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Book review:

TOPICS IN THE MATHEMATICAL MODELLING OF COMPOSITE MATERIALS

by

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The composite materials are made of a mixture of various simple materials. The size of the heterogeneities is supposed to be small compared to the macroscopic size of the sample. We are dealing therefore, with two scales: the macroscopic one, i.e., the size of the domain occupied by the material, and the microscopic one, i.e., the size of the heterogeneities. The macroscopic behavior of microscopically heterogenous materials has attracted the attention of researchers in applied mathematics, material science and applied mechanics. This area of investigations has many applications involving optimal design of structures, design of composite materials, description of coherent phase transitions. The following topics are discussed in the book: 1. existence of solutions to homogenized equations describing macroscopic properties of the material; 2. evaluation of the bounds for coefficients of homogenized equations; 3. existence of optimal solutions to domain optimization problems; 4. formulation of necessary optimality conditions for relaxed optimization problem for homogenized systems.

The book contains an arbitrary but relatively coherent selection of fundamental papers written by leading experts in the field. These papers were written originally in French or in Russian. They were circulating either as notes or as preprints in scientific community. They were not accessible for newcomers to the domain. The aim of the volume's editors was to publish the selected papers in a single readily accessible volume in English translation and accelerate the progress in research on mathematical foundations of the field.

The book consists of eight chapters. Chapter 1 entitled "On the Control of Coefficients in Partial Differential Equations" by F. Murat and L. Tartar deals with ill-posed domain optimization problem for the Dirichlet system where the coefficient a of the second order elliptic partial differential equation is a control variable. This coefficient is equal to a in 0_1 and is equal to β in 0_2, where $0_1 \cup 0_2 = 0$. Due to the lack of sufficient regularity of the domain O this problem, in general, may have no optimal solution. To assure the existence of optimal solutions a new Dirichlet problem is being introduced where the original coefficient a is replaced by a new one, a^0, obtained as a limit of the minimizing subsequence a_n of the original coefficients. Moreover, the solution u_n of the original Dirichlet problem corresponding to the coefficient a_n tends to a solution u^0 of the Dirichlet problem with the coefficient a^0. The characterization of the set of coefficients a^0 is not completely solved except for the one- and two-dimensional problems. In the paper the necessary optimality condition for the relaxed problem is formulated.

Chapter 2 "Estimation of Homogenized Coefficients" by L. Tartar is concerned with obtaining formulae for effective coefficients of equations satisfied by macroscopic quantities. The homogenization method gives the possibility of finding equations satisfied by macroscopic quantities from equations satisfied by the physical quantities and from the information on the microscopic composition. The mathematical method is based on the study of solutions to partial differential equations with highly oscillating coefficients. These solutions are very close, in a weak topology sense, to the solution of the homogenized equation with rather smooth coefficients. The homogenized equation can be very different from the original one. Formulae in the case of periodic coefficients and laminated material are given. The paper deals with the case where the partial information about the microscopic structure is available only. The inequalities satisfied by the homogenized coefficients in this case are provided.

Chapter 3 entitled "H-Convergence" by F. Murat and L. Tartar deals with the H-convergence theory as a tool for analysis of composites in complete generality without geometrical hypotheses such as periodicity or randomness. When specified to selfadjoint case it becomes equivalent to G-convergence. The authors study the existence of solutions to homogenized elliptic equations of the second order. The second order elliptic equation with coefficients A' depending on parameter E is introduced. The coefficients A' are bounded. The bounded solution u' to this equation describes the microscopic properties of the material. The authors show that the limit u^0 of u' with E passing to zero satisfies the equation of the same type as is satisfied by u'. Their approach is based on the notion of H-convergence. The sequence {A'} of coefficients A' is H-convergent to a coefficient A^0 if and only if u' tends to u^0 as well as A' grad u' tends to A^0 grad u^0 when E is passing to zero and u^0 is a solution of the homogenized equation with the coefficients A^0 (u^0 describes macroscopic properties of the material). The authors use also compensated compactness phenomenon, i.e., the fact that the scalar product of two weakly convergent sequences converges, when E is passing to zero, to the scalar product of the limits of the sequences in suitable topology. Using H-convergence notion as well as the compensated compactness property the authors show the existence of the homogenized energy, characterize the conditions the coefficient A^0 has to satisfy and discuss the approximation of grad u' by a corrector matrix.

Chapter 4 entitled "A Strange Term Coming From Nowhere" by D. Cioranescu and F. Murat is devoted to study the solution of a Dirichlet problem in a domain perforated by holes. The authors consider the domain perforated by the increasing number of holes, regularly distributed with decreasing diameter. The most interesting case is when the holes reach a critical size depending on their number and distribution. In this case the limit of the sequence of solutions of microscopic equation is the solution of a Dirichlet problem in unperforated domain with another operator which is the sum of the initial one and of an additional term coming in from the holes. The behavior of solutions of variational inequalities with highly oscillating obstacles is also studied. These inequalities are associated with the preceding Dirichlet problems through suitable definition of obstacles. The energy method based on construction of suitable test functions is used for study these homogenized problems.

