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Transformation into anti-manipulation method in voting. Changes in properties*

by

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Abstract: This paper examines the properties of the anti-manipulation method in voting. Such a method can be used by committees and similar bodies to ensure that votes reflect genuine preferences. The anti-manipulation method is based on the Borda Count and discourages strategic voting by excluding scores that deviate excessively from the mean. The method does not eliminate strategic voting but diminishes the motivation to apply it. We compare the properties of the Borda Count and the anti-manipulation method. The properties, which are most often found in the literature, were chosen for comparison. Thus, the following properties are examined: consistency, vulnerability to the no-show paradox, vulnerability to the subset choice condition, homogeneity, monotonicity, and vulnerability to the reversal bias paradox as well as the Condorcet winner and loser paradoxes. The anti-manipulation method fails to satisfy most of these properties. A real data example, the voting of a certain jury, is used as a counterexample in most cases.

Keywords: voting, manipulation, strategic voting, Borda count, properties, classical music competitions

1. Introduction

This paper examines voting in committees and similar bodies. It is observed that voters sometimes vote strategically in order to increase the probability that their favorite candidate will win, such votes being contrary to or at least inconsistent with their genuine preferences. Only the dictatorial method is immune to manipulation (see Gibbard, 1974; Satherwaitte, 1975). Some methods of

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determining the voting outcome are less vulnerable to this particular kind of manipulation than others.

The present study has its origin in an analysis of the voting methods employed by the juries in classical music competitions. A variety of voting methods are used, but certain methods, based on the Borda count, can be singled out. Sometimes a straight Borda count is used. In such situations some strategic voting is possible. A voter can increase the probability of his/her favorite candidate winning by giving a lower score to a competitor. The Borda count was used in the finals of the XV International Henryk Wieniawski Violin Competition, held in Poznań, Poland, in 2016. During this competition, one of the main Polish newspapers wrote about a "war of jurors". Having analyzed the results of this particular competition, Kontek and Sosnowska (2020) devised an anti-manipulation method of their own. While this method cannot prevent manipulation, it may discourage strategic voting. The method involves excluding voters whose scores deviate excessively from the mean. The algorithm for computing the winner was published in Ramsza and Sosnowska (2020) and is applied in the computations, whose results are presented in this paper. The analysis of juror cliques in the Wieniawski competition, which is based on social networks theory, can be found in Sosnowska and Zawislak (2019). It must be noted that excluding voters constitutes an intervention into the electorate. Such interventions are, however, used in music competitions (see the last Eurovision Competition, where votes of the countries suspected of manipulation were replaced by those for the region). The method, proposed by Kontek and Sosnowska was used during the Chopin Competition for Amateurs (2021) and mentioned in the magazine of the International Chopin Competition in 2021.

The present paper compares the properties of the Borda Count and the anti-manipulation method, based on it. These properties were chosen by axiomatization of the Borda count and scoring methods (Young, 1974, 1975), and those most often discussed in the literature, see Nurmi (2004), and Felsenthal and Nurmi (2017, 2018, 2019), namely:

consistency, vulnerability to the no-show paradox, vulnerability to subset choice condition, homogeneity, monotonicity, vulnerability to reversal bias paradox and to the Condorcet winner and loser paradoxes.

The literature on each of these properties is cited here whereever applicable. The most applied counterexample in this paper is the one based on the Wieniawski competition. It is very important that the counterexample comes from reality. In computational social choice, such real-life counterexamples are much more powerful than sophisticated counterexamples that never actually arise.

The paper is constructed as follows. Section 2 presents the anti-manipulation method. Section 3 describes the Wieniawski competition. Sections 4–10 examine consistency, vulnerability to the no-show paradox, vulnerability to the subset choice condition, homogeneity, monotonicity, vulnerability to the reversal bias paradox and the Condorcet winner and loser paradoxes. In Section 11 other methods of diminishing strategic voting are presented. Section 12 contains the conclusions.

2. The anti-manipulation method

Kontek and Sosnowska (2020) presented a method of voting, which is meant to discourage the potential strategic voting. The method is based on the Borda Count and was devised after having observed the actual voting of the members of the jury during the XV Wieniawski violin competition in Poznan in 2016. The respective observations are presented in Section 3 of this paper. The method designed can be used for voting in committees, e.g., exactly in the juries of the classical music competitions.

