

Neural network-based allocation and self-improved
firefly-based optimal sizing of fuel cells in distributed
generation systems

by

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Abstract: The notion of Distributed Generation (DG) refers to the production of power at the level of consumption. Production of energy on-site, instead of offering it centrally, reduces the cost, internal dependencies, difficulties, inefficiencies, and risks that are related to transmission and distribution systems. In case DG is realized with fuel cells, several issues exist in respect to allocating and sizing of these fuel cells in the system. For solving those issues, dual stage intelligent technique is employed in this paper. First, the Neural Networks (NN) technique is adopted for determining the required location to place the fuel cells. Secondly, an enhanced version of Self Improved Fire-Fly (SIFF) algorithm is adopted for finding the optimal size of the fuel cells. The implemented technique is simulated in four IEEE benchmark test bus systems, and the respective performance analysis along with statistical analysis serves for validation purposes. The here proposed technique is compared with six other known algorithms, namely Particle Swarm Optimization (PSO), Firefly (FF) algorithm, Artificial Bee colony (ABC) algorithm, Improved Artificial Bee colony algorithm (IABC), Genetic Algorithm (GA) and Global Search Optimizer (GSO). The results obtained from the comparative analysis show the enhanced performance of the proposed mechanism.

Keywords: DG system, fuel cells, location, sizing, multiple objectives, firefly algorithm, self-improved firefly algorithm

1. Introduction

The improvement in DG systems has become a matter of high concern due to the environmental as well as economic causes (see, e.g., Vaziri et al., 2011). The capacities of the DG systems range from few kW to 50 MW (Moradi and Abedini, 2012), and they are developed and utilised largely owing to the improvement in the small-scale green generation methods (Siemens, 2015). The

kinds of implemented DGs depend on the terminal properties, including injection of the real power, the real and reactive power, real but consumed reactive power, etc. These, and other characteristics influence the potential design of a concrete system. By designing and implementing appropriate installations, the DGs may gain broader applications, considering the requirements, concerning reliability, power quality, shorter construction schedules, low cost of maintenance, peak shaving power and loss reduction (Brown et al., 2001; Mendez et al., 2006; Arya and Koshti, 2012). One of the essential design issues is the assignment of the units of DG. Hence, several optimization techniques have been adopted, whose purposes can be classified into optimal location and optimal size determination (Subramanyam, Ram and Subrahmanyam, 2016, 2018; Rao, Ram and Subrahmanyam, 2017, 2018). The analytical, heuristic and deterministic methods are employed for optimization. The techniques employed include Tabu search (Lee et al., 2002; Gandomkar, Vakiliyan and Ehsan, 2005), ant colony optimization (Foliage and Haghifam, 2007), particle swarm optimization (Raj et al., 2008), as well as genetic algorithms (Cali and Pillow, 2001; Aghtaie and Deghanian, 2011). With time, combinations of various kinds of algorithms have been applied to solve the problems in question (see, e.g., Moradi and Abedini, 2012). The need for advanced optimization methods to be used results from the specific features of the DG units, such as fluctuations at the circuit level, security-wise vulnerability, complexity, location and sizing issues, voltage instability, and the level of penetration. These characteristics have to be considered, and many experiments are going on for finding solutions, which are optimal.

The paper contributes to the issue of optimal location and sizing of the fuel cells in DG systems. The desired location is determined using the NN (neural network) algorithm, and the proposed SIFF (self-improved firefly) algorithm determines the optimal sizing. The proposed method provides better locations and sizing when compared with the other techniques used for similar purposes. This paper is organized as follows. Section 2 surveys the related work and assesses the work done on this topic. Section 3 depicts the design of the power system with optimal placement of distributed fuel cells. Section 4 is devoted to the verification of the NN based optimal location and SIFF-based sizing of distributed fuel cells. Results and discussions are contained in Section 5.

2. Literature review

2.1. Related work

In 2013, Mojarrad et al. (2013) proposed a novel algorithm, involving a fuzzy satisfying method, meant to solve the problem of multi-objective optimal placement in the distribution network and DG units sizing. The there presented improved version of the approach is associated with the hybrid modified shuffled frog leaping algorithm (MSFLA), whose the target is to minimize the electrical energy loss, pollutants emission, and the energy consumption. To enhance the

search and to find a solution for premature convergence, a hybrid algorithm, linking MSFLA with differential evolution is utilized. The MSFLA is used for optimizing purposes, and differential evolution is applied to maintain the population diversity. In order to verify the optimal placement of the distribution units, an evaluation was carried out using the IEEE69 bus system.

Then, Moradi, Tousi and Abedini (2014) proposed the multi-objective Pareto Frontier Differential Evolution (PFDE) algorithm for finding the optimal solution for the distribution planning issue depending upon the characteristics of the distributed generation (DG) sources. For finding a solution, three factors are considered, namely voltage stability, network voltage fluctuations, and power loss. The multi-objective optimization considers the optimal DG integration problems, location, and sizing of DG. The fitness sharing method was integrated with non-dominated ranking in PFDE technique for maintaining the population diversity. The respective experiment was carried out with the IEEE test system containing 69 buses and 33 buses, and the results show that the proposed methods are better in determining the correct solutions.

Following this, Poornazaryan et al. (2016) carried out the experiments, related to solving of the problems of power loss and voltage consistency. Taking this problem into account, they presented a novel method, which is an enhanced version of the Imperialistic Competitive Algorithm (ICA) that finds a solution for the optimization issues in discrete variables. Hence, they analyzed the network scheduling at various loads, and with these changes, the optimal size and location of the DG system are calculated. The optimal DG system is established using the curve fitting technique and the cuckoo search algorithm. The results were evaluated using IEEE 34 bus and 69 bus test systems.

Kansal, Kumar and Tyagi (2016) developed a hybridization method, depending on an heuristic search technique for detecting the optimal location of various DGs with the goal to minimize the loss of power. The distribution system (DS) size is optimized with the analytical method, and the locations, related to the distribution system are determined with the heuristic PSO method. Here, the mathematical model is used to determine the exact optimal size of the DS. The implemented method was evaluated on the 33 bus and the 69 bus test systems and the results were compared with those of other available methods.

