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Inventory models with multiple production and remanufacturing batches under shortages*

by

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Abstract: Owing to the ecological requirements and regulations, an enormous concern is being paid to the product re-processing. In the established literature, researchers considered that the remanufactured items are as good as the new ones. Yet, such an assumption is not convenient, as in many real situations the recycled products are considered by the customers to be of secondary quality. Further, the classical studies mainly addressed the inventory models without shortages, and this is not applicable in many practical business situations. This paper extends the reverse logistics inventory models with finite production and remanufacturing rate along with the assumption that newly produced and repaired (remanufactured) objects are not of same characteristics. Shortages are allowed and numerous stock-out cases are discussed. The collected used items are remanufactured (repaired) and non-repaired products are disposed off. The proposed models are illustrated with some numerical examples and their results are discussed.

Keywords: inventory models, production, remanufacturing, shortages

1. Introduction

Inventory management in reverse logistics, which incorporates joint manufacturing and remanufacturing options, has been receiving increasing attention in recent years. However, fast developments in technology and mass appearance of new industrial products, which are coming to the market, have resulted in an increasing number of idle products and caused growing environmental problems worldwide. Therefore, increasing ecological concerns, end user awareness, economic considerations, and legislation, related to waste disposal, encourage manufacturers to take back products after customer have used them. Recently, growing interest and realisations in the reverse logistics processes, such as the

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recovery of the returned products, have become one of the ways, in which businesses endeavor to retain and increase competitiveness in the global market.

Schrady (1967) first explored a deterministic reverse logistic Economic Order Quantity (EOQ) model for repairable items with multiple repair cycles and one production cycle. The model of Schrady (1967) was extended by Nahmias and Rivera (1979) with inclusion of the case of finite repair rate. Richter (1996a, b) proposed an EOQ model with waste disposal and looked over the optimal figure of production and remanufacturing batches, depending on the rate of return. Richter (1997), Richter and Dobos (1999) investigated whether a policy of either total waste disposal or no waste disposal is optimal. Teunter (2001) considered multiple production and remanufacturing cycles and generalized the results from Schrady (1967). Dobos and Richter (2003) developed a production/recycling setup with constant demand that is satisfied by non-instantaneous production and recycling with a single repair and a single production batch in an interval of time. Later on, Dobos and Richter (2004) generalized their earlier model (Dobos and Richter, 2003) by considering multiple refurbish/repair and production batches in a time interval. Along the same line of study, Dobos and Richter (2006) further extended the model and assumed that the quality of collected used/returned items is not always suitable for further recycling. Later on, Jaber and El Saadany (2009) extended the work of Richter (1996a, b) by assuming that the remanufactured items are considered by the customers to be of lower quality than the new ones. Alamri (2011) put forward a general reverse logistics inventory model for deteriorating items by considering the acceptable returned quantity as a decision variable. Singh and Saxena (2012) proposed a reverse logistics inventory model allowing for back-orders. Hasanov et al. (2012) extended the work of Jaber and El Saadany (2009) by assuming that unfulfilled demand of remanufactured and produced items is either fully or partially back-ordered. Singh et al. (2012) developed an economic production lot-size (EPLS) model with rework and flexibility under allowable shortages. Singh and Sharma (2013a) developed a global optimizing policy for decaying items with ramp-type demand rate under two-level trade credit financing, taking into account a preservation technology. El Saadany et al. (2013) discussed an inventory model with the question as to how many times a product can be remanufactured. Singh and Sharma (2013b) explored an integrated model with variable production and demand rates under inflation. Later, Singh and Sharma (2014) proposed an optimal trade-credit policy for perishable items, assuming imperfect production and stock dependent demand. Recently, Singh and Sharma (2016) established a production reliability model for deteriorating products with random demand and inflation, and Bazan et al. (2016) presented a comprehensive review of mathematical inventory models for reverse logistics.

In the existing literature, most of the research articles are developed with the assumption that the produced and recovered items are not of different quality. In many practical business situations this hypothesis is not adequate, as the repaired (remanufactured) items are considered of secondary quality by the customers. In addition, infinite or instantaneous production and remanufacturing rates are assumed in many previous reverse logistic inventory models. Therefore, in this study, reverse logistics models with finite production, remanufacturing and several stock-out cases are developed. It is assumed that newly produced and remanufactured items are different in quality. This paper is an extension of the work of Hasanov et al. (2012) for the case of finite production and remanufacturing. We have also considered disposal cost for the disposed items. Numerical experiments and sensitivity analysis are provided to illustrate the proposed models. The behaviors of the total average cost functions for different stock-out cases are shown with respective graphs.

2. Assumptions and notations

In this section, assumptions and notations used in the proposed model are given. These assumptions and notations are based on Dobos and Richter (2004) and Jaber and El Saadany (2009).

2.1. Assumptions

1. Finite production and remanufacturing rates.
2. Remanufactured items are not as good as new.
3. Demands for produced and remanufactured items are known, constant but different.
4. Lead time is zero and unlimited storage capacity is available.
5. Constant but different collection rates for previously used manufactured and remanufactured items.
6. A single product case.
7. Infinite planning horizon.

2.2. Notations

Decision variables:

m	Number of remanufacturing batches
n	Number of production batches
γ_r	Collection percentage of available returns of previously remanufactured items ($0 \leqslant \gamma_r \leqslant 1$)
γ_p	Collection percentage of available returns of newly produced items ($0 < \gamma_p \leqslant 1$)
θ_r	Proportion of maximum inventory in a cycle of used/repaired items consumed in the remanufacturing segment of $T(0 \leqslant \theta_r \leqslant 1)$
θ_p	Proportion of maximum inventory in a cycle of newly produced items consumed in the production segment of $T(0 \leqslant \theta_p \leqslant 1)$

Input parameters:

D_p	Demand rate for newly produced items (units/ unit of time)
D_r	Demand rate for remanufactured items (units/ unit of time), where D_r is not necessarily equal to D_p

D_p/η Production rate ($0 < \eta < 1$)
D_r/δ Remanufacturing rate ($0 < \delta < 1$)
S_p Setup cost for a production cycle (\$)
S_r Setup cost for a remanufacturing cycle (\$)
h_p Holding cost per unit per unit of time of a produced item (\$/unit/unit of time)
h_r Holding cost per unit per unit of time of a remanufactured item (\$/unit/unit of time)
h_u Holding cost per unit per unit of time of a used item (\$/unit/unit of time)
c_p Per unit production cost (\$)
c_r Per unit remanufacturing cost (\$)
c_w Per unit disposal cost (\$)
β_p Percentage of available returns from the primary market for produced items
β_r Percentage of available returns from the secondary market for remanufactured items ($0 \leqslant \beta_r \leqslant \beta_p \leqslant 1$), where $(1 - \beta_r)$ and $(1 - \beta_p)$ are the waste disposal rates
l_r Lost sale cost for a remanufactured item (\$/unit)
l_p Lost sale cost for a produced item (\$/unit)
b_r Backorder cost for a remanufactured item (\$/unit/unit of time)
b_p Backorder cost for a produced item (\$/unit/unit of time)
v Proportion of D_p that is backordered ($0 < v < 1$), and (1-v) is the proportion of D_p that is lost
s Proportion of D_r that is backordered ($0 < s < 1$), and (1-s) is the proportion of D_r that is lost

Decision variable dependent parameters:

T Cycle length
T_R One remanufacturing cycle length
T_P One production cycle length
T_R^P Length of the period, for which the inventory of produced items is positive during the remanufacturing process
T_P^R Length of the period, for which the inventory of remanufactured items is positive during the production process
t_r Length of an incomplete segment of a remanufacturing cycle during the remanufacturing process
t_p Length of an incomplete segment of a production cycle during the production process
T_1 The period, in which remanufacturing starts and shortages for remanufactured (or repaired) items (which occurred during production process) are backordered and demand for remanufactured items, which occurs during this period, is also satisfied

T_2 The period in which production starts and shortages for newly produced items (which occurred during remanufacturing process) are backordered and demand for new items which occurs during this period is also fulfilled.

3. Mathematical formulation and solution

The production, remanufacturing and waste disposal model, described in Hasanov et al. (2012), is depicted in Fig. 1. Similarly as in the paper by Richter (1996b), there are two shops in the system. In the first shop (serviceable stock), newly produced and remanufactured (repaired) items are accumulated, while in the second shop (repairable stock), returned/used items are stored. Collected used items are screened, and those considered to be non-repairable are disposed off. In this model, it is assumed that the newly produced items are sold on the primary market, while, on the other hand, the remanufactured items are sold on the secondary market at a reduced price. There are multiple remanufacturing and production cycles in an interval of length T.

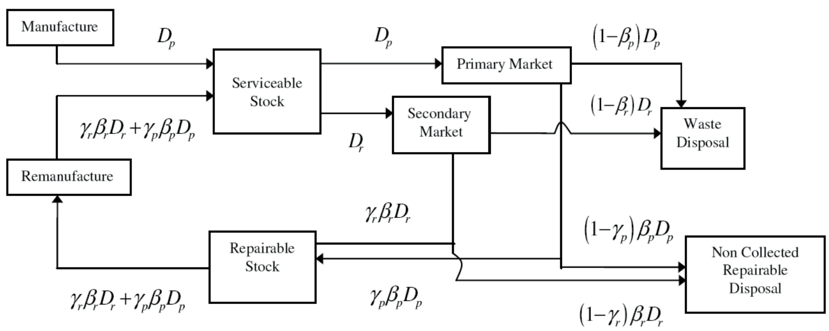

Figure 1. Material flow for a production and remanufacturing system

3.1. Scenario 1

3.1.1.Case 1: Partial backordering

The Case 1 of Scenario 1, where unfulfilled demands of remanufactured and produced items are partially backordered, is illustrated in Fig. 2. In this case, some sales are considered to be lost, as all the customers will not wait for the next batch, when the shortages can be backordered. Specifically, a percentage (s) of demand for remanufactured items is backordered during the remanufacturing period T_1 and a percentage of demand (1-s) is lost. Quite analogously, a percentage (v) of the demand for newly produced items is backordered during the period T_2, while a percentage $(1-v)$ for new items is lost.