The method proposed is constructed as follows. Assume that there are k voters (say, members of the jury) and n alternatives (say, participants of the competition). First, the voters vote as in the Borda Count, i.e., they order the alternatives from the most preferred one (n points assigned) to the least preferred one (1 point assigned). The mean score over the voters for each of the alternatives is then computed. This produces altogether $k+1$ vectors: a vector of the scores for each of the k voters and a vector of the means for each of the alternatives. The value of "distance" of each vector of scores for the individual voters from the vector of means is then calculated. At this point, 20% of jurors, ranked starting from those, whose scores deviate most from the vector of means, are eliminated. Finally, the mean of the scores from the remaining voters is calculated for each of the alternatives. The alternative with the highest mean, obtained in this manner, is the winner. The 20% threshold is justified by the mathematical computations, which are provided in Kontek and Sosnowska (2020).

In the above procedure the Manhattan distance is used. Voters are being eliminated according to the following principles. If 20% of the voters is not an integer number, then the nearest integer not exceeding 20% is used. The voters are then sorted in the descending order of the distance values of their score vectors from the mean vector. If, after eliminating the integer component of the 20% of voters, whose differences are the biggest ones, there are m voters with the same distance from the vector of means, then these voters are not eliminated, but the proportion of 1-1/m of their scores are used when calculating the mean of the voters ultimately taken into account. The following example illustrates the method.

Example 1 There are 17 jurors. Jurors and distances of their vector of scores from the mean are presented in Table 1.

Table 1. Distance of jurors' vector of scores from the mean

Jurors	Distances from the mean	Weights of scores
J1	10	0
J2	9	0
J3	8	2/3
J4	8	2/3
J5	8	2/3
J6	7	1
J7	7	1
J8	6	1
J9	6	1
J10	5	1
J11	5	1
J12	4	1
J13	4	1
J14	3	1
J15	3	1
J16	3	1
J17	3	1

Source: Ramsza and Sosnowska (2020)

In this particular case (17-person jury) 20% of jurors is 3.4, of which the whole part is equal 3. Three jurors should therefore be eliminated. We eliminate jurors *J1* and *J2*, whose scores were distanced from the mean by 10 and 9, respectively), and partly eliminate jurors *J3, J4* and *J5*, each with a distance of 8. Thus, according to the principles of the method, 1/3 of weight of each juror *J3, J4*, and *J5* is eliminated, meaning that their scores are multiplied by 2/3. The vector of weights is now: (0, 0, 2/3, 2/3, 2/3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1).

Kontek and Sosnowska (2020) described an experiment, in which they analyzed the manipulability level that they had defined by specifying the method. In particular, the anti-manipulation method devised was compared with the trimmed mean method. In the trimmed mean method, a percentage of the lowest and highest scores was removed for each juror. The Borda method, the trimmed mean rule, and the anti-manipulation method were employed and their manipulation levels were compared. The manipulation level was the lowest for the anti-manipulation method.

There are some similarities between the anti-manipulation method and the trimmed mean method. In both methods, some of the appreciations are removed. In the trimmed mean method, for each contestant, the extreme jurors' appreciations are removed. So, there may be removed appreciations from different voters for different contestants. In the anti-manipulation method, all of the scores from the selected voters are removed. Appreciations of the voters, producing altogether extreme appreciations, are removed for all contestants. The trimmed mean method is also considered to be a method, which protects against manipulation (see, e.g., Louis, Nunez and Xefteris, 2023).

In the paper by Kontek and Sosnowska (2020) a certain manipulation index was defined. The index was computed for some voting experiments and the conclusion therefrom was that it was lower for the anti-manipulation method than for the trimmed mean method.

