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Social choice, stable outcomes and deliberative democracy*

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Abstract: It has turned out that all voting rules fail on some intuitively plausible desiderata. This has led some political scientists to argue that the notion of the will of the people is profoundly ambiguous and the absence of voting equilibria a generic state of affairs. As a constructive remedy to this some authors have introduced the idea of *deliberative democracy*. This view of democracy has much to recommend itself, most importantly the emphasis on individuals in devising the decision alternatives. Some empirical evidence also suggests that the deliberative institutions provide an escape from some of the most notorious incompatibility results in social choice theory. We shall critically examine this suggestion. The view emerging from this examination is that social choice theory and deliberative democracy are complementary, not competing approaches to democratic decision making.

Keywords: choice desiderata, domain restriction, incompatibility result, single-peakedness, voting rule

1. Introduction

About 40 years ago William H. Riker published one of his main works: *Liberalism Against Populism: A Confrontation Between the Theory of Democracy and the Theory of Social Choice* (Riker, 1982). The book distinguishes between two concepts of democracy: the liberal and populist one. The former sees the possibility of changing the rulers (presidents, parliaments, councils etc.) in elections held at regular intervals as the crucial hallmark of democracy, while the latter

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equates the electoral outcomes with the 'will of the people' in a more substantive way. Riker rejects the populist account by arguing that the will of the people is an empty notion because there are distributions of the voters' opinions over alternatives such that different voting rules result in different outcomes. Thus, the will of the people would seem to depend not only on the opinions of voters but also on the voting rule being applied. Moreover, all voting rules seem to have at least one implausible property that is incompatible with the idea that voting outcomes reflect the popular will.

Riker's view of the superiority of the liberal theory of democracy gets additional support from the many negative results in social choice theory, which typically amount to the theoretical incompatibility of several choice desiderata. The best-known of these is undoubtedly due to Arrow (Arrow, 1963), but there are others of similar severity, e.g. Gibbard's, Satterthwaite's and Moulin's incompatibility theorems (Gibbard, 1973; Kelly, 1978; Moulin, 1988; Satterthwaite, 1975). In what follows we shall first briefly remind ourselves of those results, whereupon we have a look at another, more recent, view of democracy, viz. the deliberative one (see Fishkin 1991, 2009). It has been suggested that deliberative institutions are conducive to creating decision situations whereby some important negative social choice results can be escaped from and the voting results are rendered more stable than those results would lead us to believe. We then evaluate the promise of deliberative institutions and argue that while some deliberative institutions may indeed create conditions that make some voting paradoxes unlikely, this in no way undermines the validity of those results as such. Furthermore, it will be argued that some deliberative processes may be conducive to more, rather than less, strategic maneuvering by voters. In a similar vein, the notion of stability of voting outcomes, adopted by deliberative theorists, is perhaps too narrow in restricting attention only to fixed electorates and overlooking participation paradoxes as sources of instability.

2. Different outcomes from identical opinions and a review of some classic incompatibility theorems

The procedure-dependence of voting outcomes is exemplified in Table 1, where the summary results are shown for the case, in which 11 voters have expressed* their preferences over six candidates and six different voting rules are being applied: (i) the plurality voting, (ii) the plurality runoff system, (iii) Copeland's rule, (iv) the Borda count, (v) the approval voting, and (vi) the range voting. Rule (i) assigns each voter one vote and the winner is the candidate that is given more votes than any of the others. Assuming that each voter is ignorant of the opinions of other voters, it makes sense to argue that all voters vote for the candidate they rank first. Under this assumption, the opinion distribution of Table 1 yields A as the winner. System (ii) elects the candidate that is

*The order of preference is from top (best) to bottom (worst).

majority preferred to its sole competitor in the contest between the two largest voter-getters in the plurality voting. This method results in D. Copeland's rule elects a candidate that would defeat more of its contestants in pairwise majority comparisons (i.e. ignoring other candidates) than any other candidate.[†] In this example the Copeland (and Condorcet) winner is E. The Borda count is based on the scores given to candidates in individual preference rankings. The candidate's points given by a voter equal the number of candidates ranked lower than it by the voter in question. Summing up the points given by all voters to a given candidate constitute the latter's Borda score. The election result under the Borda count is the ranking of candidates in the order of their Borda scores, the lower, the better. In this example the Borda winner is C.