Mohandas, Balamurugan and Lakshminarasimman (2015) used a Chaotic Artificial Bee Colony (CABC) algorithm for resolving the problems of voltage inconsistency and power loss. For reducing the loss of power, it is vital to enhance the sizing and the location of the DG systems. The optimization, related to assigning the location and to sizing is performed with the multi-objective performance index (MOPI) for better voltage consistency. Specifically, constant power load model and other voltage-dependent load models, such as industrial, residential, and commercial, are also considered. The efficiency of the imple-

mented method is confirmed by conducting the experiments in 38-node and the 69-node radial distribution system.

Murty and Kumar (2015) carried out experiments on the DS and compared the power loss sensitivity, power stability index, and developed voltage stability index for determining the optimal sizing and location of the DG in the radial network. The power consistency index is employed for detecting the most problematic bus in the system, causing voltage inconsistency, and the DG is allocated to the most sensitive bus. The method presented was evaluated for its effectiveness against the background of other methods in terms of optimal DG location as well as optimal sizing of the DGs. The experiments were evaluated with the use of the IEEE 12-bus, modified 12-bus, 69-bus and 85-bus test systems.

2.2. Assessment

Localizing the optimal size and point for assigning the fuel cell in a DG system is a non-convex problem. Several bio-inspired optimization techniques (Mojarrad et al., 2013; Kansal, Kumar and Tyagi, 2016; Mohandas, Balamurugan and Lakshminarasimman, 2015; Moradi, Tousi and Abedini, 2014) and algorithms depending on nonlinear programming (Poornazaryan et al., 2016), meant to deal with these problems, have been described in the literature. The analyses encompassed global detecting ability, local identifying ability, the appropriate rate of convergence, capacity of evading from local optima, minimized computing cost, less tuning of parameters, adaptivity with respect real surroundings, etc. Anyhow, it is not possible for any algorithm to cope effectively with all of the above-mentioned issues and variables. The efficiency of the method is determined on the basis of its performance and advantages. The qualities and the challenges, associated with the state-of-the-art methods, considered in the here summarised survey, are illustrated in Table 1.

In the context of this review, it can be said that the frog leaping algorithm (Mojarrad et al., 2013) has several advantages, concerning its computing performance, global search capacity, etc. Anyhow, the requirements, related to adaptivity for the real time performance and the issues of nonlinearity have not been discussed in this work. Further, the method faces the problems caused by premature convergence and it can hardly evade being stuck in a local optimum. The differential evolution method (Moradi, Tousi and Abedini, 2014) is an evolutionary algorithm, employed in various optimization problems. It works well with fewer parameters and is also applicable to high dimensional complex optimization problems and hence it is utilized to optimize the locations of fuel cells. However, it displays instability in convergence. Further, the PSO (see Kansal, Kumar and Tyagi, 2016), which is a classical swarm intelligence technique, with reduced computational complexity, was also proposed.

It should be noted, though, that these algorithms are still subject to prob-

lems in multimodal environments. The application of the ABC algorithm and chaos theory jointly (see Mohands, Balamurugan and Lakshminarasimman, 2015) can lower the significance of the problems caused by the sticking to local optima. They usually try to skip the second order information related to the problem, and, as a result, the rate of convergence is poor. The customized techniques, such as imperialistic competitive algorithm (Poornazaryan et al., 2016) can reduce most of the mentioned problems, but the necessity of multiple initializations and the presence of penalty constraints reduce the performance of the optimization algorithm and the reliability of the solutions.

The here considered issue belongs among the bi-level optimization problems. The fuel cells, constituting the objects of optimization, have to be both optimally located and the problem of sizing has to be appropriately solved, with the preservation of the requirements and limitations of the respective power system. Hence, a well-developed optimization algorithm is required to deal with such complex problem aspects.

3. Problem formulation with optimal placement of distributed fuel cells

3.1. Quality parameters

This section presents the way of calculating the here considered quality parameters of the DG system.

Power loss index (PLI): A system, exhibiting enhanced performance will certainly experience a lower power loss. The power loss, which associated with the DG, is considered to be crucial. Hence, it is treated as an objective for the model, to be minimized, as this is shown in Eqs. (1) and (2). Here, fitness, f_1 , corresponds to minimization of power loss (P_{loss}). The value of P_{loss} is defined through Eq. (2), where N_B denotes the total number of buses, Y_{ij} stands for the magnitude of the ij^{th} element of the bus admittance matrix Y ; V_{mi} is the magnitude of the complex voltage (pu) of the mi bus; then, δ_{mi} is the phase angle of voltage at bus mi , and analogously for δ_{mj} :

$$f_1 = \min(P_{loss}) \quad (1)$$

$$P_{loss} = \sum_{i=1}^{N_B} \sum_{j>1}^{N_B} \{Y_{ij}\} [V_{mi}^2 + V_j^2 - 2V_{mi}V_{mj} \cos(\delta_{mi} - \delta_{mj})] \quad (2)$$

Then, the real and reactive power loss indices can be defined as in Eqs. (3) and (4), respectively. In these equations, the terms P_{LDG} and Q_{LDG} denote the whole real and the reactive power losses of the distribution system after the inclusion of the DG. Notations P_L and Q_L indicate the entire losses in real and reactive power, respectively, occurring due to the reduction in DG. These losses can be

Table 1. Summary of the literature review of state-of the art methodologies

<i>Reference</i>	<i>Adopted methodology</i>	<i>Advantages</i>	<i>Disadvantages</i>
Mojarrad et al. (2013)	Frog leaping	<ul style="list-style-type: none"> - Efficient computations - Good global search capability - Less computational cost 	<ul style="list-style-type: none"> - Non-uniform initial population - Poor convergence ratio. - Limitations in local search ability - Premature convergence - Complex non-linear optimization problems
Moradi, Tousi and Abedini (2014)	Differential evolution	<ul style="list-style-type: none"> - Requires fewer parameters - High computing performance - Applicable to high-dimensional complex optimization problems 	<ul style="list-style-type: none"> - Unstable convergence - Easily drops into regional optimum
Poornazaryan et al. (2016)	Imperialistic competitive algorithm	<ul style="list-style-type: none"> - No need for gradient function in optimization - Ability to deal with different optimization problems. - Good convergence - Global minimum achievement 	<ul style="list-style-type: none"> - Multiple initialization phases with penalty constraints
Kansal, Kumar and Tyagi (2016)	PSO	<ul style="list-style-type: none"> - Fast convergence. - No problem of particle stagnancy - Less computational complexity 	<ul style="list-style-type: none"> - Real value data assumption - Getting stuck at local optimum under multimodal scenarios
Mohandas, Balamurugan and Lakshminarasimman (2015)	Chaos theory and ABC	<ul style="list-style-type: none"> - Easy to implement. - Broad applicability in complex functions - High flexibility - Global optimizer 	<ul style="list-style-type: none"> - Lack of use of secondary information about the problem - Needs new fitness tests - Possibility of losing relevant information

minimized, when the DGs are located and/or sized in an optimum manner.