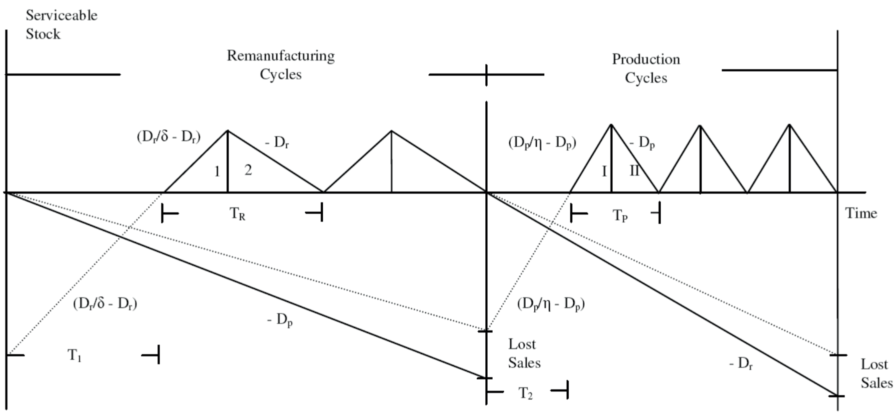

Figure 2. The inventory status for the system with partial backordering

There are multiple remanufacturing and production batches in an interval of length T, so we have

$$mT_R + T_1 + nT_P + T_2 = T. \tag{1}$$

Now, the remanufacturing quantity is

$$D_r\left(mT_R + T_1\right) + s\left(nT_P + T_2\right)D_r = \left(mT_R + T_1\right)\gamma_r\beta_r D_r + \left(nT_P + T_2\right)\gamma_p\beta_p D_p$$

$$\Rightarrow \left(1 - \gamma_r\beta_r\right)D_r\left(mT_R + T_1\right) = \left(nT_P + T_2\right)\left(\gamma_p\beta_p D_p - sD_r\right). \tag{2}$$

Since the shortages are partially backordered, during T_1 and T_2, so:

For the time period T_1, we have

$$\frac{D_r}{\delta}T_1 = sD_r\left(nT_P + T_2\right) + D_r T_1$$

$$\Rightarrow \frac{\left(1-\delta\right)T_1}{s\delta} = \left(nT_P + T_2\right). \tag{3}$$

Similarly, for the period T_2, we have

$$\frac{D_p}{\eta}T_2 = vD_p\left(mT_R + T_1\right) + D_p T_2$$

$$\Rightarrow \frac{\left(1-\eta\right)T_1}{v\eta} = \left(mT_R + T_1\right). \tag{4}$$

From equations (2), (3) and (4) we get

$$T_1 = \frac{s\delta\left(1-\eta\right)\left(1-\gamma_r\beta_r\right)D_r T_2}{v\eta\left(1-\delta\right)\left(\gamma_p\beta_p D_p - sD_r\right)}. \tag{5}$$

Now, from equations (1), (3) and (4), we have

$$\frac{(1-\eta)\,T_2}{v\eta}+\frac{(1-\delta)\,T_1}{s\delta}=T.$$

By putting the value of T_1 from equation (5), we get

$$T_2=\frac{v\eta\left(\gamma_p\beta_pD_p-sD_r\right)T}{\left(g-sD_r\right)(1-\eta)},\text{ where } g=\left(D_r+\gamma_p\beta_pD_p-\gamma_r\beta_rD_r\right). \tag{6}$$

Hence, from equations (5) and (6), we have

$$T_1=\frac{s\delta\left(1-\gamma_r\beta_r\right)D_rT}{(1-\delta)\left(g-sD_r\right)}. \tag{7}$$

By simplifying equations (4), (6) and (7), we obtain

$$T_R=\frac{\left[\alpha-(1-\delta)\,sD_r\right]T}{m\,(1-\delta)\left(g-sD_r\right)},\text{ where } \alpha=\left[(1-\delta)\,\gamma_p\beta_pD_p-\left(1-\gamma_r\beta_r\right)s\delta D_r\right]. \tag{8}$$

Again, from equations (3), (6) and (7), we get

$$T_P=\frac{T\left(\xi+vs\eta D_r\right)}{n\,(1-\eta)\left(g-sD_r\right)},\text{ where } \xi=\left[(1-\eta)\left(1-\gamma_r\beta_r\right)D_r-v\eta\gamma_p\beta_pD_p\right]. \tag{9}$$

The inventory holding cost expressions for the newly produced, remanufactured and returned items are given, respectively, as

$$H_P=\frac{nh_p\,(1-\eta)\,D_pT_P^2}{2}=\frac{h_p\left(\xi+vs\eta D_r\right)^2D_pT^2}{2n\,(1-\eta)\left(g-sD_r\right)^2} \tag{10}$$

$$H_R=\frac{mh_r\,(1-\delta)\,D_rT_R^2}{2}=\frac{h_r\left[\alpha-(1-\delta)\,sD_r\right]^2D_rT^2}{2m\,(1-\delta)\left(g-sD_r\right)^2} \tag{11}$$

$$\begin{aligned}H_r=h_u\Bigg[&\frac{mD_rT_R^2}{2}\left\{\delta+\gamma_r\beta_r-2\delta\gamma_r\beta_r+(m-1)\ \left(1-\gamma_r\beta_r\right)\right\}+\frac{\gamma_p\beta_pD_pT_2^2}{2}\\&+\frac{\left(1-\delta\gamma_r\beta_r\right)D_rT_1^2}{2\delta}+\gamma_r\beta_rD_r\,(1-\delta)\,T_RT_2+(m-1)\left(1-\gamma_r\beta_r\right)\times\\&D_rT_RT_1\ +\left(1-\delta\gamma_r\beta_r\right)D_rT_RT_1+\frac{\gamma_p\beta_pD_pn^2T_P^2}{2}+\\&\left\{\gamma_p\beta_pD_pT_2+\gamma_r\beta_rD_r\,(1-\delta)\,T_R\right\}nT_P\Bigg]\end{aligned}$$

$$\Rightarrow H_r =$$

$$\frac{h_u T^2}{2\left(g-sD_r\right)^2}\left[\frac{D_r\left\{\alpha-(1-\delta)\,sD_r\right\}^2}{m\,(1-\delta)^2}\left\{\delta+\gamma_r\beta_r\,(1-2\delta)+(m-1)\,(1-\gamma_r\beta_r)\right\}\right.$$
$$+\frac{\gamma_p\beta_p D_p v^2\eta^2\left(\gamma_p\beta_p D_p-sD_r\right)^2}{(1-\eta)^2}+\frac{s^2\delta D_r^3\,(1-\delta\gamma_r\beta_r)\,(1-\gamma_r\beta_r)^2}{(1-\delta)^2}$$
$$+\frac{2v\eta\gamma_r\beta_r D_r\left\{\alpha-(1-\delta)\,sD_r\right\}\left(\gamma_p\beta_p D_p-sD_r\right)}{m\,(1-\eta)}+\frac{\gamma_p\beta_p D_p v^2\eta^2\left(\xi+vs\eta D_r\right)^2}{(1-\eta)^2}$$
$$+\frac{2s\delta D_r^2\,(1-\gamma_r\beta_r)\left\{\alpha-(1-\delta)\,sD_r\right\}\left\{(m-1)\,(1-\gamma_r\beta_r)+(1-\delta\gamma_r\beta_r)\right\}}{m\,(1-\delta)^2}$$
$$\left.+\left\{\frac{2\gamma_p\beta_p D_p v\eta\left(\gamma_p\beta_p D_p-sD_r\right)}{(1-\eta)}+\frac{2\gamma_r\beta_r D_r\left\{\alpha-(1-\delta)\,sD_r\right\}}{m}\right\}\frac{\left(\xi+vs\eta D_r\right)}{(1-\eta)}\right]. \tag{12}$$

See Appendices 1 and 2 for the derivations of the holding costs expressions. The total holding cost per unit of time is

$$H_T=\frac{H_P+H_R+H_r}{T}$$

$$\text{or } H_T=T\psi\left(m,n,\gamma_r,\gamma_p\right), \tag{13}$$

where

$$\psi\left(m,n,\gamma_r,\gamma_p\right)=\frac{H_P+H_R+H_r}{T^2}. \tag{14}$$

The set up cost per unit time is

$$S_c=\frac{\left(mS_r+nS_p\right)}{T}. \tag{15}$$

The disposal cost per unit of time is

$$D_c=\frac{c_w}{T}\left[\left(D_p-\gamma_p\beta_p D_p\right)\left(nT_P+T_2\right)+\left(D_r-\gamma_r\beta_r D_r\right)\left(mT_R+T_1\right)\right].$$

By introducing the values of T_2, T_1, T_R and T_p from equations (6)-(9), respectively, and then solving, we get

$$D_c=\frac{c_w\left(1-\gamma_r\beta_r\right)D_r\left(D_p-sD_r\right)}{\left(g-sD_r\right)}. \tag{16}$$

The remanufacturing cost per unit of time (including the purchasing cost of used item) is

$$R_c=\frac{c_r}{T}\left[\frac{D_r}{\delta}T_1+\frac{D_r}{\delta}\left(\delta T_R\right)m\right].$$

By introducing the values of T_1 and T_R from equations (7)-(8), respectively, and then solving, we get

$$R_c = \frac{c_r D_r \left[\alpha + (\delta - \gamma_r \beta_r) s D_r\right]}{(1-\delta)(g - sD_r)} \tag{17}$$

The production cost per unit of time is

$$P_c = \frac{c_p}{T}\left[\frac{D_p}{\eta} T_2 + \frac{D_p}{\eta}(\eta T_P) n\right].$$

By putting the values of T_2 and T_P from equations (6) and (9), respectively, and then solving, we obtain

$$P_c = \frac{c_p D_p \left[\xi + vs\eta D_r + v(\gamma_p \beta_p D_p - sD_r)\right]}{(1-\eta)(g - sD_r)}. \tag{18}$$

The total backordering cost per unit of time for newly produced items is

$$BC_p = \frac{b_p}{T}\left[\frac{vD_p}{2}(mT_R + T_1)(mT_R + T_1) + \frac{vD_p}{2}(mT_R + T_1) T_2\right].$$

After solving, we get

$$BC_p = \frac{b_p v D_p (1 - \eta + v\eta)(\gamma_p \beta_p D_p - sD_r)^2 T}{2(1-\eta)(g - sD_r)^2}. \tag{19}$$

The total backordering cost per unit of time for remanufactured items is

$$BC_r = \frac{b_r}{T}\left[\frac{sD_r}{2}(nT_P + T_2)(nT_P + T_2) + \frac{sD_r}{2}(nT_P + T_2) T_1\right].$$

After solving, we obtain

$$BC_r = \frac{b_r (1 - \delta + s\delta)(1 - \gamma_r \beta_r)^2 s D_r^3 T}{2(1-\delta)(g - sD_r)^2}. \tag{20}$$

The total backordering cost per unit of time is

$$BC_{PR} = BC_{pr}(\gamma_r, \gamma_p) T \tag{21}$$

where

$$BC_{pr}(\gamma_r, \gamma_p) = \left[\frac{b_p v D_p (1 - \eta + v\eta)(\gamma_p \beta_p D_p - sD_r)^2}{2(1-\eta)(g - sD_r)^2} + \frac{b_r (1 - \delta + s\delta)(1 - \gamma_r \beta_r)^2 s D_r^3}{2(1-\delta)(g - sD_r)^2}\right]. \tag{22}$$

The lost sales cost per unit of time for newly produced items is

$$LC_p = \frac{l_p}{T}\left[(1-v)\, D_p\,(mT_R + T_1)\right] = \frac{l_p\,(1-v)\, D_p\,(\gamma_p\beta_p D_p - sD_r)}{(g - sD_r)}. \tag{23}$$

The lost sales cost per unit of time for remanufactured (repaired) items is

$$LC_r = \frac{l_r}{T}\left[(1-s)\, D_r\,(nT_P + T_2)\right] = \frac{l_r\,(1-s)\, D_p\,(1-\gamma_r\beta_r)\, D_r^2}{(g - sD_r)}. \tag{24}$$

Therefore, the total cost per unit of time for the inventory system is

$$\begin{aligned} C\,(m,n,\gamma_r,\gamma_p,T) = &\left[\frac{(mS_r + nS_p)}{T} + \psi T + BC_{pr}T + \frac{c_w\,(1-\gamma_r\beta_r)\, D_r}{(g - sD_r)}\times\right.\\ &(D_p - sD_r) + \frac{c_r D_r\left[\alpha + (\delta - \gamma_r\beta_r)\, sD_r\right]}{(1-\delta)\,(g - sD_r)}\\ &+\frac{c_p D_p}{(1-\eta)\,(g - sD_r)}\Big(\xi + vs\eta D_r + v\,(\gamma_p\beta_p D_p - sD_r)\Big)\\ &\left.+\frac{l_p\,(1-v)\, D_p\,(\gamma_p\beta_p D_p - sD_r)}{(g - sD_r)} + \frac{l_r\,(1-s)\, D_p\,(1-\gamma_r\beta_r)\, D_r^2}{(g - sD_r)}\right] \end{aligned} \tag{25}$$

where $\psi(m,\, n,\, \gamma_r,\, \gamma_p)$ and $BC_{pr}(\gamma_r,\, \gamma_p)$ are given by equations (14) and (22), respectively.