The last two rules require more information from the voters than just their preference rankings. The approval voting, which elects the candidate that has more approvals than any other, calls for the voters to single out those candidates they approve of. Assuming sincere voting strategies, this amounts to requiring that the voters provide a cut-point such that all candidates above the point get one approval vote from the voter, while no candidate below the point gets any approvals from the voter in question. The approval voting gives each voter for each candidate a choice between two options: to approve the candidate or not to approve the candidate. In the profile of Table 1 we make the following - purely *ad hoc* - assumption: the group consisting of three voters approves of their three top-ranked candidates, while the remaining voters approve of only the first-ranked candidate. Under this additional assumption, the approval voting winner is B. Rule (vi), range voting, goes under several names, but the version dealt with here is based on the voters' assigning a score to each candidate. For each candidate the scores given by the voters are summed up and the candidate with the highest score sum is declared the winner. Normally, a range of scores is predetermined, e.g. integers in the $[0, 10]$ interval. Let us assume, for the sake of argument, that the nine left-most voters assign scores to candidates in the same way as in the Borda count, but the two right-most voters assign ten points to their first ranked F and 0 points to the others. Upon summing the scores we find that F is the winner in this scheme.

Each of the six candidates can be the winner by varying the voting rule in the Table 1 profile. In the case of the approval and range voting we have made additional assumptions to make the point. The intended message is that profiles with maximal procedure-dependence can be constructed. And yet, each procedure looks intuitively reasonably democratic. So, given this multiplicity of outcomes ensuing from the same profile, the interpretation of the result as the

[†]For brevity we shall not dwell on cases that involve ties in comparisons. Copeland's rule (Riker, 1982, p. 76) is discussed here as a specimen of Condorcet extensions, which are rules that always elect the Condorcet winner when one exists. The Condorcet winner is a candidate that would defeat all other candidates in pairwise majority comparisons (or in other pairwise contests, such as sports tournaments with unambiguous rules for determining the winners of pairwise comparisons (such as, for instance, the number of goals or point sums etc.).

Table 1. Six candidates, six winners

4 voters	2 voters	3 voters	2 voters
A	B	D	F
E	E	C	C
C	C	B	D
F	F	E	E
D	D	F	B
B	A	A	A

will of the people seems difficult to sustain. At least some serious work on the details of the procedures is called for.

Indeed, such work has been ongoing for more than two centuries. A major change of focus occurred some twenty years before the launching of *Control and Cybernetics* with the publication of Arrow's book, see Arrow (1963). Instead of studying the pros and cons of specific voting rules, the norms or desiderata characterizing the rules as well as their compatibilities, took the central stage. Arrow's theorem is still today the best-known result representing this new genre. Its 1963 version is stated here once more.

Theorem 1 (Arrow 1963) *No social welfare function satisfies the following conditions:*

1. *unrestricted domain (U)*
2. *independence of irrelevant alternatives (IIA)*
3. *Pareto (P)*
4. *non-dictatorship (D) .*

It should be noted that the theorem deals with social welfare functions, i.e. mappings from the set of n-tuples of connected and transitive individual preference relations to the set of (collective) connected and transitive preference relations. In other words, Arrow aimed at a ranking of alternatives that characterizes the collectivity as a whole.

Another famous incompatibility result pertains to social decision functions or, more specifically, resolute (that is, singleton-valued) social choice functions. The latter associate to each set of candidates and a profile of individual preference relations a subset of candidates, viz. the winners. Thus, the social decision functions are rules that always end up with a single winner. To state the theorem one needs two definitions.

Definition 1 *A social choice function is manipulable (by individuals) if and only if there is a situation and an individual such that the latter can bring about a preferable outcome by preference misrepresentation rather than by truthful revelation of his/her preference ranking,* ceteris paribus.