3. XV International Henryk Wieniawski Violin Competition

Table 2. Voting of jurors in the finals of the Wieniawski 2016 competition

Contestants	Jurors										
	J1	*J2*	*J3*	*J4*	*J5*	*J6*	*J7*	*J8*	*J9*	*J10*	*J11*
A	7	3	2	7	7	4	3	7	7	7	7
B	4	7	7	2	2	7	7	2	5	6	5
C	5	5	5	3	6	6	5	5	6	1	6
D	3	6	4	5	1	5	4	4	3	5	1
E	1	4	6	1	3	3	6	3	4	3	4
F	6	2	1	6	4	2	1	6	1	2	2
G	2	1	3	4	5	1	2	1	2	4	3

Source: own calculations based on data obtained from the Organizing Committee

The XVth International Henryk Wieniawski Violin Competition (the Wieniawski Competition) was held in Poznań, Poland, in 2016. The competition consisted of three stages and the finals. We analyse the results for the finals. There were 7 contestants in the finals. The Borda Count (in the reversed form, i.e., 1 was the highest score and 7 was the lowest) was used in the final. The voting of jurors in the finals is presented in Table 2. The scores in the table are converted to the standard form of the Borda Count (i.e., 7 being the highest score and 1 the lowest). These two forms are, of course, isomorphic.

The contestants are denoted by the letters $A - G$ and the jurors by *J1-J11*. Contestants A and B were the two favorites. Jurors *J4, J5* and *J8* gave the highest score (7) to contestant A and a truly low score of 2 to contestant B.

Conversely, jurors *J2, J3* and *J7* gave the score of 7 to contestant B and scores 2 or 3 to contestant A. This is why the Polish daily *Gazeta Wyborcza* wrote about a "war of jurors" and in Poland's leading music journal, *Ruch Muzyczny*, a paper by Januszkiewicz and Chorościak (2016) mooted the possibility that jurors had formed cliques.

Contestant A won the competition. If the anti-manipulation method had been employed, contestant B would have won. Contestant A is also the Condorcet winner. Therefore, the anti-manipulation winner is not necessarily the Condorcet winner.

4. Consistency

Consistency is one of the axioms in Young's axiomatization of the Borda Count, see Young (1974), and, more generally, of the scoring methods, Young (1975). Consider two disjoint sets of voters, V' and V'', and a social function f, defined on the sets of profiles over $V', V'', V = V' \cup V''$, respectively. Let w' be a profile over V', w'' a profile over V'' and (w', w'') a profile over V. The function f is consistent if the following condition holds:

$$\text{if } f(w') \cap f(w'') \text{ is nonempty, then } f(w') \cap f(w'') = f(w\text{''},w\text{''}).$$

Ramsza and Sosnowska (2020) show that the anti-manipulation method is not consistent. They find the respective sets V' and V'' in the finals of the Wieniawski Competition using a special computer program written in R. Let V' ={*J7, J9, J11*}, V'' ={*J1, J2, J3, J4, J5, J6, J8, J10*}, the winner over V is B, the winner over V' and V'' is A.

So, the Borda Count is consistent, while the anti-manipulation method is not consistent. Another proof of the inconsistency of the anti-manipulation method is provided in the next Section 5.

5. The No-Show Paradox

The no-show paradox appears when abstaining from voting is more advantageous than voting according to one's genuine preferences, see Fishburn and Brams (1983) or Woodall (1994). Methods that are invulnerable to the no-show paradox are said to satisfy the so-called participation criterion. The Borda Count is invulnerable to the no-show paradox, see Felsenthal and Nurmi (2018).

The anti-manipulation method is vulnerable to the no-show paradox. Consider, for the here analysed Wieniawski Competition, the case where juror *J5* abstains from voting. Jurors, *J1-J4* and *J6-J11* vote, and contestant A is the winner. Contestant A is juror *J5*'s preferred contestant, so *J5* gets a better result by abstaining than by voting, in which case B wins, which is worse than A for juror $J5$.

Note the following consistency counterexample. $V' = \{J5\}$, $V'' = \{J1, J2, J3, J4, J6, J7, J8, J9, J10, J11\}$. Contestant A is the winner over V' and V'', but not over $V = V' \cup V''$. This discussion may be applied to any example of the no-show paradox, so that if the method is vulnerable to the no-show paradox, it is not consistent.

Hence, the Borda Count is invulnerable to the no-show paradox and the anti-manipulation method is vulnerable to it.

6. The Subset Choice Condition

A voting method fulfils the subset choice condition if the winner, x, is the winner of every subset of the set of alternatives, of which x is a member and where voter preferences are unchanged, see Fishburn (1974). This principle is also known as the "heritage principle", see Aizerman and Malishevski (1981), and "property alpha", see Sen (1970). Further information on the subset choice condition can be found in Nurmi (1987) and Felsenthal and Nurmi (2019).