$$ILP = \frac{[P_{LDG}]}{[P_L]} \quad (3)$$

$$ILQ = \frac{[Q_{LDG}]}{[Q_L]} \quad (4)$$

Voltage Stability Index (VSI): The capacity of the power system to maintain the voltages, which are assigned to numerous network buses, within the allowed ranges, after the occurrence of a disruption, is called voltage stability. Normally, inconsistency is caused due to the incapability of the system of handling the loads with required reactive power (Aghaie and Dehghanian, 2011). The voltage stability index, which is related with the radial DS, is obtained from the power flow method. After the voltages at each bus are considered for the load flow study, the VSI for all the receiving and buses of radial distribution systems can be easily calculated, using Eq. (9). In turn, Eq. (9) is realised by means of Eqs. (5) through (8). Here, m indicates branch number, $I(m)$ is the current of branch m , $V(s)$ and $\delta(s)$ denote the voltage and phase angle at node s , $V(m)$ and $\delta(m)$ are the voltage and phase angle of branch m , while $r(m)$ and $x(m)$ denote the resistance and reactance, respectively, of the branch m . In Eq. (8), $P(m)$ and $Q(m)$ indicate, respectively, the total real and reactive power load fed at appropriate locations of the system.

$$I(m) = \frac{V(s) \angle \delta(s) - V(m) \angle \delta(m)}{r(m) + jx(m)} \quad (5)$$

$$r(m) = \text{Real} [(V_m \angle \delta_m - V_s \angle \delta_s) / I(m)] \quad (6)$$

$$x(m) = \text{Imag} [(V_m \angle \delta_m - V_s \angle \delta_s) / I(m)] \quad (7)$$

$$P(m) - jQ(m) = V^*(m) I(m) \quad (8)$$

$$\left. \begin{aligned} |V(m)|^4 - (|V(s)|^2 - 2P(m)r(m) - 2Q(m)x(m)) |V(m)|^2 \\ + (P^2(m) + Q^2(m)) (r^2(m) + x^2(m)) \end{aligned} \right\} = 0. \quad (9)$$

Let us consider (see Chakravorty and Das, 2001), in the context of Eqs. (5) through (9),

$$b(m) = (|V(s)|^2 - 2P(m)r(m) - 2Q(m)x(m)) \quad (10)$$

$$c(m) = (P^2(m) + Q^2(m)) (r^2(m) + x^2(m)). \quad (11)$$

By using (10) and (11), (9) can be expressed in the form of Eq. (12). Eq. (12) clearly shows that the convergence of load flow in the radial distribution system can take place under the condition provided in Eq. (13).

$$|V(m)|^4 - b(m) |V(m)|^2 + c(m) = 0 \quad (12)$$

$$b(m)^2 - 4.0c(m) \geq 0. \quad (13)$$

Substitution of (10) and (11) in (13) results in the following:

$$\left. \begin{aligned} &|V(s)|^4 - 4.0(P(m)x(m) - Q(m)r(m))^2 \\ &- 4.0(P(m)r(m) + Q(m)x(m))|V(s)|^2 \end{aligned} \right\} \geq 0. \quad (14)$$

Let us assume that

$$VSI(m) = \left\{ \begin{aligned} &|V(s)|^4 - 4.0(P(m)x(m) - Q(m)r(m))^2 \\ &- 4.0(P(m)r(m) + Q(m)x(m))|V(s)|^2 \end{aligned} \right. . \quad (15)$$

The radial distribution systems always function in a stable mode, if Eq.(16), below, is satisfied.

$$VSI(m) \geq 0, \quad \text{for } m = 2, 3, \dots, N_B \quad (16)$$

There are mainly two benefits, which arise from this model.

- (i) Measurement of the whole set of requirements
- (ii) Possibility of attaining maximum speed of calculations in the real-time environments.

If the voltage stability index, related with each node in the network is determined, the calculation of the consistency of voltage, associated with the whole system, can be performed. The node displaying high sensitivity is the one, for which a low value of VSI is observed. Then, a novel index, representing the voltage consistency, associated with the entire network for distribution, can be given as defined in Eq. (17). Note that the network shows a larger instability of voltage range, when $OVSI$ (Overall Voltage Stability Index) takes on a higher value, implying the requirement of minimizing it.

$$f_2 = OVSI = \sum_{m=2}^{NB} [VSI(m)] \quad (17)$$

Voltage Profile: It is clear that the load reduction has a positive effect on the voltage profile of the network, while load increase aggravates it. The definition of the voltage profile with respect to the IVD (voltage profile index) is provided in Eq. (18), where NN represents the number of nodes. This expression assumes the value of 1.03 p.u. and 1.00 p.u. for the radial type of distribution including 38 nodes and 69 nodes, respectively.

$$f_3 = IVD = \max_{i=2}^{NN} \left(\frac{|\bar{V}_{no\,minimal}| - |\bar{V}_i|}{|\bar{V}_{no\,minimal}|} \right) \quad (18)$$

Usually, every individual bus has its own range of voltage, such as ($V_{\min} \leq V_i \leq V_{\max}$). Owing to these technical constraints, a small allowed range of IVD values can be obtained.