Now, by putting the first order partial derivative of equation (25) equal to zero and solving for T, we get

$$T = \sqrt{\frac{(mS_r + nS_p)}{\psi\,(m,n,\gamma_r,\gamma_p) + BC_{pr}\,(\gamma_r,\gamma_p)}}. \tag{26}$$

Putting the value of T from equation (25) into (26), we obtain

$$\begin{aligned} C\,(m,n,\gamma_r,\gamma_p) = \frac{1}{(g - sD_r)} &\left[2\sqrt{(mS_r + nS_p)\,(\psi + BC_{pr})} + c_w\,(1-\gamma_r\beta_r)\times\right.\\ &D_r\,(D_p - sD_r) + \frac{c_r D_r\left[\alpha + (\delta - \gamma_r\beta_r)\, sD_r\right]}{(1-\delta)} + \frac{c_p D_p}{(1-\eta)}\times\\ &\left[\xi + vs\eta D_r + v\,(\gamma_p\beta_p D_p - sD_r)\right] + l_p\,(1-v)\, D_p\times\\ &\left.(\gamma_p\beta_p D_p - sD_r) + l_r\,(1-s)\, D_p\,(1-\gamma_r\beta_r)\, D_r^2\right] \end{aligned} \tag{27}$$

3.1.2. Case 2: Full backordering

The Case 2 of Scenario 1, in which unfulfilled demands for remanufactured and produced items are fully backordered, is illustrated in Fig. 3. During the remanufacturing period T_1, the shortages for remanufactured items are backordered, while shortages for new items are backordered during the production period T_2.

See Appendix 3 for the expression of total cost per unit of time.

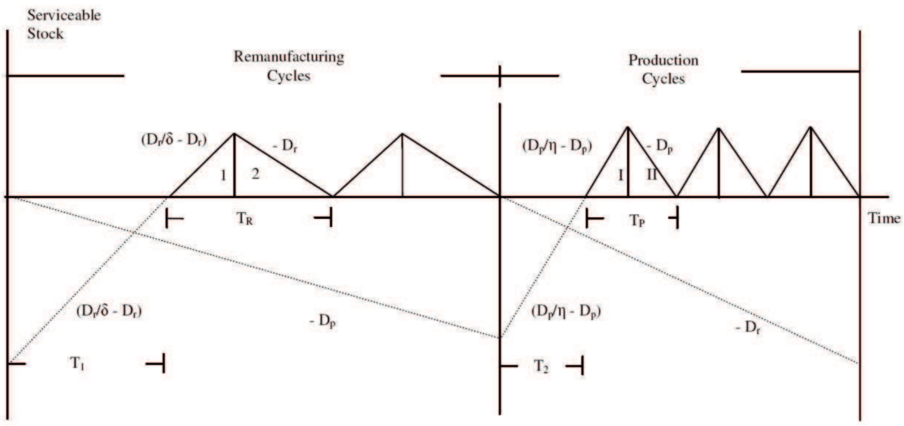

Figure 3. The inventory status for the system with pure backordering

3.2. Scenario 2

During the Scenario of overlapping, there are multiple remanufacturing and production cycls in the time period of lenght T. Consequently, we have

$$[(m-1)\,T_R + \delta T_R + t_r + T_1] + [(n-1)\,T_P + \eta T_R + t_p + T_2] = T. \tag{28}$$

Also, we have

$$t_p = \frac{\theta_p}{D_p}\left(\frac{D_p}{\eta} - D_p\right)\eta T_P$$

$$\Rightarrow t_p = \theta_p\,(1-\eta)\,T_P. \tag{29}$$

and

$$t_r = \frac{\theta_r}{D_r}\left(\frac{D_r}{\delta} - D_r\right)\delta T_R$$

$$\Rightarrow t_r = \theta_r\,(1-\delta)\,T_R. \tag{30}$$

Since

$$t_p + T_R^P = (1-\eta)\,T_P,$$

so, by putting the value of t_p from equation (29), and then solving, we get

$$T_R^P = (1-\theta_p)\,(1-\eta)\,T_P. \tag{31}$$

Similarly,

$$t_r + T_P^R = (1-\delta)\,T_R.$$

By putting the value of t_r from equation (30), and then solving, we obtain

$$T_P^R = (1-\theta_r)(1-\delta)\, T_R. \tag{32}$$

Now, the remanufacturing quantity is

$$D_r(mT_R + T_1) = (mT_R + T_1)\gamma_r\beta_r D_r + (nT_P + T_2)\gamma_p\beta_p D_p$$

$$\Rightarrow (1-\gamma_r\beta_r) D_r (mT_R + T_1) = (nT_P + T_2)\gamma_p\beta_p D_p. \tag{33}$$

3.2.1. Case 1: Overlapping and Partial backordering

The Case 1 of Scenario 2 is the one, where the fraction of the last remanufacturing run partially extends beyond the respective cycle interval, with the starting production segment of the same cycle, and the last production run of the current (previous) cycle going beyond relative to the starting remanufacturing segment of the next (current) cycle. In this case, unfulfilled demands of remanufactured and newly produced items are partially backordered, as illustrated in Fig. 4. The unmet demand is lost at a cost. Note, that no more than one (production or remanufacturing) portion is allowed as the remenufacturing and production are performed in sequence on the same facility.

Here, the remanufacturing quantity is

$$\begin{aligned}(mT_R + T_1)\gamma_r\beta_r D_r + (nT_P + T_2)\gamma_p\beta_p D_p \\ = D_r(mT_R + T_1) + \left[(n-1)T_P + \eta T_P + t_p + T_2 - T_P^R\right] sD_r.\end{aligned} \tag{34}$$

Since, during the period T_2, the shortages (for new items), which have occurred during the remanufacturing process, are partially backordered, so, we have

$$\left[(m-1)T_R + \delta T_R + t_r + T_1 - T_R^P\right] vD_p = \frac{D_pT_2}{\eta} - D_pT_2$$

$$\Rightarrow [(m-1) + \delta + \theta_r(1-\delta)]\, vT_R - v(1-\theta_p)(1-\eta)T_P + vT_1 - \frac{(1-\eta)T_2}{\eta} = 0. \tag{35}$$

Similarly, for the time period T_1, we have

$$\left[(n-1)T_P + \eta T_P + t_p + T_2 - T_P^R\right] sD_r = \frac{D_rT_1}{\delta} - D_rT_1$$

$$\Rightarrow [(n-1) + \eta + \theta_p(1-\eta)]\, sT_P - s(1-\theta_r)(1-\delta)T_R + sT_2 - \frac{(1-\delta)T_1}{\delta} = 0. \tag{36}$$

By solving equations (35) and (36), we get

$$T_1 = \frac{s\delta}{[(1-\eta)(1-\delta) - vs\eta\delta]} \left[\{mv\eta + (1-\delta)(\theta_r - 1)(1-\eta+v\eta)\} T_R + (1-\eta)\{n + (\theta_p - 1)(1-\eta+v\eta)\} T_P\right] \quad (37)$$

and

$$T_2 = \frac{v\eta}{[(1-\eta)(1-\delta) - vs\eta\delta]} \left[(1-\delta)\{m + (\theta_r - 1)(1-\delta+s\delta)\} T_R + \{ns\delta + (1-\eta)(\theta_p - 1)(1-\delta+s\delta)\} T_P\right] \quad (38)$$

From equations (34), (37) and (38), we find

$$T_R = \frac{L}{M} T_P. \quad (39)$$

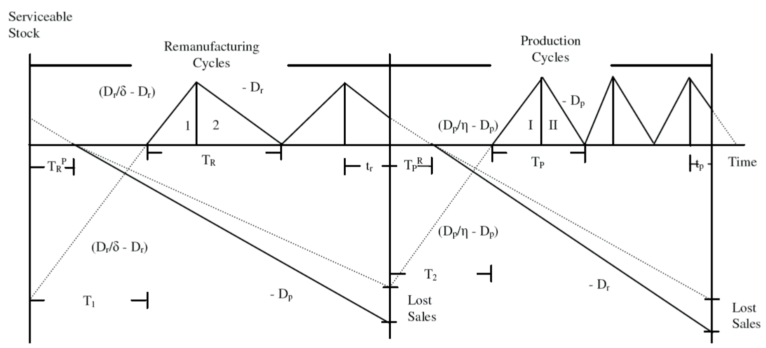

Figure 4. The inventory status for the system with overlapping and partial backordering

In equation (39):

$$\begin{aligned} L = [&v\eta(\gamma_p\beta_p D_p - sD_r)\{ns\delta + (1-\eta)(\theta_p - 1)(1-\delta+s\delta)\} \\ &- s\delta(1-\gamma_r\beta_r) D_r (1-\eta)\{n + (\theta_p - 1)(1-\eta+v\eta)\} \\ &- sD_r\{(1-\eta)(1-\delta) - vs\eta\delta\}\{(n-1) + \eta + \theta_p(1-\eta) - n\gamma_p\beta_p D_p\}] \end{aligned} \quad (40)$$

and

$$\begin{aligned} lM = [&D_r\{m(1-\gamma_r\beta_r) + s(\theta_r - 1)(1-\delta)\}\{(1-\eta)(1-\delta) - vs\eta\delta\} \\ &+ s\delta(1-\gamma_r\beta_r) D_r\{mv\eta + (1-\delta)(\theta_r - 1)(1-\eta+v\eta)\} \\ &- sD_r\{(1-\eta)(1-\delta) - vs\eta\delta\}\{(n-1) + \eta + \theta_p(1-\eta) - n\gamma_p\beta_p D_p\}] \end{aligned} \quad (41)$$