Consider the Wieniawski Competition data here considered and a subset of the set of all the contestants, in which A and B are the only members. When the anti-manipulation method is applied to this subset, the winner is contestant A, even though contestant B is the overall winner. Therefore, the anti-manipulation method does not satisfy the subset choice condition. Nor does it satisfy the independence of irrelevant alternatives criterion.

Consequently, the Borda Count fulfils the subset choice condition, but the anti-manipulation method does not fulfil it.

7. Homogeneity

A voting method is homogeneous if multiplying the preferences of each voter by a constant factor (i.e., maintaining the proportions of preferences) preserves the winner, see Fishburn (1977) and Nurmi (2004). The Borda Count is homogeneous. The anti-manipulation method is not homogeneous. For example, if the preferences of the Wieniawski competition jurors had been multiplied by 20, contestant A would have won instead of contestant B.

However, when the Wieniawski competition preferences are multiplied by 10, contestant B remains the winner. It can be shown that the anti-manipulation method is not only not consistent, but that it exhibits "strong inconsistency". The sets of voters V' and V'' are identical, as the profiles w' and w''. Some methods are inconsistent but not "strongly inconsistent". The Copeland method (points for comparing pairs; 1 for winning, 1/2 for a tie, 0 for losing) is inconsistent but not strongly inconsistent. A non-homogeneous method is inconsistent. Let $J1, ..., Jk$ form a jury and A be the winner of their voting. Let r be such a number that multiplication of a jury by r gives the smallest multiplication yielding a winer different from A. Then V' is the jury, V'' is the $r-1$ multiplication of the jury. If so, A is the winner for V' and V'', but is not the winner for $V' \cup V''$, this being the r multiplication of the jury.

Hence, the Borda Count is homogeneous, while the anti-manipulation method is not homogeneous.

8. Monotonicity

A method is monotonic if an improvement in a winner's ranking, *ceteris paribus*, does not make another contestant the winner. The Borda Count is monotonic, see Fishburn (1977, 1982). By *ceteris paribus* it meant that the order of the remaining contestants is not changed. The anti-manipulation method is not monotonic. The results for the Wieniawski competition finals serve as a counterexample. Assume Juror *J5* changes his score for the winner, contestant B, from 2 to 5, while preserving the order of the scores assigned to the remaining contestants. The changes are shown in brackets in Table 3. This change would make contestant A the winner.

Table 3. Lack of monotonicity in the case of the Wieniawski Competition finals

Contestants	Jurors										
	J1	*J2*	*J3*	*J4*	*J5*	*J6*	*J7*	*J8*	*J9*	*J10*	*J11*
A (winner)	7	3	2	7	7 (7)	4	3	7	7	7	7
B winner	4	7	7	2	2 (5)	7	7	2	5	6	5
C	5	5	5	3	6 (6)	6	5	5	6	1	6
D	3	6	4	5	1 (1)	5	4	4	3	5	1
E	1	4	6	1	3 (2)	3	6	3	4	3	4
F	6	2	1	6	4 (3)	2	1	6	1	2	2
G	2	1	3	4	5 (4)	1	2	1	2	4	3

Source: own calculations.

So, the Borda Count is monotonic, and the anti-manipulation method is not monotonic.

9. Reversal Bias

Reversal bias, also known as the preference inversion paradox, occurs when the same alternative is ranked first by a certain profile of rankings, called the initial profile, and by the profile of reversal rankings, see Saari and Barney (2003). The Borda Count is immune to reversal bias.

The anti-manipulation method is immune to reversal bias. If the preferences are inverted, the new vector of means is the inverted initial vector of means. The distances between the inverted vectors of the score and the inverted vector of means remain unchanged. This results in the same voters being eliminated. The preferences of the remaining voters are the inverted preferences of the initial remaining voters. The Borda Count is immune to reversal bias, so it cannot give the same result for the initial profile and the profile of reversed rankings. The profiles are constructed for the set of remaining jurors.

So, both the Borda Count and the anti-manipulation method are immune to reversal bias.