Power flow constraints: The equations of power flow, which are nonlinear, are treated as the equality constraints, meant to secure the preservation of the entire real and reactive powers, which are related with DS. Eq. (19) and Eq. (20) represent these flow of power constraints.

$$P_{gni} = P_{dni} - V_{ni} \sum_{j=1}^N V_{nj} Y_{nj} \cos(\delta_{ni} - \delta_{nj} - \theta_{nj}) \quad (19)$$

$$Q_{gni} = Q_{dni} - V_{ni} \sum_{j=1}^N V_{nj} Y_{nj} \sin(\delta_{ni} - \delta_{nj} - \theta_{nj}) \quad (20)$$

Line flow constraints: The power flow at various network locations can change due to the insertion of DGs in the system. The line flow should be maintained well within the permitted range, so that the occurrences of line overloading are avoided. The line flows remain within the allowed ranges, when the IC index, defined here, takes on a value lower than one. If the IC index exceeds unity, the line flows do no longer exhibit the desired control property. Calculation of the IC value can be done using Eq. (21), in which $\overline{S_{ij}}$ denotes the maximum apparent power flow (MVA), associated with the line that connects the buses i and j ; $\overline{CS_{ij}}$ denotes the capacity of the apparent power flow (MVA), related to the lines i and j ; finally, NL denotes the total number of lines.

$$IC = \max_{i=1}^{NL} \left(\frac{|\overline{S_{ij}}|}{|\overline{CS_{ij}}|} \right) \quad (21)$$

The unified model: The optimal solutions are generated with respect to the line flow and the power flow constraints. The unified design assumes as objectives the loss of power, the VSI and the voltage profile (IVD). Thus, the formulation, related to the unified design is as provided in Eq. (3.1), where, λ_1 , λ_2 and λ_3 are weights selected in such a way that $\sum \lambda_i = 1$, β_1 and β_2 are the weighting factors, taken here to be $\beta_1 = \beta_2 = 0.5$, in general, though, $\beta_2 = (1 - \beta_1)$.

$$F^{obj} = \exp \left(\lambda_1 P_{loss} + \lambda_2 OVSI + \frac{\lambda_3}{IVD} \right) + \left\{ \begin{array}{l} [\max(\min(V_i) - V_{\max}), 0] \\ + [\max((V_{\min}) - \min(V_i)), 0] \end{array} \right\} \beta_1 + \{\max(|\min(S)| - |S^{\max}|), 0\} \beta_2 \quad (22)$$

This design objective is supposed to be achieved by the implemented algorithm, leading to the fuel cell location and size optimization.

3.2. Proposed architecture

In the implemented two-stage technique, as outlined in Fig. 1, the ANN model uses the data from the DS and the data on a set of fuel cells to perform optimization of the accurate location, at which the fuel cells are to be placed. After the

determination of the optimal locations of fuel cells by the ANN model, the optimal size, assigned the fuel cells is determined through the use of an improved algorithm, called SIFF algorithm. The results obtained from the SIFF algorithm are evaluated in the light of the quality indices here introduced, namely related to voltage profile, power flow, and stability. For determining the optimal sizing of the fuel cells, the details associated with the optimal location are also considered.

4. The procedure for optimum location and sizing of distributed fuel cells

4.1. The ANN-based optimal location of distributed fuel cells

To determine the location of the fuel cells, the artificial neural network (ANN or NN) model (see Kobayashi, 2017) accesses the training library, which consists of fuel cells that are linked to the corresponding locations. The locations are chosen in such a way that they reduce the power loss of fuel cells before training, using the proposed Self Improved Firefly algorithm, SIFF. Hence the respective model for describing the optimal location can be represented as in Eq. (23).

$$B_d^* = \arg \min_{B_d} P_{loss} \quad (23)$$

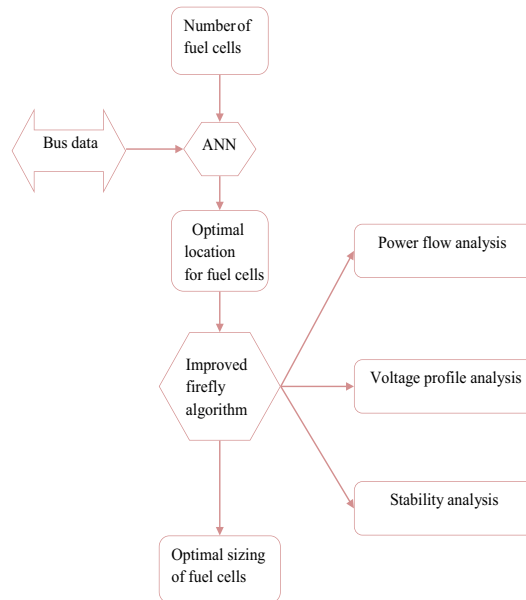


Figure 1. Overview of the DS design with optimal fuel cell allocation and sizing

The here proposed SIFF algorithm intends to solve the problem with the previously defined multi-aspect objective function for some definite configuration, established by the current system data. Here, the optimal locations are identified and the library is built up as $[\bar{F} B_d]$, where \bar{F} represents the fuel cell's number, for the cell, which is supposed to be linked with the system and B_d contains the accurate locations, where the fuel cells are to be linked with the buses. The term \bar{F} is represented by a column vector of $N_f \times 1$ dimensions, addressing N_f fuel cells and B_d is the binary matrix of dimensions $N_f \times N_f$, in which values equal 1 denote the links of fuel cells to a bus, and zeros indicate the absence of fuel cell connections. B_d is set up in a way provided in Eq. (24), where f indicates the link of fuel cell with the f^{th} degree.

$$\sum_{f=1}^{N_f} B_d(f) = \bar{F}(f) \quad (24)$$

The training library is associated with the NN model for the purpose of training, and the model gains the knowledge, related to the library by utilizing the nonlinear function as provided in Eq. (25),

$$B_d(f) = \omega_0 + \sum_{h=1}^{N_H} \frac{1}{1 + \exp(-\bar{F}\omega_h)} \quad (25)$$

where N_H refers to the total number of hidden neurons and the network weights are indicated by ω .

The adopted NN is generally of feed forward character, and the training is done by using the back propagation basis.

4.2. SIFF-based sizing of distributed fuel cells

For determining the optimal sizing of the distributed fuel cells, the here proposed SIFF algorithm has been developed. The original FF (firefly) algorithm (see Yang, 2010; Fister, Fister and Brest, 2013) refers to the spectacular group of insect species. The flashing light of fireflies is a wonderful sight in the summer sky in the temperate and tropical regions. There are almost two thousand kinds of fireflies, and most of them generate rhythmic and short flashes. The flash pattern is specific for definite species. The flashing light is generated by a bioluminescence process, and the actual working of the respective processes is still debated. Anyhow, two basic functions of flashes are: to obtain the adequate concentration of mating partners (interaction) and to attract the desired prey. In addition, flashing serves as a warning mechanism for protection.