By solving equations (28), (35), (36), (37) and (38) we get

$$\frac{(1-\delta) QT_R}{[(1-\eta)(1-\delta) - vs\eta\delta]} + \frac{(1-\eta) RT_P}{[(1-\eta)(1-\delta) - vs\eta\delta]} = T \quad (42)$$

where

$$Q=[(1-\eta)\{m+(\theta_r-1)(1-\delta+s\delta)\}+\{mv\eta+(1-\delta)(\theta_r-1)(1-\eta+v\eta)\} \\ -\{(1-\eta)(1-\delta)-vs\eta\delta\}(\theta_r-1)] \tag{43}$$

and

$$R=[(1-\delta)\{n+(\theta_p-1)(1-\eta+v\eta)\}+\{ns\delta+(1-\eta)(\theta_p-1)(1-\delta+s\delta)\} \\ -\{(1-\eta)(1-\delta)-vs\eta\delta\}(\theta_p-1)]. \tag{44}$$

By putting the value of T_R from equation (39), we get

$$T_P=\frac{[(1-\eta)(1-\delta)-vs\eta\delta]\,MT}{[(1-\delta)\,QL+(1-\eta)\,RM]}. \tag{45}$$

From equations (39) and (45), we deduce

$$T_R=\frac{[(1-\eta)(1-\delta)-vs\eta\delta]\,LT}{[(1-\delta)\,QL+(1-\eta)\,RM]}. \tag{46}$$

From equations (37), (45) and (46), we get

$$T_1=\frac{s\delta XT}{[(1-\delta)\,QL+(1-\eta)\,RM]} \tag{47}$$

where

$$X=[L\{mv\eta+(1-\delta)(\theta_r-1)(1-\eta+v\eta)\} \\ +M(1-\eta)\{n+(\theta_p-1)(1-\eta+v\eta)\}]. \tag{48}$$

Again, from equations (38), (45) and (46), we obtain

$$T_2=\frac{v\eta YT}{[(1-\delta)\,QL+(1-\eta)\,RM]} \tag{49}$$

where

$$Y=[M\{ns\delta+(1-\eta)(\theta_p-1)(1-\delta+s\delta)\} \\ +L(1-\delta)\{m+(\theta_r-1)(1-\delta+s\delta)\}]. \tag{50}$$

The inventory holding cost expressions for the newly produced, remanufactured and returned items are given, respectively, as

$$H_P=\frac{nh_p(1-\eta)\,D_pT_P^2}{2}=\frac{nh_p(1-\eta)\,D_p\,[(1-\eta)(1-\delta)-vs\eta\delta]^2\,M^2T^2}{2\,[(1-\delta)\,QL+(1-\eta)\,RM]^2} \tag{51}$$

$$H_R=\frac{mh_r(1-\delta)\,D_rT_R^2}{2}=\frac{mh_r(1-\delta)\,D_r\,[(1-\eta)(1-\delta)-vs\eta\delta]^2\,L^2T^2}{2\,[(1-\delta)\,QL+(1-\eta)\,RM]^2} \tag{52}$$

$$H_r = h_u \left[\frac{m D_r T_R^2}{2} \{\delta + (1-2\delta)\gamma_r\beta_r + (m-1)\ (1-\gamma_r\beta_r)\} + \frac{\gamma_p\beta_p D_p T_2^2}{2} \right.$$
$$+ (1-\delta\gamma_r\beta_r) D_r T_1 \left(\frac{T_1}{2\delta} + T_R\right) + \gamma_r\beta_r D_r (1-\delta) T_R T_2 + (m-1)(1-\gamma_r\beta_r)$$
$$\left. \times D_r T_R T_1 + \frac{\gamma_p\beta_p D_p n^2 T_P^2}{2} + \{\gamma_p\beta_p D_p T_2 + \gamma_r\beta_r D_r (1-\delta) T_R\} n T_P \right]$$

$$\Rightarrow H_r = \frac{h_u T^2}{2\left[(1-\delta) QL + (1-\eta) RM\right]^2} \left[m L^2 D_r \left[(1-\eta)(1-\delta) - vs\eta\delta\right]^2 \times \right.$$
$$\{\delta + \gamma_r\beta_r (1-2\delta) + (m-1)(1-\gamma_r\beta_r)\} + \gamma_p\beta_p D_p v^2\eta^2 Y^2 + (1-\delta\gamma_r\beta_r) \times$$
$$D_r s^2 \delta X^2 + 2\gamma_r\beta_r D_r v\eta L Y (1-\delta)\left[(1-\eta)(1-\delta) - vs\eta\delta\right] + 2sL\delta X D_r \times$$
$$\{(m-1)(1-\gamma_r\beta_r) + (1-\delta\gamma_r\beta_r)\}\left[(1-\eta)(1-\delta) - vs\eta\delta\right]$$
$$+ n^2 M^2 \gamma_p\beta_p D_p \left[(1-\eta)(1-\delta) - vs\eta\delta\right]^2 + 2nM\left[\gamma_p\beta_p D_p v\eta Y\right.$$
$$\left. + \gamma_r\beta_r D_r L (1-\delta)\left[(1-\eta)(1-\delta) - vs\eta\delta\right]\right]\left[(1-\eta)(1-\delta) - vs\eta\delta\right]\Big] \quad (53)$$

The total holding cost per unit of time is

$$H_T = \frac{H_P + H_R + H_r}{T}$$
$$H_T = T\psi(m, n, \gamma_r, \gamma_p, \theta_r, \theta_p) \quad (54)$$

where

$$\psi(m, n, \gamma_r, \gamma_p, \theta_r, \theta_p) = \frac{H_P + H_R + H_r}{T^2}. \quad (55)$$

The set up cost per unit time is

$$S_c = \frac{(mS_r + nS_p)}{T}. \quad (56)$$

The disposal cost per unit of time is

$$D_c = \frac{c_w}{T}\left[(D_p - \gamma_p\beta_p D_p)(nT_P + T_2) + (D_r - \gamma_r\beta_r D_r)(mT_R + T_1)\right].$$

By inserting the values of T_P, T_R, T_1 and T_2 from equations (45), (46), (47) and (49), respectively, and then solving, we get

$$D_c = \frac{c_w}{\left[(1-\delta) QL + (1-\eta) RM\right]}\left[D_p (1-\gamma_p\beta_p)\left[nM\{(1-\eta)(1-\delta) - vs\eta\delta\}\right.\right.$$
$$\left.\left. + v\eta Y\right] + D_r (1-\gamma_r\beta_r)\left[mL\{(1-\eta)(1-\delta) - vs\eta\delta\} + s\delta X\right]\right] \quad (57)$$

The remanufacturing cost per unit of time (including the purchasing cost of used items)is

$$R_c = \frac{c_r}{T}\left[\frac{D_r}{\delta} T_1 + \frac{D_r}{\delta}(\delta T_R) m\right].$$

By putting the values of T_R and T_1 from equations (46) and (47), respectively, and then solving, we obtain

$$R_c = \frac{c_r D_r \left[sX + mL \left\{(1-\eta)(1-\delta) - vs\eta\delta\right\}\right]}{\left[(1-\delta) QL + (1-\eta) RM\right]} \tag{58}$$

The production cost per unit of time is

$$P_c = \frac{c_p}{T}\left[\frac{D_p}{\eta}T_2 + \frac{D_p}{\eta}(\eta T_P) n\right].$$

By introducing the values of T_P and T_2 from equations (45) and (49), respectively, and then solving, we get

$$P_c = \frac{c_p D_p \left[vY + nM \left\{(1-\eta)(1-\delta) - vs\eta\delta\right\}\right]}{\left[(1-\delta) QL + (1-\eta) RM\right]}. \tag{59}$$

The total backordering cost per unit of time for newly produced items is

$$\begin{aligned} BC_p = \frac{b_p}{T}\Bigg[&\frac{vD_p}{2}\left[(m-1)T_R + \delta T_R + t_r + T_1 - T_R^P\right]\left[(m-1)T_R + \delta T_R\right. \\ &\left. + t_r + T_1 - T_R^P\right] + \frac{vD_p}{2}\left[(m-1)T_R + \delta T_R + t_r + T_1 - T_R^P\right]T_2\Bigg] \end{aligned}$$

After solving, we obtain

$$BC_p = \frac{b_p D_p (1-\eta)(1-\eta+v\eta) vY^2 T}{2\left[(1-\delta) QL + (1-\eta) RM\right]^2} \tag{60}$$

The total backordering cost per unit of time for remanufactured items is

$$\begin{aligned} BC_r = \frac{b_r}{T}\Bigg[&\frac{sD_r}{2}\left[(n-1)T_P + \eta T_P + t_p + T_2 - T_P^R\right]\left[(n-1)T_P + \eta T_P\right. \\ &\left. + t_p + T_2 - T_P^R\right] + \frac{sD_r}{2}\left[(n-1)T_P + \eta T_P + t_p + T_2 - T_P^R\right]T_1\Bigg] \end{aligned}$$

After solving, we get

$$BC_r = \frac{b_r D_r (1-\delta)(1-\delta+s\delta) sX^2 T}{2\left[(1-\delta) QL + (1-\eta) RM\right]^2}. \tag{61}$$

The total backordering cost per unit of time is

$$BC_{PR} = BC_{pr}(m, n, \gamma_r, \gamma_p, \theta_r, \theta_p) T \tag{62}$$

where

$$\begin{aligned} &BC_{pr}(m, n, \gamma_r, \gamma_p, \theta_r, \theta_p) \\ &= \left[\frac{b_p D_p (1-\eta)(1-\eta+v\eta) vY^2 T}{2\left[(1-\delta) QL + (1-\eta) RM\right]^2} + \frac{b_r D_r (1-\delta)(1-\delta+s\delta) sX^2 T}{2\left[(1-\delta) QL + (1-\eta) RM\right]^2}\right]. \end{aligned} \tag{63}$$

The lost sales cost per unit time for newly produced items is

$$LC_p = \frac{l_p}{T}\left[(1-v)\,D_p\left\{(m-1)\,T_R + \delta T_R + t_r + T_1 - T_R^P\right\}\right]$$

$$\Rightarrow LC_p = \frac{l_p D_p Y\,(1-v)\,(1-\eta)}{[(1-\delta)\,QL + (1-\eta)\,RM]}. \tag{64}$$

The lost sales cost per unit time for remanufactured items is

$$LC_r = \frac{l_r}{T}\left[(1-s)\,D_r\left\{(n-1)\,T_P + \eta T_P + t_p + T_2 - T_P^R\right\}\right]$$

$$\Rightarrow LC_r = \frac{l_r D_r X\,(1-s)\,(1-\delta)}{[(1-\delta)\,QL + (1-\eta)\,RM]}. \tag{65}$$

Therefore, the total cost per unit of time is

$$\begin{aligned}
&C\,(m,n,\gamma_r,\gamma_p,\theta_r,\theta_p,T)\\
&= \Bigg[\frac{(mS_r + nS_p)}{T} + \psi T + BC_{pr}T + \frac{1}{[(1-\delta)\,QL + (1-\eta)\,RM]}\times\\
&[c_w\,[D_p\,(1-\gamma_p\beta_p)\,\{nM\,\{(1-\eta)\,(1-\delta) - vs\eta\delta\} + v\eta Y\}\\
&+D_r\,(1-\gamma_r\beta_r)\,\{mL\,\{(1-\eta)\,(1-\delta) - vs\eta\delta\} + s\delta X\}]\\
&+c_r D_r\,[sX + mL\,\{(1-\eta)\,(1-\delta) - vs\eta\delta\}] + c_p D_p\times\\
&[vY + nM\,\{(1-\eta)\,(1-\delta) - vs\eta\delta\}] + l_p D_p Y\,(1-v)\,(1-\eta)\\
&+ l_r D_r X\,(1-s)\,(1-\delta)\ \Bigg]
\end{aligned} \tag{66}$$