DEFINITION 2 *A social choice function is non-trivial (non-degenerate) if and only if for each candidate x, there is such a preference profile that x is chosen.*

THEOREM 2 (GIBBARD, 1973, AND SATTERTHWAITE, 1975) *Every universal and non-trivial resolute social choice function is either manipulable or dictatorial.*

Another way of saying this is that all non-dictatorial, non-trivial and singleton-valued social choice functions may end up with situations, where the sincere voting strategies do not lead to Nash-equilibria. Thus, it is not necessarily in the voters' best interest to act in accordance with their preference rankings over the candidates.

The third theorem dealt with here pertains to the incentive to participate. Again two definitions are presented.

DEFINITION 3 *A voting rule satisfies participation condition if a voter never loses by joining the electorate and reporting truthfully his/her preference (as opposed to abstaining),* ceteris paribus.

DEFINITION 4 *A rule satisfies Condorcet consistency if it always elects the Condorcet winner when one exists.*

THEOREM 3 (MOULIN, 1988) *If there are more than three candidates and at least 25 voters, no voting rule satisfies both the Condorcet consistency and the participation condition.*[‡]

3. Deliberative institutions and the incompatibility of social choice of desiderata

3.1. Deliberation and domain restrictions

Deliberative democracy is a style of governance whereby the collective decision making is described as a process of interaction between the voters, facilitators and election officers for finding out the collective will on the issues to be decided upon. The process involves individual decision making, information gathering, negotiation, bargaining, construction of decision alternatives and modifying those over time. It is typically applied in small group settings, where face-to-face interactions between voters are made possible and similarly the information sources can be debated.[§] In fact, the specific voting rule possibly applied is in a secondary role to be resorted to as a last resort. The advocates

[‡]Brandt, Geist and Peters (2017) have extended the incompatibility result to cover the situations involving at least four candidates and no less than 12 voters.

[§]In such settings there is a risk that a social dictator or an oligarchy emerges, i.e. an individual or a group of individuals, whose views always determine the outcome of the deliberation.

of deliberative democracy deem their approach superior to representative forms of governance and argue that it can help in solving some thorny problems of the theory of voting, see Dryzek and List (2003).

It is frequent that the verbs 'escape' and 'avoid' are used in this context. In other words, it is argued that the deliberative institutions escape or avoid some of the voting paradoxes and/or incompatibility results, such as those listed above. List et al. (2013) have found empirical evidence suggesting that

> ' ... deliberation *can* robustly protect against majority cycles ... by moving preferences toward single-peakedness.'

Single-peakedness of preferences – it will be recalled – means that there is a degree of unanimity among the voters, viz. about which candidate is not the worst. If this amount of unanimity prevails in the electorate concerning all triplets of candidates, the simple majority pairwise comparisons lead to an outcome that is stable (Black, 1948).¶ This is one sufficient condition for avoiding cyclic majorities. The finding according to which people tend to modify their preferences towards single-peaked profiles is interesting and potentially important for practical purposes. It has, however, nothing to do with escaping Arrow's theorem. The theorem deals with 'given' preferences, while the empirical findings pertain to observed modifications of preferences over time – supposedly as a result of deliberations, negotiations, consultations etc. By the time when the deliberations have terminated, we are given with a set of modified preferences. These may or may not be single-peaked. If they are, the pairwise majority rule leads to non-cyclic majority collective preference relation, If they are not, then the rule may lead to a cyclic collective preference relation. By saying that the cyclic majority relation – the 'voting paradox', as it is sometimes called – is avoided, we are in fact saying that under restricted domains the rule may work well. However, we have violated the unrestricted domain condition. Thus, the validity of Arrow's theorem is not at stake at all. In his book from 1951 Arrow (see Arrow, 1963, pp. 7-8) explicitly states that

> ' ... we will ... assume in the present study that individual values are taken as data and not capable of being altered by the decision process itself ... If individual values can themselves be affected by the method of social choice, it becomes much more difficult to learn what is meant by one method's being preferable to another'.