In the firefly optimization technique, the intensity of light I at a particular location B , can be chosen as $I(B) \propto f(B)$, while the intensity of light I decreases as the distance p increases, in accordance with the following equation:

$$I(p) = I_0 e^{-\gamma p^2} \quad (26)$$

where I_0 denotes the maximum intensity of light, and the light absorption coefficient is represented by γ .

Hence, an equation, which is same as Eq. (26), can be formulated, to explain the attractiveness, β , where β_0 is the attractiveness at $p = 0$. The attractiveness β and the intensity of light I are similar in some aspects, which explains the similarity of the respective expressions.

$$\beta = \beta_0 e^{-\gamma p^2}. \quad (27)$$

The distance between two fireflies B_i and B_j is represented as shown in Eq. (28), where n indicates the length of the chromosome (the number of parameters to be optimized). The movements of the i^{th} firefly are influenced by the attraction to another firefly, j , which is more attractive.

$$p_{ij} = \|B_i - B_j\| = \sqrt{\sum_{k=1}^{k=n} (B_{ik} - B_{jk})^2} \chi. \quad (28)$$

In this context, the result from Eq. (28) can be applied in Eq. (29), where ε_i is a random number, obtained from the Gaussian distribution.

$$B_{i+1} = B_i + \beta_0 e^{-\gamma p_{ij}^2} (B_j - B_i) + \alpha \varepsilon_i. \quad (29)$$

The mobility of fireflies includes three terms, the present position of the i^{th} firefly, attraction towards other fireflies that are more attractive, and a random walk that includes a parameter, denoted α , which is a random number, whose values range from 0 to 1. When attractiveness $\beta_0 = 0$, the movement is based only on the random walk. The parameter γ has a great influence on the speed of convergence. Even if the parameter γ can have any value in the range of $\gamma \in [0, \infty]$, theoretically; its construction depends on the actual optimization problem. In most cases, it ranges from 0.1 to 10.

The updating process towards the optimized solution can be realised by multiplying the third term of Eq. (29) by a change of fitness value, τ :

$$B_{i+1} = m_i + \beta_0 e^{-\gamma p_{ij}^2} (B_j - B_i) + \alpha \varepsilon_i \times \tau \quad (30)$$

$$\text{where } \tau = \frac{f(t-1) - f(t)}{f(t-1)}. \quad (31)$$

Here, $f(t-1)$ represents the previous fitness function value, while $f(t)$ denotes the current fitness function value.

The description of the proposed method is provided in the following subsequent steps (see also the pseudocode of Algorithm 1 and Fig. 2):

1. The population of fireflies is initially generated.
2. The light intensity of all the fireflies is measured using Eq. (26).
3. The distance of the fireflies is varied corresponding to the attractiveness index.

	Algorithm 1: SIFF-based optimal sizing of distributed fuel cells		
Step1	Initialize the population of fireflies		
Step2	Calculate the intensity of light		
Step3	First iteration, $t = 1$		
Step4	While $t \leq t_{max}$		
Step5		For every i from 1 to n	
			For every j from 1 to n
			Move firefly i towards j in a certain dimension
			Vary distance with respect to attractiveness.
			Evaluate the intensity of light in the new solutions.
Step6	Determine the value of τ using Eq. (31)		
Step7	Update the best position using Eq. (30)		
	$t = t + 1$		
	Return B^*		

4. The best light intensity is measured and updated using Eq. (27).
5. The change in the fitness value is evaluated using Eq. (31).
6. Iterations are continued until SIFF algorithm obtains the best (optimal) solution within a given number of iterations.

5. Results and discussion

5.1. Simulation procedure

The experimentation, regarding the location and sizing of fuel cells, was performed in MATLAB. The simulation process was carried out in four IEEE standard bus systems, namely, 9 bus test system, 12 bus test system, 33 bus test system, and 69 bus test system.

The computational capability to determine the locations and the sizing of fuel cells using the implemented SIFF algorithm was verified through the analysis focussed on convergence. It was meant to check the convergence property of the proposed algorithm for the entire iterative process. Analysis was also conducted with regard to the reduction of cost, achieved with each of the optimization algorithms, used in the comparative study. In other words, the cost models were accounted for, related to power loss, voltage stability and voltage profile, as well as the final cost model, i.e. the combination of the aforesaid cost models. The total number of iterations for the here proposed algorithm was assigned as 100. The results, obtained with this algorithm, were compared with those for six other algorithms, namely GA (Martinez-Cañada et al., 2017), PSO (Chen et al., 2017), ABC (Fister and Brest, 2013), FF (Yang, 2010; Fister, Fister and Brest, 2013), IABC (Subramanyam, Ram and Subrahmanyam, 2018), and GSO (Basu, 2015). The results, obtained in the respective experiments, are

analyzed in the following sub-sections.

5.2. Convergence analysis

The convergence analysis of the cost function values for the sizing of the fuel cells in a DG system for IEEE 33 bus system using SIFF algorithm is illustrated in Fig. 4 (a). It can be easily seen in this figure that all the cost functions are getting reduced with the increase in the number of iterations. At iteration number 100, the proposed technique is better than FF by 25% and than ABC by 90%. This demonstrates that the proposed technique reduces the cost function better than the other techniques. Similarly, the course of the cost function along the iterations for IEEE 12 bus system is illustrated in Fig 4 (b). Again, it can be noted that all the cost functions decrease with the increase in the number of iterations. However, by the iteration number 40, the proposed technique has reduced the cost function more than GSO by 21%, than FF by 1%, than GA by 71%, than ABC by 1%, and than PSO by 36%/. The courses of the cost functions for the IEEE 9 bus system are shown in Fig. 4 (c). Here also it is seen that all the techniques have the capacity to reduce the cost along iterations. Anyhow, at the 60th iteration, the proposed method is better than IABC by 15.7%, than ABC by 55.7%, than GA by 55%, and than FF by 2.8%. Hence, it can be concluded that the implemented SIFF technique has the capability to reduce the cost function better than the existing techniques through the optimal sizing of the fuel cells in the DG system.