Now, putting the first order partial derivative of equation (66) equal to zero and solving for T, we get

$$T = \sqrt{\frac{(mS_r + nS_p)}{\psi\,(m,n,\gamma_r,\gamma_p,\theta_r,\theta_p) + BC_{pr}\,(m,n,\gamma_r,\gamma_p,\theta_r,\theta_p)}}. \tag{67}$$

Putting the value of T from equation (67) in (66), we get

$$C\left(m,n,\gamma_r,\gamma_p,\theta_r,\theta_p\right)$$

$$= \frac{1}{\left[\left(1-\delta\right)QL+\left(1-\eta\right)RM\right]}\left[2\sqrt{\left(mS_r+nS_p\right)\left(\psi+BC_{pr}\right)}\times\right.$$

$$\left[\left(1-\delta\right)QL+\left(1-\eta\right)RM\right]+l_pD_pY\left(1-v\right)\left(1-\eta\right)$$

$$+c_w\left[D_p\left(1-\gamma_p\beta_p\right)\left\{nM\left\{\left(1-\eta\right)\left(1-\delta\right)-vs\eta\delta\right\}+v\eta Y\right\}\right.$$

$$+D_r\left(1-\gamma_r\beta_r\right)\left\{mL\left\{\left(1-\eta\right)\left(1-\delta\right)-vs\eta\delta\right\}+s\delta X\right\}\Big]$$

$$+c_rD_r\left[sX+mL\left\{\left(1-\eta\right)\left(1-\delta\right)-vs\eta\delta\right\}\right]+c_pD_p\times$$

$$\left.\left[vY+nM\left\{\left(1-\eta\right)\left(1-\delta\right)-vs\eta\delta\right\}\right]+l_rD_rX\left(1-s\right)\left(1-\delta\right)\right] \tag{68}$$

3.2.2. Case 2: Overlapping and pure backordering

The case 2 of Scenario 2, described in Fig. 5 is similar to that of Fig. 4, except that it assumes for pure backordering over a portion of the remanufacturing and production segments.

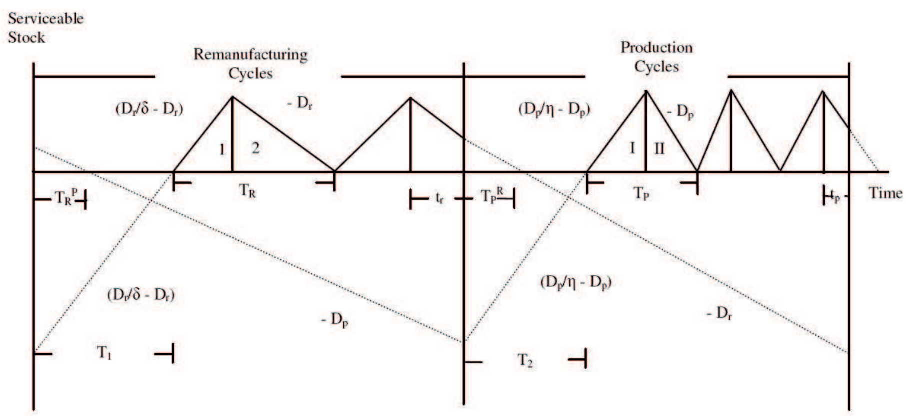

Figure 5. The inventory status for the system with overlapping and pure backordering

See Appendix 4 for the expression of total cost per unit of time.

3.3. Solution procedure

Input the values of the parameters D$_r$, D$_p$, S$_r$, S$_p$, h$_p$, h$_r$, h$_u$, c$_w$, c$_p$, c$_r$, β_p, β_r, b$_p$, b$_r$, l$_p$, l$_r$, v, s, δ and η. Then proceed like in Jaber and El Saadany (2009), that is, as follows:

Step 1. Set n=1, m=1, and optimize C(1,1,γ_r,γ_p). Record the values of C(1,1,γ_r,γ_p), $\gamma^*_{r(1,1)}$ and $\gamma^*_{p(1,1)}$.

Step 2. Repeat Step 1 for m=2, n=1, and record C(2,1,γ_r,γ_p), $\gamma^*_{r(2,1)}$ and $\gamma^*_{p(2,1)}$. Compare C(1,1,γ_r,γ_p) and C(2,1,γ_r,γ_p). If C(1,1,γ_r,γ_p) <C(2,1,γ_r,γ_p), terminate the search for n=1 and record the value of C(1,1,γ_r,γ_p). If C(1,1,γ_r, γ_p) >C(2,1,γ_r,γ_p), repeat step 1 for m=3, m=4, etc. Terminate once C(m$^*_1$-1,1,γ_r,γ_p) >C(m$^*_1$,1,γ_r,γ_p) <C(m$^*_1$+1,1,γ_r,γ_p), where m$^*_1$ is the optimal value for the number of remanufacturing cycles when there is one production cycle. Record the value of C(m$^*_1$,1,γ_r,γ_p), m$^*_1$, $\gamma^*_{r\left(m^*_1,1\right)}$and $\gamma^*_{r\left(m^*_1,1\right)}$.

Step 3. Repeat Steps 1 and 2 for n=2. Compare C(m$^*_1$,1,γ_r,γ_p) and $C(m^*_2$,2, γ_r,γ_p). If $C(m^*_1, 1, \gamma_r, \gamma_p) < C(m^*_2, 2, \gamma_r, \gamma_p)$, terminate the search and $C(m^*_1, 1, \gamma_r, \gamma_p)$ is the optimum solution. If $C(m^*_1, 1, \gamma_r, \gamma_p) > C(m^*_2, 2, \gamma_r, \gamma_p)$, then leave the value of $C(m^*_1, 1, \gamma_r, \gamma_p)$ and repeat the steps 1 and 2 for n=3.

Step 4. Terminate the search, once $C(m^*_{i-1}, i-1, \gamma_r,\gamma_p) \geqslant C(m^*_i, i, \gamma_r, \gamma_p)$ $<C(m^*_{i+1}, i{+}1, \gamma_r, \gamma_p)$, where i is the optimal value for the number of production cycles when there are m$^*_i$ remanufacturing cycles at the cost of $C(m^*_i, i, \gamma_r, \gamma_p)$.

A similar solution procedure can be used to find the optimal solution for Scenario 2.

4. Numerical examples and sensitivity analysis

In this section, we have provided four numerical examples to illustrate the behaviour of the models developed in the previous section.

Example 1 (*Scenario 1-* (*case 1: Partial Backordering*)) We consider the following parameter values on the basis of previous study: $D_r = 10$, $D_p = 10$, $S_p = 400$, $S_r = 200$, $h_p = 4$, $h_r = 2$, $h_u = 2$, $\beta_p = 0.667$, $\beta_r = 0.667$, $\gamma_{min} = 0.01$ (Jaber and El Saadany, 2009, p. 120), $c_w = 0.8$, $c_p = 15$, $c_r = 8$, $\delta = 0.45$, $\eta = 0.5$, $b_p = 10$, $b_r = 5$, $l_p = 3$, $l_r = 1.5$, $v = 0.3$ and $s = 0.3$. All the computations are performed with the help of software MATHEMATICA 8.0. From Table 1 it can be seen that the optimal solution is $m = 1$, $n = 1$, $\gamma_p = 0.889$, $\gamma_r = 1$ and $C(m, n, \gamma_r, \gamma_p) = 349.726$. The behavior of the total cost function with respect to the parameters γ_r and γ_p is presented in Fig. 6. Tables 2 and 3 show the results of sensitivity analysis conducted with respect to the key parameters of the inventory system.

From Table 2, the following interesting findings can be deduced, which are summarized below as:

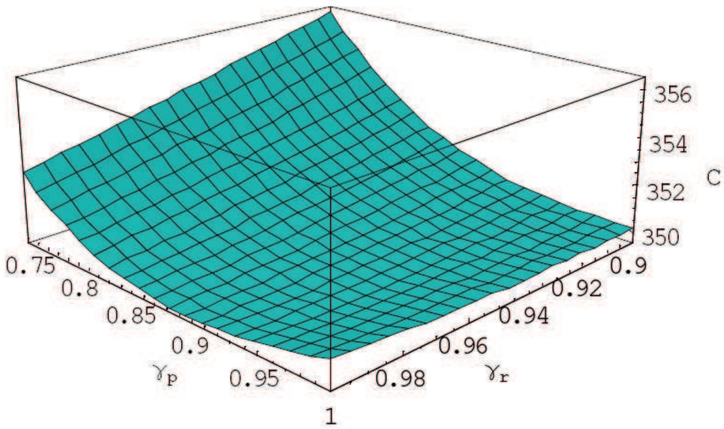

Figure 6. Behavior of the total cost function with respect to γ_r and γ_p for case 1 of Scenario 1

Table 4. Optimal policy for Example 1

Trial	m	n	γ_p	γ_r	C
1*	1*	1*	0.889*	1*	349.726*
2	2	1	0.955	1	367.393
3	1	2	0.761	1	392.969
4	2	2	0.831	1	404.263

(1) The sensitivity table shows that when demand parameter 'D_r' varies from 1 to 12, the value of γ_r =1 and γ_p lies between 0.092 and 1 for the optimal solution; this implies that the most favorable strategy is to collect the maximum available used products from secondary market and partially from primary market. Even as γ_p =1 and γ_r decreases down to 0 when $12 \leqslant D_r < 36$, it is economically beneficial to collect the used items partially from the secondary market and all the available returns from the primary market. After that, when $36 \leqslant D_r \leqslant 50$, the optimal solution exists for γ_r =0 and γ_p =1, and in this case, it is beneficial to collect all the available returns from the primary market and no returns from the secondary market.

(2) When $1 \leqslant D_p < 5$, the best strategy takes place for γ_p =1 while γ_r increases from 0 to 1, so it is economically advantageous to collect all the available used products from the primary market and partially from the secondary market. On the other hand, when $5 \leqslant D_p \leqslant 50$, the optimal solution exists for γ_r =1, and for reducing the value of γ_p down to 0.309, so it is encouraging to accumulate all the available returns from the secondary market and partially

Table 5. The effect of the changing values of the system parameters on the optimal policies

Parameter	value	m	n	γ_p	γ_r	C
D_r	1	1	1	0.092	1	245.944
	12	1	1	1	1	370.834
	36	1	1	1	0	671.212
	50	1	1	1	0	780.863
D_p	1	1	1	1	0	262.391
	5	1	1	1	1	295.220
	50	1	1	0.309	1	901.095
S_p	1	1	7	0.777	1	269.371
	11.05	1	1	0.864	1	275.951
	500	1	1	0.880	1	364.160
S_r	1	10	1	1	1	303.582
	49.97	1	1	0.986	1	325.539
	400	1	1	0.873	1	377.582
c_w	0.1	1	1	0.876	1	347.101
	2	1	1	0.913	1	354.140
β_p	0.01	1	1	1	0	549.462
	0.295	1	1	1	1	419.995
	0.667	1	1	0.889	1	349.726
β_r	0.01	1	1	1	1	369.663
	0.667	1	1	0.889	1	349.726

from the primary market.