10. Condorcet winner and loser paradoxes

It has been shown in Section 2 of this paper that the Condorcet winner is not necessarily the anti-manipulation winner. This illustrates the Condorcet winner paradox. Moreover, however, the Condorcet loser may be the anti-manipulation winner. This property, though, is not fulfilled for the Borda Count method, see Nurmi (2004).

The anti-manipulation method makes the Condorcet loser the winner. This is shown in Table 4.

Table 4. The Condorcet loser is the anti-manipulation winner

10 jurors	8 jurors	7 jurors	4 jurors
D	B	C	D
A	C	A	C
B	A	B	A
C	D	D	B

Source: Nurmi (2004)

The profile of preferences is presented in Table 4, following Nurmi (2004). Contestant D is the Condorcet loser and the anti-manipulation winner. The Condorcet loser paradox therefore occurs.

So, both the Borda Count and the anti-manipulation methods are vulnerable to the Condorcet winner paradox. The anti-manipulation method is vulnerable to the Condorcet loser paradox, while the Borda Count method is not vulnerable to it.

11. Comparison with other methods which may diminish manipulation

There are two main methods, which diminish the role of extreme scores, the trimmed mean method and winsorizing.

In the trimmed mean method some extreme values are removed. In the classical music competitions, the Olympic Mean is usually applied, where the lowest score and the highest score are removed. This method was used in XIV and XVI editions of the International Henryk Wieniawski Violin Competition.

In winsorizing, the extreme values are limited to some distance from the mean. This method was used in the International Fryderyk Chopin Piano Competitions. The correcting procedure was applied in such a way that the scores lower than the mean by more than a were raised to mean minus a, and scores higher than the mean by more than a were reduced to the mean plus a. The number a is a parameter, different in different competitions, usually dependent on the scale of possible scores. In the XVI Chopin Competition a was equal to 10, 8, 6, and 5, depending on the stage of the competition. In competitions nos. XVII and XVIII a was equal to 2 or 3, again depending on the stage of competition.

As it is well known, the result of the competition depends not only on jurors' preferences, but also on the method of voting and vote aggregation applied. Let us study the following example.

Example 2 Voting preferences, presented in columns *J1* – *J6* of Table 5 are considered. In the 9^{th} column, the sum of scores computed by the Olympic Mean is presented. The Olympic Mean is obtained after dividing this sum by 4. In the 10^{th} column, the sum of scores computed by winsorizing with parameter $a = 3$ is presented. The winsorizing mean is obtained after dividing this sum by 6. The order of the means is the same as the order of the sums. So, using the Olympic Mean contestant V wins, using the winsorizing contestant W wins, and applying the anti-manipulation method (where juror *J3* is removed) contestant T wins. So, jurors' preferences are the same, but each voting rule leads to different results.

Table 5. Dependence of results of voting on the chosen rule of voting

Contestants	Jurors						Mean	Sum Olympic mean	Sum Winsorizing (3)
	J1	*J2*	*J3*	*J4*	*J5*	*J6*			
V	61	61	60	60	56	57	59.17	238	355.17
W	54	54	61	60	62	62	58.83	237	356.32
T	62	62	40	58	58	58	56.33	236	351.09
Distance from the mean	12.33	12.33	19.33	3.67	8.00	7.00			

Source: own calculations

12. Conclusions

Let us summarize our considerations in Table 6. In this table "+" means that the method fulfils a property or is non-vulnerable to a paradox, while "–" means that the method is vulnerable to a given paradox or does not fulfil a given property.

Table 6 shows that the anti-manipulation method does not fulfil most of the properties, which are fulfilled by the Borda Count. So, the deep intervention into the electorate, which leads to formation of the anti-manipulation method from the Borda Count changes almost all properties. After such an intervention, we obtain a method, which discourages strategic voting, but altogether has worse properties. It is implied by the goals of the voting body, which method it would prefer to use.

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Table 6. Comparison of the properties of the Borda Count and the anti-manipulation method

No.	Property	Borda Count	Anti-manipulation method
1	Consistency	+	-
2	No-show paradox	+	-
3	Subset Choice Condition	-	-
4	Homogeneity	+	-
5	Monotonicity	+	-
6	Reversal Bias	+	+
7	Condorcet Winner Paradox	-	-
8	Condorcet Loser Paradox	+	-

Source: own investigations

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