The effects of the optimum sizing of fuel cells for the IEEE 69 bus system in terms of the cost function are shown in Fig. 4 (d). The cost function values for all methods are reduced along the number of iterations. Yet, for instance, at iteration 100, the proposed technique yields the cost function reduction bigger than IABC by 75%, than PSO by 80.9%, than FF by 86.6%, than ABC by 87%, and than GA by 87.8%. So, the previously reached conclusion appears to hold in this case, as well.

5.3. Performance analysis

This section is devoted to performance analysis of sizing of fuel cells in DG system using the SIFF algorithm. Here the three objective functions, namely P_{loss} , OVS and IVD , are analysed individually along with the total objective functions, the respective results being provided in Tables 2 though 5 for the IEEE 33 bus system, IEEE 12 bus system, IEEE 9 bus system, and IEEE 69 bus system, respectively.

In Table 2, for the first objective function, it can be seen that the implemented technique exhibits better performance by 18.2% more than PSO, by 16.6% more than GSO, by 37.07% more than GA, by 3.8% more than ABC, by 9.8% more than FF, and by 16.03% more than IABC. Similarly, for the IEEE 12 bus system, as this is shown in Table 3, the SIFF technique is better than PSO by 6.64%, than GSO by 2.65%, than GA by 18.65%, than ABC by 20.27%, than FF

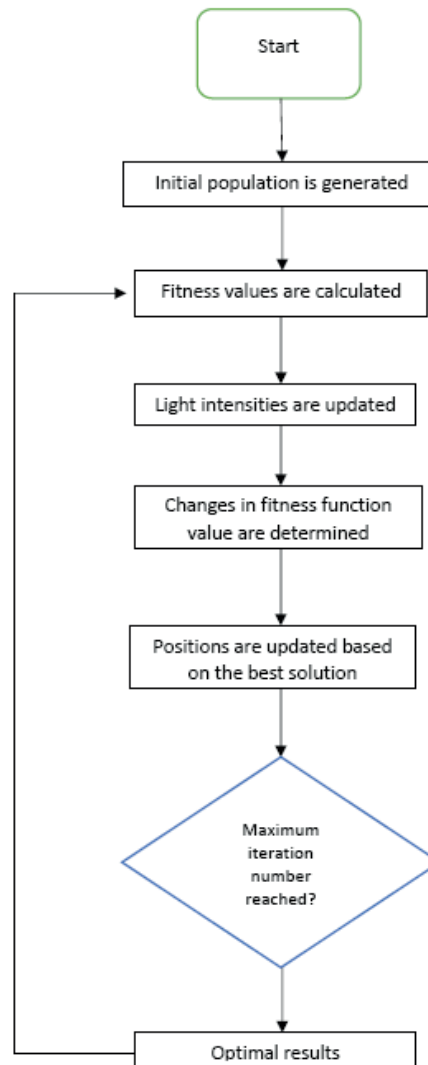


Figure 2. Flowchart describing the sizing of fuel cells using SIFF algorithm

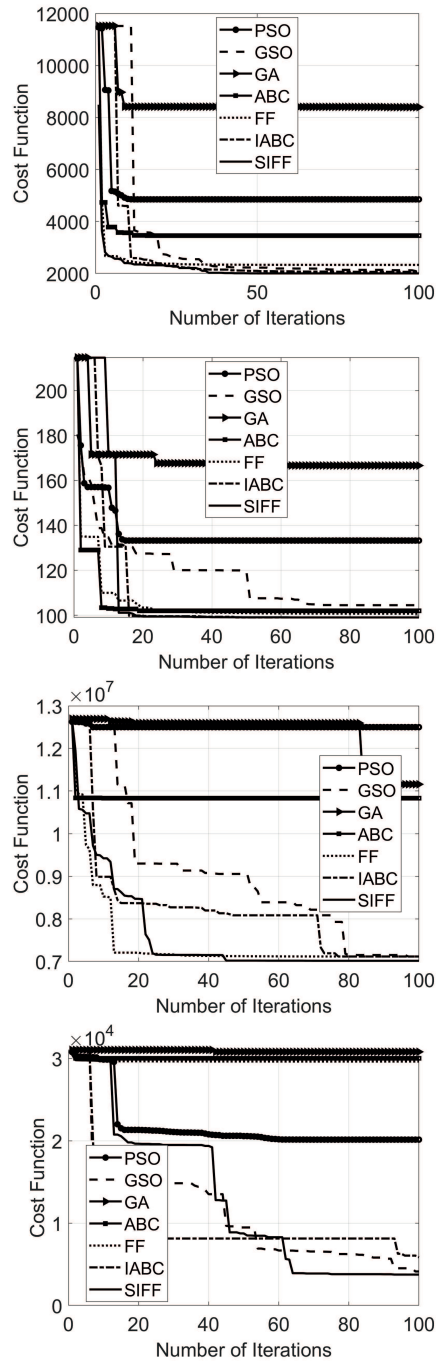


Figure 3. Convergence analysis for allocating fuel cells using SIFF algorithm (a) IEEE 33 bus system (b) IEEE 12 bus system (c) IEEE 9 bus system (d) IEEE 69 bus system

by 1.18%, and than IABC by 0.31%. In the case of IEEE 9 bus system, the simulation results, provided in Table 4, show that SIFF is better than PSO by 11.28%, than GSO by 1.84%, than GA by 7.1%, than ABC by 8.03%, than FF by 0.39%, and than IABC by 1.81%, respectively. The results, obtained for the IEEE 69 bus system, as provided in Table 5, show that the proposed SIFF method is better than PSO by 61.06%, than GSO by 5.48%, than GA by 53.79%, than ABC by 94.15%, than FF by 88.2%, and than IABC by 10.04%, respectively. Thereby, the superior quality of the new technique appears to be demonstrated.

Table 2. Performance analysis for all the objective functions for IEEE 33 bus system*

Methods	P_{loss}	$OVSI$	IVD	Total cost
PSO	199.6947	0.143064	-5.7E-05	5046.609
GSO	140.7127	0.073418	-0.00012	2227.117
GA	231.3078	0.16669	-3.5E-05	8624.495
ABC	175.2202	0.112183	-6.7E-05	3627.49
FF	152.0933	0.06875	-9.5E-05	2478.489
IABC	141.6783	0.0677	-0.00012	2200.427
SIFF	168.7431	0.092093	-0.0001	2168.439

*Here, and further on in reporting the results, the respective techniques were implemented in accordance with appropriate reference items, namely PSO: Chen et al. (2017), GSO: Basu (2015); GA: Martinez-Cañada et al. (2017); ABC: Yang (2010); FF: Fister, Fister and Brest (2013); IABC: Subramanyam, Ram and Subrahmanyam (2018).