(3) When S_p varies from 1 to 11.05, then the optimal policy takes place for m=1, while n varies from 7 to 1. While the value of γ_p varies from 0.777 to 0.864 and γ_r =1 for the optimal policy, the best strategy is to collect all the available returns from the secondary market and partially from the primary market. After that, when 11.05 $<S_p \leqslant 500$, γ_p varies from 0.864 to 0.880 and γ_r =1 for the optimal policy, and in this case the best strategy is to collect all the available returns from the secondary market and partially from the primary market.

(4) When S_r varies from 1 to 49.97, then solution exists for n=1, while m varies from 10 to 1. While the value of γ_p varies from 1 to 0.986 and γ_r =1 for the optimal solution, the best strategy is to collect maximum available returns from the primary and secondary markets. After that, when $49.97 \leqslant S_r \leqslant 400$, then γ_p varies from 0.986 to 0.873 and γ_r =1 for the optimal policy, therefore the best approach is to collect all the available returns from the secondary market and partially from the primary market.

(5) When c_w varies from 0.1 to 2, then γ_r =1 and the value of γ_p varies from 0.876 to 0.913 for the most favorable policy, which suggests taking all the

available used items from the secondary market and the maximum from the primary market.

(6) When $0.01 \leqslant \beta_p < 0.295$, then the best possible solution exists for $\gamma_p = 1$ and $0 \leqslant \gamma_r \leqslant 1$, so it is economically beneficial to take all the available used products from the primary market and partially from the secondary market. After that, when $0.295 < \beta_p \leqslant 0.667$ and $0.01 \leqslant \beta_r \leqslant 0.667$, then the optimal strategy takes place for $\gamma_r = 1$ and the value of γ_p varying from 1 to 0.889, so it is preferable to accumulate all the available returns from the secondary market and partially from the primary market.

Table 6. The effect of changing values of the backordering cost parameters on the optimal strategy

b_p	b_r	m	n	γ_p	γ_r	C
7	2	1	1	0.702	1	286.394
	7	1	1	0.958	1	312.542
10	2	1	1	0.641	1	291.408
	7	1	1	0.836	1	322.233

From Table 3 it can be observed that when $b_p = 7$, $2 \leqslant b_r \leqslant 7$, the optimal policy exists for $0.702 \leqslant \gamma_p \leqslant 0.958$ and $\gamma_r = 1$, and therefore it is efficient to collect all the available returns from the secondary market and partially from the primary market. Similarly, when $b_p = 10$ and $2 \leqslant b_r \leqslant 7$, the optimal solution takes place for $0.641 \leq \gamma_p \leq 0.836$ and $\gamma_r = 1$, so it is beneficial to collect all the available returns from the secondary market and partially from the primary market.

Example 2 (*Scenario 1*- (*case 2: Full Backordering*)) Consider the case of Example 1, except for the values $D_r = 4$, $v = 1$ and $s = 1$. From Table 4 it can be seen that the optimal policy is $m = 1$, $n = 1$, $\gamma_r = 1$, $\gamma_p = 0.669$ and $C(m, n, \gamma_r, \gamma_p) = 417.073$. The behavior of the total average cost function with respect to γ_r and γ_p is shown in Fig. 7.

Table 7. Optimal policy for Example 2

Trial	m	n	γ_p	γ_r	C
1*	1*	1*	0.669*	1*	417.073*
2	2	1	0.672	1	455.888
3	1	2	0.657	1	475.287
4	2	2	0.660	1	507.382

Example 3 (*Scenario 2*- (*case 1: Overlapping and pure backordering*))

On the basis of previous study the selected parameter values are as follows: $c_w = 0.8$, $c_p = 12$, $c_r = 7$, $D_r = 10$, $D_p = 10$, $S_p = 400$, $S_r = 200$, $h_p = 2$,

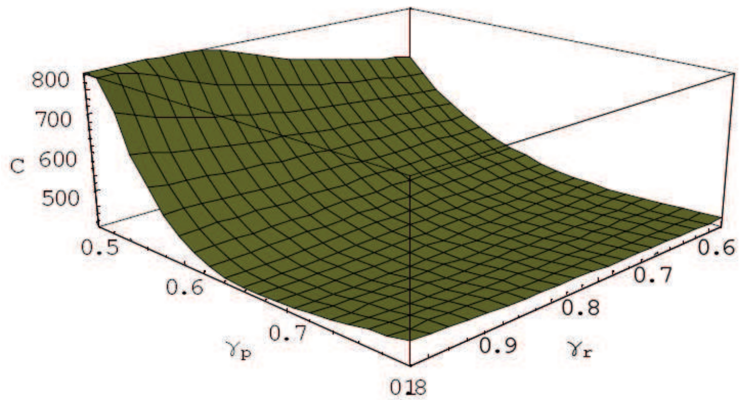

Figure 7. Behavior of the total cost function with respect to γ_r and γ_p for case 2 of Scenario 1

$h_r = 1$, $h_u = 1$, $b_p = 10$, $b_r = 5$, $l_p = 7$, $l_r = 2$, $v = 0.3$, $s = 0.3$, $\beta_p = 0.667$, $\beta_r = 0.667$, $\delta = 0.45$, $\eta = 0.5$.

From Table 5 it can be seen that the optimal policy is m=1, n=1, γ_r =1, γ_p =0.246, θ_p =1, θ_r =0.653 and $C(m, n, \gamma_r, \gamma_p, \theta_r, \theta_p) = 305.479$. The effect of the changes in parameter values on the optimal policy is shown in Tables 6 and 7. The behavior of the total average cost function with respect to θ_r and θ_p is shown in Fig. 8.

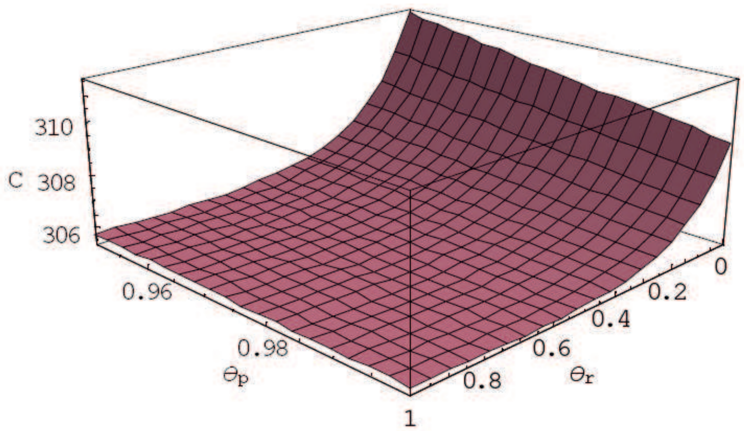

Figure 8. Behavior of the total cost function with respect to θ_p and θ_r for case 1 of Scenario 2

From Table 6 some important conclusions can be drawn, as follows: (1) When $c_w = 0.1$, then the optimal solution exists for $\gamma_p = 0.464$, $\gamma_r = 0$, $\theta_p = 1$ and $\theta_r = 0.110$; this suggests to collect no returns from the secondary market

Table 8. Optimal strategy for Example 3.

Trial	m	n	γ_p	γ_r	θ_p	θ_r	C
1*	1*	1*	0.246*	1*	1*	0.653*	305.479*
2	2	1	0.253	1	0.835	0	322.946
3	1	2	0.420	0	0.703	0	333.743
4	2	2	0.213	1	0.003	0	343.545

and only partially from the primary market. In this case, the fraction of the remanufacturing cycle almost completely overlaps, while there is no overlapping of the fraction of the production cycle. When $0.1 < c_w \leqslant 0.47$, then the optimal solution takes place for the increasing value of γ_r $(0 \leqslant \gamma_r \leqslant 1)$ and the decreasing value of γ_p down to 0.243, which shows that it is reasonable to collect the available returns partially from the primary and secondary markets. After that, the best solution exists for $\gamma_r = 1$ and $0.243 \leqslant \gamma_p \leqslant 0.256$ when c_w lies between 0.47 and 2. When $0.1 < c_w \leqslant 2$, then overlapping of the fraction of the remanufacturing cycle shifts from partial to no overlapping, whereas there is no overlapping of the fraction of the production cycle.

(2) When $1 \leqslant D_r \leqslant 50$, then the optimal solution exists for $0.171 \leqslant \gamma_p \leqslant 0.202$, $\gamma_r = 1$, $\theta_p = 1$ and $0.336 \leqslant \theta_r \leqslant 0$, so it is preferable to accumulate a lesser amount of used products from the primary market and the entirety of the used products from the secondary market. In addition, no overlapping of the fraction of the production cycle takes place, while the overlapping of the fraction of the remanufacturing cycle shifts from partial to complete overlapping.

(3) When $1 \leqslant D_p \leqslant 50$, then the best possible solution exists for $\gamma_r = 1$, $1 \leqslant \gamma_p \leqslant 0.054$, $0.301 \leqslant \theta_p \leqslant 1$ and $\theta_r = 1$, and therefore it is beneficial to collect all the available returns from the primary and secondary markets, which is the best policy when $D_p = 1$. After that, when $1 < D_p \leqslant 50$, then it is reasonable to collect all the available returns from the secondary market and a small amount from the primary market. On the other hand, there is no overlapping of the fraction of remanufacturing cycle when $1 \leqslant D_p \leqslant 50$, while the overlapping of the fraction of the production cycle turns from partial to no overlapping.

(4) When $1 \leqslant S_p \leqslant 28.1$, then for the optimal solution $m = 1$ and n reduces from 9 to 1; after that, when $28.1 \leqslant S_p \leqslant 500$, the optimal solution takes place for $m = 1$ and $n = 1$. It is practical to collect all the available returns from the secondary market and partially from the primary market when $1 \leqslant S_p \leqslant 500$, also, there is no overlapping of the fraction of the production cycle for $1 \leqslant S_p \leqslant 500$. While for $S_p = 1$ there is complete overlapping of the fraction of the remanufacturing cycle, it turns into no overlapping when $1 < S_p \leqslant 28.1$, and again it changes to partial overlapping when $28.1 < S_p \leqslant 500$.