So, the theorem and the deliberative experiments deal with different settings and thus the latter cannot refute the former. In fact, the theorem is an analytic result immune to empirical testing.‖ It is, however, important to point the specific 'spot' in the proof of the theorem where the unrestricted domain condition kicks in. In Sen's proof (Sen, 1970, pp. 41-46) this condition is invoked in the

¶For a discussion on single-peakedness and its variants, see Dummett and Farquharson (1961) and Ballester and Haeringer (2011)

‖This applies, of course, also to other ways of escaping' Arrow's theorem, for example the one recently introduced by Holliday and Pacuit (2021), which amounts to replacing Arrow's IIA with another similar condition.

Table 2. Non-single-peaked profile with a Condorcet winner

3 voters	1 voter	1 voter
A	B	C
C	C	A
B	A	B

proof of the proposition that given the conditions U, P and IIA, there has to be an individual, who is almost decisive with respect to some ordered pair of alternatives (candidates). In this proof a specific triplet of voter groups is assumed on the grounds that by condition U this is possible. Without this step of invoking the unrestricted domain assumption, the proof of Arrow's theorem would not go through.

The results of List et al. (2013) suggest that deliberative institutions are associated with specific types of domain restrictions, viz. those leading towards single-peaked profiles. Theoretically, single-peaked profiles are a specific type of a wider class, viz. those containing a Condorcet winner. All single-peaked profiles contain a Condorcet winner, while the converse does not hold since there exist profiles that are not single-peaked, but still contain a Condorcet winner. Table 2 illustrates this.

The finding that deliberative institutions are conducive to single-peaked profiles is, of course, encouraging, as it seems to suggest that those institutions lead to complete and transitive preference relations excluding instabilities due to cyclic majorities. Non-transitive collective preferences are, however, but one potential source of instability of voting outcomes. Another such source appears when one considers variable electorates. A word of caution is in order for those readers who read Smith's path-breaking article published nearly 50 years ago (Smith, 1973). Smith consider electorates with changing profiles, but of invariable size, whereas our focus here is on electorates that are formed by adding voter groups to or removing voters from an existing electorate. Thus, in the terminology of this paper, Smith's electorates are fixed (in size) rather than variable.

3.2. Stability in changing electorates

Moulin's theorem applies specifically to electorates that are changing in size. In fact, the theorem uses a thought experiment: given the outcome of the rule in an electorate, consider what would happen if the electorate had been smaller due to the absence of some voters with identical preferences, while the remaining voters had kept their preferences intact. If the outcome after this change had been better for the absentees, then we have an instance of violation of the participation condition. If voting is viewed as the game where each voter has two strategies – to vote or to abstain – then the possibility of violation of participation means that casting a vote may not necessarily lead to a Nash equilibrium in pure strategies.

A couple of specific types of failure of the participation condition have been deemed of special interest (see Fishburn and Brams, 1983; and Woodall 1996):

1. the no-show paradox, which occurs when a group of identically-minded voters, when joining the electorate *ceteris paribus*, changes the outcome from X to Y, where Y is the candidate the group ranks last (lowest),
2. the more-is-less paradox, which occurs when a group of identically-minded voters all ranking X first, by joining the electorate, *ceteris paribus*, changes the outcome from X to some other candidate (which they rank below X).

These are also known as negative strong no show paradox (NegSNSP) and positive strong no show paradox (PosSNSP), respectively (see Perez, 2001, pp. 605-606).

Unlike the instabilities due to majority cycles, deliberative processes apparently provide no escape routes from participation paradoxes. The latter rely on retrospective 'what if' type of reasonings and may cause the voters to regret their decision to vote at all. But what if the voting outcome is a stable one in the sense of singling out the Condorcet winner as the outcome? Would the retrospective thinking still give some voters a reason to regret to have voted? In other words, could the voting outcome that is stable in one sense be instable in another? Yes, it could. Table 3 summarizes the performance of ten Condorcet extension methods – i.e. methods that always elect the Condorcet winner when one exists – in terms of two participation-related criteria: vulnerability to the NegSNSP and vulnerability to the PosSNSP. The analysis focuses on the Condorcet domain, that is, on the class of profiles where there is a Condorcet winner. By definition, the Condorcet extensions end up with the existing Condorcet winner in every profile in this class. So, the stability of the outcome is guaranteed in the traditional sense.