Table 3. Performance analysis for all the objective functions for IEEE 12 bus system

Methods	P_{loss}	$OVSI$	IVD	Total cost
PSO	17.18137	0.001443	-0.00096	149.3599
GSO	16.53983	0.000686	-0.0012	119.9612
GA	13.10472	0.000577	-0.00114	178.6658
ABC	12.84478	5.34E-05	-0.0014	113.7786
FF	15.92066	0.00042	-0.00123	115.5763
IABC	16.11029	0.000543	-0.00123	114.0995
SIFF	16.1108	0.000544	-0.00123	114.0998

On the other hand, Table 6 shows the performance analysis related to the sizing of fuel cells in DG system at various load conditions, namely 50%, 100%, and 150%, using SIFF algorithm and other compared techniques.

Table 4. Performance analysis for all the objective functions for IEEE 9 bus system

Methods	P_{loss}	$OVSI$	IVD	Total cost
PSO	2490.133	0.063976	-1.9E-08	12502697
GSO	2196.443	0.08744	-2.4E-08	7116818
GA	2396.606	0.069191	-2E-08	11160991
ABC	2417.534	0.067123	-2E-08	10837087
FF	2246.485	0.080531	-2.3E-08	7117567
IABC	2197.181	0.08735	-2.4E-08	7117699
SIFF	2237.688	0.081489	-2.4E-08	7016646

Table 5. Performance analysis for all the objective functions for IEEE 69 bus system

Methods	P_{loss}	$OVSI$	IVD	Total cost
PSO	223.8844	0.086695	-2.3E-05	20366.72
GSO	146.6318	0.066699	-0.0001	4268.38
GA	300.8341	0.157761	-1.5E-05	31112.5
ABC	269.8942	0.123422	-1.5E-05	30300.87
FF	261.677	0.102188	-1.5E-05	29897.42
IABC	152.9687	0.064002	-7.3E-05	6003.58
SIFF	139.0064	0.062777	-0.00011	3883.91

It can be seen that for the IEEE 33 bus system, at the load of 50%, the proposed technique is by 18% better than PSO, by 17% better than GSO, by 86% better than GA, by 97% better than ABC, by 71% better than FF, and by 12% better than IABC. Then, for the IEEE 12 bus system, the proposed technique is by 79% superior to PSO, by 72% superior to GSO, by 77% superior to GA, by 76% superior to ABC, by 7.5% superior to FF, and by 7.6% superior to IABC. For the IEEE 9 bus system, the implemented method is better than PSO by 56%, than GSO by 3.5 %, than GA by 57%, than ABC by 55%, than FF by 3.6%, and than IABC by 3.7%. For the IEEE 69 bus system, the implemented method is by 80% better than PSO, by 32% better than GSO, by 83% better than GA, by 80% better than ABC, by 40% better than FF, and by 79% better than IABC. Hence it can be concluded that also in this respect the proposed technique fares better than those compared with it.

5.4. Statistical analysis

As the all of the here compared methods are metaheuristics, they are stochastic in nature. This is why these techniques were executed five times each, and the best and worst solutions were determined. Table 7 shows the respective results

for all of the benchmark IEEE bus systems considered.

With respect to the IEEE 33 bus system, the proposed method is better than PSO by 0.24%, than GSO by 4.35 %, than GA by 75.7%, than ABC by 23.6%, than FF by 11.43%, and than IABC by 5.57%, respectively, thus showing the improved capability of the new method. For the IEEE 12 bus system, the proposed method is by 44.1% superior to PSO, by 0.28% superior to GSO, by 50.69% superior to GA, by 32.91% superior to ABC, by 3.44% superior to FF, and by 3.96% superior to IABC. For the IEEE 9 bus system, the proposed method is better than PSO by 37.4%, than GSO by 0.32%, than GA by 57.53%, than ABC by 48.58%, than FF by 17.11%, and than IABC by 0.2%. Finally, for the IEEE 69 bus system, the proposed method is better than PSO by 99.11%, than GSO by 95.3%, than GA by 64.4%, than ABC by 61.3%, than FF by 99.3%, and than IABC by 96.77%, respectively. This provides another kind of evidence for the superiority of the new method.

5.5. Effect of multiple objectives

The effect of consideration of multiple objectives for the sizing of fuel cells for various IEEE test bus systems in terms of value scattering is shown in Fig. 4. Here, the implemented mechanism provides the capability for more wholesome distribution of the multiple objective values for the IEEE bus systems. At the same time, the existing techniques do not offer such kind of globally oriented distribution, they rather tend to be tersely distributed. This shows, again, that the solution distribution is well enhanced by the proposed technique.

6. Conclusions

This paper introduces a two-stage intelligent technique for solving the issues, associated with location and sizing of fuel cells in DG system. These issues include spontaneous current fluctuations, security, complexity, optimal location and sizing, voltage instability and level of penetration. In the proposed two-stage technique, an NN model was first adopted for determining the required location to place the fuel cells. Secondly, the SIFF algorithm, derived from the firefly algorithm, was adopted for finding the optimal size of the fuel cells.

The proposed method was simulated for four IEEE benchmark test bus systems, and the appropriate performance analysis was carried out. In the framework of this analysis, the proposed technique was compared with six algorithms, known from the respective literature, namely the PSO, FF, ABC, IABC, GA and GSO algorithms. From the simulations, it could be concluded that the implemented technique was better than PSO by 80%, than GSO by 72%, than IABC by 53%, than GA by 86%, than ABC by 97% and than FF by 80%, respectively, in terms of the cost function. Thus, the experimental results, obtained from the simulation, show the enhanced performance of the new technique in locating and sizing the fuel cells in the DG systems.