(5) When $1 \leqslant S_r \leqslant 20.4$, then for the optimal solution $n = 1$ and m reduces from 9 to 1, and after that, when $20.4 \leqslant S_r \leqslant 400$, the optimal solution occurs

Table 9. The effect of changes in the values of the system parameters on the optimal policies

Parameter	value	m	n	γ_p	γ_r	θ_p	θ_r	C
c_w	0.10	1	1	0.464	0	1	0.110	299.664
	0.47	1	1	0.243	1	1	0.568	303.178
	2.00	1	1	0.256	1	1	1	313.641
D_r	1	1	1	0.171	1	1	0.336	222.017
	50	1	1	0.202	1	1	0	600.564
D_p	1	1	1	1	1	0.301	1	190.736
	50	1	1	0.054	1	1	1	856.136
S_p	1	1	9	0.205	1	1	0	236.266
	28.1	1	1	0.252	1	1	1	249.915
	500	1	1	0.244	1	1	0.529	316.972
S_r	1	9	1	0.258	1	0.453	0	274.849
	20.4	1	1	0.250	1	1	1	281.929
	400	1	1	0.242	1	1	0.441	327.631
β_p	0.01	1	1	1	0	1	0	366.694
	0.1	1	1	1	1	0.759	1	332.606
	0.667	1	1	0.246	1	1	0.653	305.479
β_r	0.01	1	1	0.463	0	1	0.256	306.037
	0.09	1	1	0.429	1	1	0.301	306.039
	0.667	1	1	0.246	1	1	0.653	305.479

for m=1 and n=1. It is practical to collect all the available returns from the secondary market and partially from the primary market when $1 \leqslant S_r \leqslant 400$, also the overlapping of the fraction of the production cycle changes from partial to no overlapping as S_r varies from 1 to 400. While for $S_r = 1$ there is complete overlapping of the fraction of theremanufacturing cycle, it turns into no overlapping when $1 < S_r \leqslant 20.4$, and then again it changes to partial overlapping when $20.4 < S_r \leqslant 400$.

(6) When $0.01 \leqslant \beta_p < 0.1$, then the best possible solution takes place for $\gamma_p = 1$ and $0 \leqslant \gamma_r < 1$, meaning that it is economically beneficial to receive all used products from the primary market and partially from the secondary market. After that, when $0.1 \leqslant \beta_p \leqslant 0.667$, then the optimal strategy takes place for $\gamma_r = 1$ and the value of γ_p varying from 1 to 0.246, so it is preferable to accumulate all the available returns from the secondary market and partially from the primary market. The fraction of the production cycle overlaps partially when $0.01 < \beta_p < 0.667$, and the fraction of the remanufacturing cycle overlaps partially when $0.01 < \beta_p \leqslant 0.667$, except at $\beta_p = 0.1$. There is no overlapping of the fraction of the production cycle at $\beta_p = 0.01$ and 0.667, while there is complete overlapping of the fraction of remanufacturing cycle at $\beta_p = 0.01$.

(7) When $0.01 \leqslant \beta_r \leqslant 0.667$, then the best possible solution exists for $0.463 \leqslant \gamma_p \leqslant 0.246$, $0 \leqslant \gamma_r \leqslant 1$, $\theta_p = 1$ and $0.256 \leqslant \theta_r \leqslant 0.653$. Consequently, it is preferable to assemble used products partially from the primary market and to ensure no overlapping of the fraction of the production cycle. The fraction of the remanufacturing cycle overlaps partially when $0.01 \leqslant \beta_r \leqslant 0.667$, and it is recommendable, therefore, to accumulate the available returns partially from the secondary market when $0.01 < \beta_r < 0.09$ (with no returns for $\beta_r = 0.01$). After that, when $0.09 \leqslant \beta_r \leqslant 0.667$, then all the used products should be collected from the secondary market.

Table 10. The effect of changes in the values of the backordering cost parameters on the optimal policy

b_p	b_r	m	n	γ_p	γ_r	θ_p	θ_r	C
7	2	1	1	0.233	1	1	1	279.283
	6.12	1	1	0.592	0	1	0.430	304.609
	7	1	1	0.622	0	1	0.330	307.514
10	2	1	1	0.191	1	1	0	282.882
	6.3	1	1	0.508	0	1	0.0839	311.320
	7	1	1	0.528	0	1	0.034	313.604

It can be observed on the basis of Table 7 that when $b_p = 7$, $2 \leqslant b_r \leqslant 7$, the optimal policy exists for $0.233 \leqslant \gamma_p \leqslant 0.622$ and $0 \leqslant \gamma_r \leqslant 1$, and therefore it is profitable to collect the available returns partially from the primary market and to ensure no overlapping of the fraction of the production cycle. Then, at $b_r = 2$ all used items should be collected and when $2 < b_r < 6.12$, the available returns should be accumulated partially, and after that, when $6.12 \leqslant b_r \leqslant 7$, no returns should be collected from the secondary market. Partial overlapping (no overlapping) of the remanufacturing cycle is preferable when $2 < b_r \leqslant 7 (b_r = 2)$. Further, when $b_p = 10$ and $2 \leqslant b_r \leqslant 7$, the the observation can be made in the similar manner as given for $b_p = 7, 2 \leq b_r \leq 7$, except that the fraction of the remanufacturing cycle almost completely overlaps when $2 \leq b_r \leq 7$.

Example 4 (*Scenario 2- (case 2: Overlapping and full backordering)*)

On the basis of the previous investigations the parameter values are specified as follows: $c_w = 0.8$, $c_p = 12$, $c_r = 7$, $D_r = 500$, $D_p = 600$, $S_p = 400$, $S_r = 200$, $h_p = 2$, $h_r = 1$, $h_u = 1$, $b_p = 10$, $b_r = 5$, $\beta_p = 0.667$, $\beta_r = 0.667$, $\delta = 0.45$, $\eta = 0.5$, v=1, s=1.

From Table 8 it can be seen that the optimal strategy is m=1, n=1, $\gamma_r = 1$, $\gamma_p = 1$, $\theta_p = 1$, $\theta_r = 0.140$, and $C = 13167.40$. The behavior of the total average cost function with respect to θ_r and θ_p is shown in Fig. 9.

Table 11. The optimal strategy for Example 4

Trial	m	n	γ_p	γ_r	θ_p	θ_r	C
1*	1*	1*	1*	1*	1*	0.140*	13167.4*
2	2	1	1	1	1	0	13627.8
3	1	2	0.01	0	1	1	13471.0
4	2	2	0.01	0	1	1	13692.7

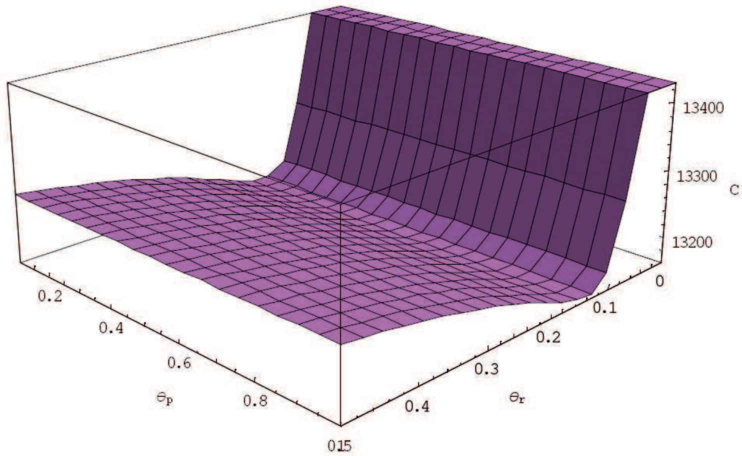

Figure 9. Behavior of the total cost function with respect to θ_p and θ_r for case 2 of Scenario 2

5. Conclusions

In this article, reverse logistics inventory models with finite production and remanufacturing rates are developed. To minimize the effects of stock-outs, the overlapping of the fraction of one production cycle and one remanufacturing cycle is taken into consideration. The cases of partial and full backordering, which are discussed in this paper, have been illustrated with some numerical experiments. This paper is more realistic and advantageous from the previous one, as it is developed with the following more practical attributes: (1) demand dependent production and remanufacturing rates are taken into account (2) disposal cost is considered, (3) newly produced and remanufactured items are considered of different quality standard, (4) returned rate is considered as a function of demand rate and (5) purchasing cost of used items is also included. In addition, from the sensitivity analysis whose results are shown in Tables 2 and 3 (for partial backordering case of Scenario 1), it is clear that when $1 \leq D_r \leq 12$, $5 \leq D_p \leq 50$, $1 \leq S_p < 500$, $1 \leq Sr < 400$, $0.1 \leq cw < 2$, $0.295 \leq \beta_p \leq 0.667$, $0.01 \leq \beta r \leq 0.667$, $bp = 7$, 10 and $2 \leq br \leq 7$, then the optimal policy is to collect all the available returns from the secondary market, otherwise partial or no collection of the used items from the secondary market is advisable. On the other hand, when $12 \leq Dr \leq 50$, $1 \leq Dp \leq 5$, $1 \leq Sr < 49.97$, $0.01 \leq \beta p \leq 0.295$ and $\beta r = 0.01$, then the most favorable policy is to collect all the available returns from the primary market, otherwise it is economically beneficial to collect available returns partially from the primary market. Again, from the sensitivity analysis, with results shown in Tables 6 and 7 (for overlapping and partial backordering case of Scenario 2), it is clear that when $0.47 \leq cw < 2$, $1 \leq Dr \leq 50$, $1 \leq Dp \leq 50$, $1 \leq Sp < 500$, $1 \leq Sr < 400$, $0.1 \leq \beta p \leq 0.667$, $0.09 \leq \beta r \leq 0.667$, $bp = 7$, 10 and $2 \leq br < 6.12$, then the optimal policy is to collect all the available returns from the secondary market, otherwise partial or no collection of the used items from the secondary market is worthwhile. On the other hand, when $D_p = 1$ and $0.01 \leq \beta p \leq 0.1$, then the most favorable policy is to collect all the available returns from the primary market, otherwise it is sensible to collect available returns partially from the primary market. The future research can include the consideration of the model in the fuzzy environment

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Appendix 1

The holding cost for the newly produced items is calculated as follows:

$$H_P = h_p n \left(the\, area\, of\, the\, triangle\, I \,+\, the\, area\, of\, the\, triangle\, II \right).$$

$$H_P = h_p n \left[\frac{1}{2} \left(\left(\frac{D_p}{\eta} - D_p \right) \eta T_P \right) \eta T_P + \frac{1}{2} \left(\left(\frac{D_p}{\eta} - D_p \right) \eta T_P \right) (1-\eta) T_P \right]$$

$$\Rightarrow H_P = \frac{h_p n (1-\eta) D_p T_P^2}{2} \tag{i}$$

The holding cost for remanufactured items is calculated as follows:

$$H_R = h_r m \left(the\, area\, of\, the\, triangle\, 1 \,+\, the\, area\, of\, the\, triangle\, 2 \right).$$

$$H_R = h_r m \left[\frac{1}{2} \left(\left(\frac{D_r}{\delta} - D_r \right) \delta T_R \right) \delta T_R + \frac{1}{2} \left(\left(\frac{D_r}{\delta} - D_R \right) \delta T_R \right) (1-\delta) T_R \right]$$

$$\Rightarrow H_R = \frac{h_r m (1-\delta) D_r T_R^2}{2} \tag{ii}$$

Appendix 2

According to Fig. 10 we calculate the holding cost for returned items as follows:
Area of part A is

$$\Delta_A = \left(\frac{1}{2}\left(\frac{D_r}{\delta} - \gamma_r\beta_r D_r\right)\delta T_R\right)\delta T_R = \frac{1}{2}\left(1 - \delta\gamma_r\beta_r\right)\delta D_r T_R^2.$$