The voting rules included in Table 3 are (see Felsenthal, 2019, pp. 10-13):

- The amendment rule: the candidates are confronted with each other in pairs in accordance with an exogenous agenda, so that for any given pair, the candidate that gets more votes than the contestant proceeds to the contest with the next candidate in the agenda, etc., until all candidates have been present in at least one pairwise comparison. The winner of the last comparison is the overall winner.
- The maximin rule: all pairwise comparisons of all candidates are considered and the candidate, whose minimum support over all contests is the largest is declared the winner.
- Dodgson's rule: given a preference profile, determine the minimum number of pairwise switches of adjacent candidates in individual voters' preference relations required for making a candidate the Condorcet winner and elect the candidate, for which this number is the smallest.

Table 3. Ten Condorcet extensions in Condorcet domains

procedure	vulnerability to NegSNSP	vulnerability to PosSNSP
amendment	yes	no
maximin	no	no
Dodgson	yes	no
Nanson	yes	no
Baldwin	yes	no
Copeland	yes	no
Black	yes	no
Kemeny	yes	no
Schwartz	yes	no
Young	no	no

- Nanson's rule: compute the Borda scores for each candidate and eliminate all those candidates with the average or smaller Borda score. Compute the new Borda scores for the remaining candidates. Continue until one candidate remains. This is the winner.
- Baldwin's rule: the same as Nanson's, but only the candidate with the smallest Borda score is eliminated at each stage.
- Copeland's rule: consider all pairs of candidates and determine the winner in each one of them. Tally the number of wins for every candidate and elect the one with the largest number of wins.
- Black's rule: elect the Condorcet winner if one exists, otherwise elect the Borda winner.
- Kemeny's (median) rule: for k candidates generate all $k!$ strict rankings. For each one of them, tally the minimum number of individual preference switches between adjacent candidates required to make the given ranking unanimously accepted. The ranking with the smallest tally is the collective ranking and its top positioned candidate the winner.
- Schwartz's rule: determine the smallest set of candidates such that no candidate outside the set defeats in pairwise contests any of the candidates inside the set. This smallest set consists of the winners.
- Young's rule: for any given candidate define a score that equals the minimum number of voters whose preferences have to be ignored in order to make this candidate the Condorcet winner. The candidate with the smallest score is the winner.

We illustrate the NegSNSP in the case of Black's rule using Table 4 (see Felsenthal and Nurmi, 2017, pp. 65, 75). Here, Black's rule yields B (the strong Condorcet winner) as the winner. Suppose now that three voters with ADBC ranking join the electorate. In the augmented electorate there is no Condorcet winner and thus the Borda count kicks in. The Black winner is now C, the lowest

Table 4. NegSNSP under Black's rule

5 voters	4 voters
B	C
C	D
D	A
A	B

ranked candidate of the three added voters. Hence, the situation depicted by Table 4 provides incentives for a sizable part of the augmented electorate to abstain.

Table 3 shows that only two out of the ten Condorcet extensions is invulnerable to the two types of participation paradoxes in Condorcet domains. In other words, the participation instability, suggested by Moulin's theorem, persists even in the domain that is intuitively most favorable to Condorcet extensions. On closer inspection, the table also shows that the participation instability is of the NegSNSP variety, while no rule is vulnerable to the PosSNSP variety in the Condorcet domain. This is pretty obvious, since if x is the winner in a profile by virtue of being the Condorcet winner, it remains the Condorcet winner after a group of identically minded voters who all rank x first joins the electorate.

The relevance of Moulin's theorem to deliberative institutions is indirect and pertains to the notion that those institutions improve the possibilities of stable outcomes. This may be so, but the underlying concept of stability is quite narrow in focusing only on the avoidance of majority cycles. Moulin's theorem points to the tension between Condorcet-inspired concept of stability and the participation-inspired stability. Hence, the strive for the former may *eo ipso* undermine the latter.