Table 6. Performance analysis on fuel cell sizing at different load conditions

Test Bus system	Methods	Base case load	Load 50 %	Load 100 %	Load 150 %
IEEE 33	PSO	2528.521	1538.83	1487.877	1768.78
	GSO	2168.477	1567.212	1490.071	1315.985
	GA	9090.405	13998.17	21046.65	33920.43
	ABC	2878.824	3719.904	4607.701	2580.57
	FF	2526.121	3232.7	2333.805	2458.633
	IABC	2176.911	1658.193	1998.194	1280.266
	SIFF	2163.487	1886.352	1486.796	1300.328
IEEE 12	PSO	228.7272	214.815	1069.013	2325.775
	GSO	117.0436	442.2616	111.7097	111.604
	GA	229.9879	526.4923	1323.61	3030.834
	ABC	223.5327	498.3668	137.2163	2832.522
	FF	115.6277	111.5111	112.153	114.9742
	IABC	112.7735	113.1645	114.9906	112.7231
	SIFF	112.5088	119.6889	111.9526	112.9265
IEEE 9	PSO	10413421	23040355	41109472	84139353
	GSO	7000757	14132960	26461148	43751937
	GA	10699300	23996952	49832772	84389173
	ABC	10430084	23316368	50167185	84817400
	FF	8179123	18671135	42913642	69611441
	IABC	6976399	14551530	26655184	44279465
	SIFF	6955387	14689150	26590401	45477240
IEEE 69	PSO	20078.02	30171.85	30231.01	30630.88
	GSO	3950.94	3903.842	3908.316	4016.732
	GA	31358.79	31425.25	31188.08	31556.71
	ABC	30486.89	30549.39	30654.88	30726.87
	FF	19746.69	29924.03	30026.99	20538.16
	IABC	8256.364	8289.095	5972.757	4131.024
	SIFF	3870.056	5767.636	4002.678	4026.566

Table 7. Statistical analysis on fuel cell sizing in DG system for different IEEE benchmark systems

Test Bus system	Methods	Best	Worst	Mean	Median	Std dev
IEEE 33	PSO	1995.688	4848.388	2686.976	2126.304	1220.912
	GSO	2087.717	2179.111	2135.373	2139.108	34.38099
	GA	8248.778	11477.14	9671.46	8764.464	1656.776
	ABC	2473.564	3453.674	2773.732	2677.442	389.3506
	FF	2229.228	2327.741	2276.229	2275.44	40.03781
	IABC	1999.391	2073.044	2036.377	2038.263	31.28523
	SIFF	2000.506	2011.374	2005.447	2004.998	4.832183
IEEE 12	PSO	98.60446	187.7077	130.2761	132.8322	36.37802
	GSO	98.89004	104.5372	101.9963	103.1342	2.856492
	GA	166.4271	212.642	176.3705	166.8702	20.31058
	ABC	101.9606	156.2784	119.3167	111.3061	21.47028
	FF	98.93345	101.352	99.90009	99.41789	1.063399
	IABC	98.65096	100.8446	99.31741	99.02256	0.875856
	SIFF	97.63969	104.8412	100.7822	99.22222	3.06823
IEEE 9	PSO	10086473	12500355	11161267	11205018	876916.2
	GSO	6945618	7114704	7008278	6997461	63225.59
	GA	10791940	11158717	11004343	11050017	147170.4
	ABC	9804807	10974432	10379049	10443695	545293.6
	FF	7115412	9376011	8427967	8323833	863502.6
	IABC	6945898	7115585	6999372	6993352	69393.97
	SIFF	6957159	7014498	6985357	6980745	26686.2
IEEE 69	PSO	19328.23	29849.72	24114.29	21681.23	5177.429
	GSO	3722.761	5888.83	4750.021	4289.636	988.5982
	GA	30804.4	31044.51	30949.96	31043.15	128.7687
	ABC	30032.68	30283.58	30150.78	30125.56	118.3787
	FF	14735.82	29689.59	22566.82	19386.26	6750.773
	IABC	4269.455	8124.137	5813.929	5424.48	1418.382
	SIFF	3746.214	3855.469	3788.575	3787.488	45.80565

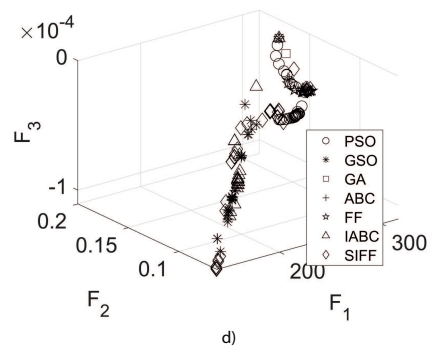
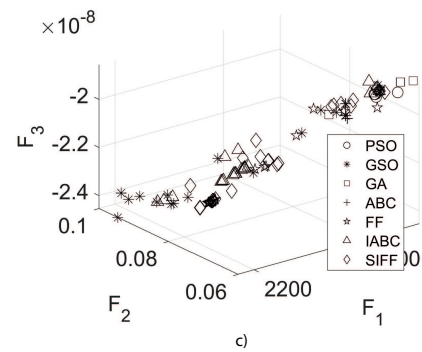
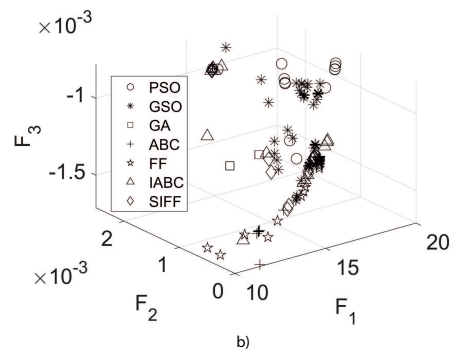
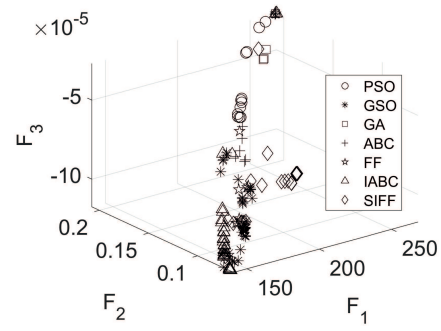


Figure 4. Impact of multiobjectivity on fuel sizing for (a) IEEE 33 bus system; (b) IEEE 12 bus system; (c) IEEE 9 bus system; (d) IEEE 69 bus system

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