. The number of replenishment cycles for the optimal profit under all unit quantity discount with franchise fees is 6. The profit of the manufacturer over the planning horizon under all unit quantity discount contract is 205,184.8, while it amounts to 345,570 in the decentralized scenario, but the distributor's and retailer's profits are higher. Although the all unit quantity discount contract resolves the channel conflict, the manufacturer faces a huge loss. On the other hand, all unit quantity discount with franchise fee eliminates channel conflict and creates the win-win opportunity for all the channel members in the franchise fees region $F_d \leq 132, 415$; $F_m \geq 140, 385$ and $F_m - F_d \leq 122, 143$. Moreover, all the channel members improve their respective profits through bargaining.

7. Conclusion

The paper analyzes pricing, replenishment policies, coordination and surplus profit division issues for a product in a dual-channel supply chain. The dual-channel consists of the manufacturer-distributor-retailer retail channel and the

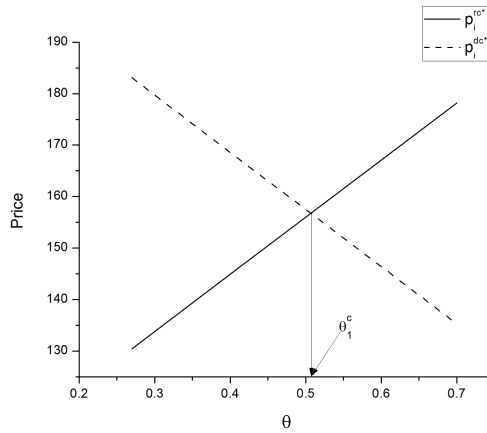


Figure 4. Nature of centralized prices with respect to product compatibility

manufacturer's online channel. The product has finite lifetime and its unit cost decreases continuously. The existing literature in this domain considers the pricing and replenishment policies for a single business entity. But the coexistence of the retail and e-tail channels in the current business scenario is quite common. The proposed model provides the direction for finding the best channel performance by determining the optimal values of the decision variables. To identify the best channel performance and win-win profits, this article uses all unit quantity discount with agreement of franchise fee and Nash bargaining. In the present model, channel coordination using contract is complex, because of different number of replenishment cycles in the centralized and decentralized decisions, besides the determination of prices in the channel. This problem is addressed and solved by the hybrid coordination contract. Moreover, the paper analyzes how channel competition intensifies for the customers' channel preferences and determines the thresholds of the product compatibility for the coexistence of profitable retail-e-tail channel. For channel competition, prices in the channels are different. Our article indicates that when the customers' retail channel preference is below a certain threshold, the manufacturer sets online price higher than the retail price, otherwise the retail price is higher. Interestingly, when the channel members cooperate, there is a competition between the channel members because of product compatibility. Thus, one should think about how cross price effect, due to product compatibility, can be reduced for improved channel performance.

Although the paper provides some interesting managerial insights, it still has definitely some limitations. The paper considers linear price dependent demand. This limitation could be relaxed by considering nonlinear function of price, this

being a decision variable. The proposed model assumes information symmetry and there is one retailer in the downstream. This model may be extended in the future by relaxing these assumptions, which are complex, but robust and reflect the dynamics of the current business scenario. Another limitation of our model is that decision variables and values of parameters are deterministic in nature. As a result, this proposed model could be extended in the future in stochastic and fuzzy environments.

Appendix

PROOF OF PROPOSITION 7

Differentiating the total profit of the retailer, given in (25) with respect to n^{ds} , i.e., $\frac{\partial \pi^{r*}}{\partial n^{ds}} = 0$, gives

$$\frac{64(b+r)s_r}{L^2}(n^{ds})^3 + \left[\left(a\theta - b\left(\alpha - \frac{\beta L}{2}\right) \right) (A - b\beta) \right] n^{ds} + \frac{L}{6}[3A(A - 2b\beta) + 2b^2\beta^2] = 0.$$

Using Cardon's method for solving the cubic equation, we get

$$n_0 = (-d + \sqrt{d^2 + b^3})^{\frac{1}{3}} + (-d - \sqrt{d^2 + b^3})^{\frac{1}{3}}$$

where $b = L^2(A - b\beta)[2a\theta - b(2\alpha - \beta L)]/384(b+r)s_r$
and $d = L^3[3A(A - 2b\beta) + 2b^2\beta^2]/768(b+r)s_r$.

The analytical solution for finding the number of replenishment cycles over the selling season L provides the optimal profit of the retailer, which can be an integer or can not be an integer. Yet, the number of replenishment cycles must be integer. It is very simple to find out the integer solution for the number for replenishment cycles for the retailer. Suppose $[n_0]$ denotes the largest integer not greater than n_0 . Then the retailer will accept $[n_0]$ if $\pi^{r/ds}([n_0]) > \pi^{r/ds}([n_0] + 1)$, otherwise $([n_0] + 1)$ is the better solution for the retailer. Hence, the optimal number of replenishment cycles for the retailer is given by

$$n_r^{ds} = \begin{cases} [n_0] & \text{if } \pi^{r*}([n_0]) > \pi^{r*}([n_0] + 1) \\ [n_0] + 1 & \text{otherwise} \end{cases}.$$

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