Area of part B is

$$\Delta_B = \left(\frac{1}{2}\gamma_r\beta_r D_r\left(1-\delta\right)T_R\right)\left(1-\delta\right)T_R = \frac{1}{2}\gamma_r\beta_r D_r\left(1-\delta\right)^2 T_R^2.$$

Area of part C is

$$\Delta_C = \left(\frac{1}{2}\gamma_p\beta_p D_p T_2\right)T_2 = \frac{1}{2}\gamma_p\beta_p D_p T_2^2.$$

Area of part D is

$$\Delta_D = \left(\gamma_r\beta_r D_r\left(1-\delta\right)T_R\right)T_2 = \gamma_r\beta_r D_r\left(1-\delta\right)T_R T_2.$$

Area of part E_i is

$$\Delta_{E_i} = \left(\left(\frac{D_r}{\delta} - \gamma_r\beta_r D_r\right)\delta T_R - \gamma_r\beta_r D_r\left(1-\delta\right)T_R\right)iT_R = \left(1-\gamma_r\beta_r\right)iD_r T_R^2.$$

Area of part F is

$$\Delta_F = \left(\frac{1}{2}\gamma_p\beta_p D_p nT_P\right)nT_P = \frac{1}{2}\gamma_p\beta_p D_p n^2 T_P^2.$$

Area of part G is

$$\Delta_G = \left(\gamma_p\beta_p D_p T_2 + \gamma_r\beta_r D_r\left(1-\delta\right)T_R\right)nT_P.$$

Area of part H is

$$\Delta_H = \left(\frac{1}{2}\left(\frac{D_r}{\delta} - \gamma_r\beta_r D_r\right)T_1\right)T_1 = \frac{1}{2\delta}\left(1-\delta\gamma_r\beta_r\right)D_r T_1^2.$$

Area of part J is

$$\Delta_J = \left(\left(\frac{D_r}{\delta} - \gamma_r\beta_r D_r\right)\delta T_R\right)T_1 = \left(1-\delta\gamma_r\beta_r\right)D_r T_R T_1.$$

Area of part K is

$$\Delta_K = \left(\left(\frac{D_r}{\delta} - \gamma_r\beta_r D_r\right)\delta T_R - \gamma_r\beta_r D_r\left(1-\delta\right)T_R\right)\left(m-1\right)T_1$$

$$= (1 - \gamma_r \beta_r)(m-1) D_r T_R T_1.$$

Therefore, the holding cost for the returned items is

$$H_r = h_u \left[m\Delta_A + m\Delta_B + \Delta_C + \Delta_D + \sum_{i=1}^{m-1} \Delta_{E_i} + \Delta_F + \Delta_G + \Delta_H + \Delta_J + \Delta_K \right]$$

$$\begin{aligned} H_r = h_u & \left[\frac{m D_r T_R^2}{2} \{\delta + \gamma_r \beta_r - 2\delta\gamma_r\beta_r + (m-1)\ (1-\gamma_r\beta_r)\} + \frac{\gamma_p \beta_p D_p T_2^2}{2} \right. \\ & + \frac{(1-\delta\gamma_r\beta_r) D_r T_1^2}{2\delta} + \gamma_r \beta_r D_r (1-\delta) T_R T_2 + (m-1)(1-\gamma_r\beta_r) \times \\ & D_r T_R T_1 + (1-\delta\gamma_r\beta_r) D_r T_R T_1 + \frac{\gamma_p \beta_p D_p n^2 T_P^2}{2} + \\ & \left. \{\gamma_p \beta_p D_p T_2 + \gamma_r \beta_r D_r (1-\delta) T_R\} n T_P \right] \end{aligned} \qquad (iii)$$

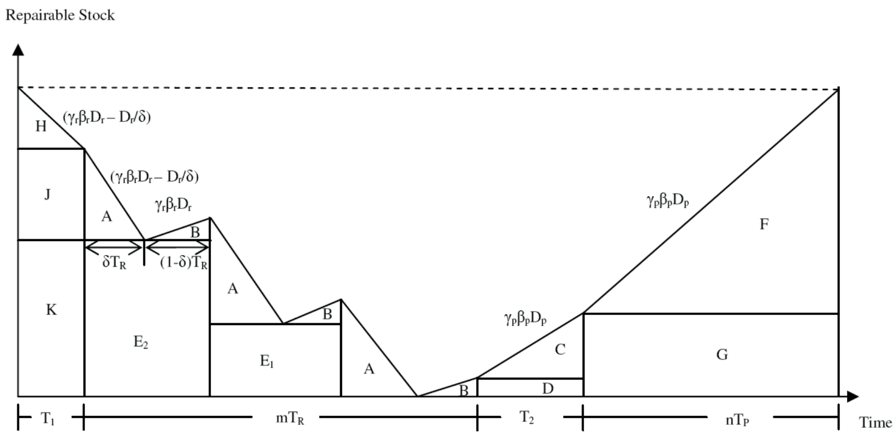

Figure 10. Inventory estimation for H$_r$

Appendix 3

To find the total cost per unit of time of the system for case 2 (full backordering) of Scenario 1 we put v=1 and s=1 in equation (27), and then we get

$$\begin{aligned} & C(m, n, \gamma_r, \gamma_p) \\ & = \frac{1}{(g - D_r)} \left[2\sqrt{(mS_r + nS_p)(\psi + BC_{pr})} + c_w (1-\gamma_r\beta_r) D_r (D_p - D_r) \right. \\ & \left. + \frac{c_r D_r [\alpha + (\delta - \gamma_r\beta_r) D_r]}{(1-\delta)} + \frac{c_p D_p [\xi + \eta D_r + (\gamma_p \beta_p D_p - D_r)]}{(1-\eta)} \right] (iv) \end{aligned}$$

Appendix 4

To find the total cost per unit of time of the system for case 2 (full backordering with overlapping) of Scenario 2 we put v=1 and s=1 in equation (68), and then we get

$$
\begin{aligned}
&C\left(m,n,\gamma_r,\gamma_p,\theta_r,\theta_p\right)\\
&\quad=\frac{1}{\left[\left(1-\delta\right)QL+\left(1-\eta\right)RM\right]}\Bigg[2\sqrt{\left(mS_r+nS_p\right)\left(\psi+BC_{pr}\right)}\times\\
&\qquad\left[\left(1-\delta\right)QL+\left(1-\eta\right)RM\right]+c_w\left[D_p\left(1-\gamma_p\beta_p\right)\{nM\times\right.\\
&\qquad\left\{\left(1-\eta\right)\left(1-\delta\right)-\eta\delta\right\}+\eta Y\}+D_r\left(1-\gamma_r\beta_r\right)\{mL\times\\
&\qquad\left.\{\left\{\left(1-\eta\right)\left(1-\delta\right)-\eta\delta\right\}+\delta X\}\right]+c_rD_r\left[X+mL\left\{\left(1-\eta\right)\times\right.\right.\\
&\qquad\left.\left.\left(1-\delta\right)-\eta\delta\right\}\right]+c_pD_p\left[vY+nM\left\{\left(1-\eta\right)\left(1-\delta\right)-vs\eta\delta\right\}\right]\Bigg].\qquad(v)
\end{aligned}
$$

References

Alamri, A.A. (2011) Theory and methodology on the global optimal solution to a general reverse logistics inventory model for deteriorating items and time-varying rates. *Computers & Industrial Engineering*, **60** (2), 236–247.

Bazan, E., Jaber, M.Y., Zanoni, S. (2016) A review of mathematical inventory models for reverse logistics and the future of its modeling: An environmental perspective. *Applied Mathematical Modelling*, **40** (5-6), 4151–4178.

Dobos, I., Richter, K., (2003) A production/recycling model with stationary demand and return rates. *Central European Journal of Operations Research*, **11** (1), 35–46.

Dobos, I., Richter, K. (2004) An extended production/recycling model with stationary demand and return rates. *International Journal of Production Economics*, **90** (3), 311–323.

Dobos, I., Richter, K., (2006) A production/recycling model with quality consideration. *International Journal of Production Economics*, **104** (2), 571–579.

El Saadany, A.M.A., Jaber, M.Y., Bonney, M. (2013) How many times to remanufacture? *International Journal of Production Economics*, 143, 598–604.

Hasanov, P., Jaber, M.Y., Zolfaghari, S. (2012) Production remanufacturing and waste disposal model for the cases of pure and partial backordering. *Applied Mathematical Modelling*, **36** (11), 5249–5261.

Jaber, M.Y., El Saadany, A.M.A. (2009) The production, remanufacture and waste disposal model with lost sales. *International Journal of Production Economics*, **120** (1), 115–124.

Nahmias, N., Rivera, H. (1979) A deterministic model for a repairable item inventory system with a finite repair rate. *International Journal of Production Research*, **17** (3), 215–221.

Richter, K. (1996a) The EOQ repair and waste disposal model with variable setup numbers. *European Journal of Operational Research*, **95** (2), 313–324.

Richter, K. (1996b) The extended EOQ repair and waste disposal model. *International Journal of Production Economics*, **45** (1–3), 443–447.

Richter, K. (1997) Pure and mixed strategies for the EOQ repair and waste disposal problem. *OR Spectrum*, **19** (2), 123–129.

Richter, K., Dobos, I. (1999) Analysis of the EOQ repair and waste disposal model with integer setup numbers. *International Journal of Production Economics*, **59** (1–3), 463–467.

Schrady, D.A. (1967) A deterministic inventory model for repairable items. *Naval Research Logistics Quarterly*, **14** (3), 391–398.

Singh, N., Vaish, B., Singh, S.R. (2012) An economic production lot-size (EPLS) model with rework and flexibility under allowable shortages. *International Journal of Procurement Management*, **5** (1), 104–122.

Singh, S.R., Saxena, N. (2012) An optimal returned policy for a reverse logistics inventory model with backorders. *Advances in Decision Sciences*, Article ID 386598, 21 pages.

Singh, S.R., Sharma, S. (2013a) A global optimizing policy for decaying items with ramp-type demand rate under two-level trade credit financing taking account of preservation technology. *Advances in Decision Sciences*, Article ID 126385, 12 pages.

Singh, S.R., Sharma, S. (2013b) An integrated model with variable production and demand rate under inflation. International Conference on Computational Intelligence: Modelling, Techniques and Applications (CIMTA-2013). *Procedia Technology*, 10, 381–391.

Singh, S.R., Sharma, S. (2014) Optimal trade-credit policy for perishable items deeming imperfect production and stock dependent demand. *International Journal of Industrial Engineering Computations*, **5** (1), 151–168.

Singh, S.R., Sharma, S. (2016) A production reliable model for deteriorating products with random demand and inflation. *International Journal of Systems Science: Operations & Logistics*, DOI: 10.1080/23302674.2016.1181221.

Teunter, R.H. (2001) Economic ordering quantities for recoverable item inventory systems. *Naval Research Logistics*, **48** (6), 484–495.