Chapter 5 entitled "Design of Composite Plates of Extremal Rigidity" by L. Gibiansky and A. Cherkaev deals with optimal design of plates. The plate is assumed to be assembled from two isotropic materials characterized by different values of their elastic moduli. The amount of each material is given. The rigidity of the plate is defined as a work performed by an external load on deflection of the points of the plate. The optimization problem consists in finding distributions of the materials to reach the extremum rigidity. The optimal distribution of the materials is given by the infinitely frequently alternating sequences of domains occupied by each of the materials. This leads to apperance of anisotropic composites. In the paper the exact bounds on the elastic energy density are obtained. The microstructures of the optimal composites are found. The effective properties of such composities are calculated. These composites extend the range of available materials. The optimal design problem for plate is formulated and solved on an extended set of design parameters. The obtained results are applied for solving optimal design problem for the variable thickness plate where the mass of the plate is given and its thickness bounded. A rule is found allowing to distinguish the cases in which the optimal composite is assembled of elements having maximal or minimal thickness only. Numerical results for the optimization problem of the clamped square plate are provided.

Chapter 6 entitled "Calculus of Variations and Homogenization" by F. Murat and L. Tartar deals with formulation of necessary optimality conditions for ill-posed optimal design problems. There are optimal design problems having no solution in the class that was considered a priori and the minimizing sequence converges to something that may be called "generalized solution" and which has to be precisely defined. Moreover, in some cases the set on which one optimizes is not convex so that one cannot make variations without enlarging the set that was considered a priori. These difficulties are solved using the approach of L. Young and Pontriagin, which consists of introducing Young's generalized functions and then formulating the problem on a convex set and calculating its derivative. The procedure that consists of constructing generalized solutions is called relaxation. In the paper the original optimization problem is replaced by the relaxed one. The necessary optimality conditions for a generalized solution to be an optimal one are studied. This leads to some characterizations and to several geometrical remarks about the solution of the relaxed problem and those of initial one if they exist.

Chapter 7 entitled "Effective Characteristics of Composite Materials and the Optimal Design of Structural Elements" by K. A. Lurie and A. Cherkaev deals with structural optimization related to a design of inhomogeneous continuous media. Using the notions of quasiconvex functions and G-closed sets G closures for selected sets are constructed. The optimal design problems for prismatic bar of extremal torsion ridigity as well as thermal flow problem are formulated. The developed methodology of constructing G - closure of admissible control set is used for solving these optimal design problems. These problems are replaced by the relaxed one. The existence of optimal solutions to the relaxed problem is shown and the optimality conditions for the relaxed problems are provided.

Chapter 7 is accompanied by the appendix "Load Characteristics of an MHD Channel in the case of Optimal Distribution of the Resistivity of the Working Medium" by K. Lurie and T. Simkina. The appendix is devoted to investigation of optimal distribution of the resistivity in the mhd channel flow to maximize net current flow when a magnetic field is applied. The optimal anisotropic resistivity distribution is realized by insertion of insulating baffles into the flow. An assemblage of locally parallel baffles creates a system of infinitely thin canals for the electric current flow. This construction allows to control the direction of current lines flows so as to maximize the net current. In the appendix the optimal solution is calculated and the current flow dependence on the resistance is discussed.

Chapter 8 entitled "Microstructures of Composites of Extremal Rigidity and Exact Bounds on the Associated Energy Density" by L. Gibiansky and A. Cherkaev is concerned with investigation of periodic structures providing minimal energy density in a given stress field. The composites are assembled from two isotropic materials taken in a prescribed proportion and may be of an arbitrary microstructure. Two sided bounds for the energy density are obtained for extremal stiffness composites. One bound is an inequality valid for all structures independently of their geometry and the second one corresponds to the energy stored in a composite with a specially chosen microstructure. For all possible external stress fields microstructures of porous composites of maximal stiffness are explicitly determined. The obtained results are applied to optimal design problems.

Since the book contains an arbitrary selection of the fundamental papers in the composite materials modelling field, written in mid seventies and mid eighties, the primary addressee of the book are PhD students and researchers entering the field. The book is essential in understanding the foundations of the field and mathematical research tools used in the field. The book is also recommended for specialists working in optimal design, and in particular - in optimization structural topology. However, from the point of view of mathematical tools used, the book may turn out too difficult to be recommmended for engineers. Each chapter of the book provides the bibliography related to the particular topic. The list of latest monographs and research papers in the field has been added by the editors to allow the interested readers to explore the literature on their own. There are still many unsolved problems in the field of composite materials modelling. The areas of current interest include bounds for multicomponent composites, bone remodeling, polycrystal plasticity and practical suboptimal design. The book provides tools to investigate these challenging problems and will accelerate progress in the field.

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