4. Deliberation and preference misrepresentation

The escape from the Gibbard-Satterthwaite theorem suggested by List and Dryzek (Dryzek and List, 2003) is basically similar to the way deliberative institutions are allegedly escaping Arrows theorem, viz. arguing that one of the conditions of the incompatibility theorem is unlikely to hold in the deliberative process. The condition in question here is manipulability. The authors argue that the deliberative process has the tendency to reveal the voters' truthful preference information in the course of debates prior to voting. Should the voters then deviate from the information they have given, they would incur costs in terms of the loss of trust and reliability. Indeed, the 'gross' benefits of preference misrepresentation would likely be diminished by these indirect costs of 'nudging' the voter behavior towards truthful representation of preferences. Similarly as in the case of Arrow's theorem, the deliberative context does not invalidate the theorem, but amounts to stating that one of the conditions shown to be incompatible in the theorem does not hold in the deliberative processes.

Whether the deliberative institutions in fact make all deviations from true preferences disadvantageous for the voter is in the end an empirical question, but to the present writer the setting assumed hereby seems nearly utopian: not only is it assumed that the discussion is substantive, balanced and civil, as stated above, but the participants exert no pressure each other, observe each other's preferences *sine ira et studio* (objectively), are willing to adjust their opinions in the course of the discussion, and are in general other-regarding (see, e.g., Mercier and Landemore, 2012, and Rasch, 2014). In other words, the deliberative setting seems to involve basically nice, considerate, polite and reasonable voters. This is a well-nigh utopian setting. It is of course at best an idealistic approximation of how actual collective decisions are made. This obvious point should not be exaggerated, though. The more relevant counterargument to the central point of the deliberative theorists is that the gradual revelation of preferences may provide *incentives for misrepresentation* under voting rules, which, in the absence of this kind of information, would be hard to manipulate. For example, the plurality runoff rule may not provide much incentive to misrepresent one's preferences if nothing is known about the other voters' opinions, but once the preference information becomes available, incentives to deviate from sincere voting increase for some voter groups (especially those having no chance of having their first ranked candidates elected). So, the preference revelation in the course of deliberation may in fact prompt, rather than discourage, the voters to preference misrepresentation. This depends, however, on the voting rule in use (see Nurmi 1987, p. 124, or Bartholdi and Orkin, 1991). Moreover, the information on other voters' preferences may actually facilitate a risk-averse voter's misrepresentation of preferences by indicating safe strategies, i.e. those that are unlikely to backfire in the sense of leading to outcomes that are worse than those ensuing from truthful representation (Slinko and White, 2008).

5. Concluding remarks

The relationship between social choice theory and deliberative democracy can best be characterized as complementary. The foci of the two approaches are different: the social choice theory studies properties of choice functions or correspondences in fixed settings, where a set of voters is considering a fixed set of candidates or alternatives, whereas the focus of deliberative democracy is on how preference profiles change spontaneously or as a result of external stimuli. The theorems discussed above express incompatibilities among several choice desiderata. They are conceptual, not empirical, truths. To establish an incompatibility between social choice properties, one needs to establish that no conceivable preference profile satisfies all the desiderata included in the theorem. To refute such a theorem all one needs is to come up with a theoretical, constructed or empirically observed example of a profile exhibiting all the desiderata that the theorem says are incompatible. Hence, the escape stratagems suggested in the deliberative democracy literature cannot refute the theorems since the latter are analytic results.

Deliberative institutions have, however, an important role in complementing social choice theorems by introducing ways, in which profiles can be and have been modified towards such opinion distributions that satisfy as many choice desiderata as possible. Such modifications, of course, constitute domain restrictions on the incompatibility theorems and as such do not refute the incompatibility theorems, but can provide important information regarding the contexts in which voting rules are likely to work reasonably well. A particularly important – in fact indispensable – role of the deliberative institutions is in the formation of the candidate or alternative set, which is typically assumed as 'given' in social choice theory. Reasonable collective decisions cannot be achieved unless the set of decision alternatives include reasonable alternatives, no matter how many social choice desiderata the voting rule satisfies.

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