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Comparative study of the parametric and analytical model of the movement of PR-02 robot arm*

by

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Abstract: The work presented in this paper concerns selected results obtained in the search for a parametric model of the movement of the end of PR-02 robot arm. The research performed involved a critical review of the literature on the subject, design and implementation of a parametric robot model, and carrying out simulations and a comparative study. First of all, it was necessary to obtain data for identification-based modeling, which result from the assumptions and experiments on the functioning of the analytical model of the movement of the end of PR-02 robot arm. Therefore, based on the functioning of the analytical model, a set of numerical data needed to identify the movement of the end of PR-02 robot arm was generated. Using the input and output data of the analytical model of the PR-02 robot arm end-motion system, additional data were generated from the environment of individual parameter values as input quantities and the coordinates of points on the motion trajectory as output quantities, within the limits of the permissible accuracy error of the individual data, in order to obtain sufficiently large amounts of data for the purposes of obtaining a parametric model of the system by identification. The obtained data were then used in identification using the ARX method to generate an identification-based model. In the preparatory research process for the research experiment, three sets of data were generated, differing in the length of the experiment duration, but always for 3 output quantities representing the coordinates of the position of the point on the trajectory of the end of the PR-02 robot arm. The data set, for which the most accurate ARX model was obtained,

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was adopted in the identification experiment, conducted on the simulation model in Simulink. Then, an appropriate model was built to perform the necessary simulations and comparative tests, meant to determine whether and to what extent the parametric model is sufficiently accurate. The comparative studies used absolute and relative errors between the analytical model and the parametric model. Moreover, the discrepancies between the trajectory plotted by the analytical model and the one plotted by the parametric model were examined, and the obtained results were discussed and conclusions and directions for further research were formulated.

Keywords: analytical modeling, identification modeling, PR-02 industrial robot, robot arm end movement, MATLAB and Simulink environment

1. Introduction

The present paper addresses the research issues, related to machine learning in the field of obtaining regression models, here for the case of the PR-02 robot end-of-arm motion system, in particular – in the field of identification and simulation of the PR-02 robot motion system. The undertaken research task therefore concerns the development of the ARX parametric model of the PR-02 robot end-of-arm motion system and its implementation in the MATLAB and Simulink environment, as well as obtaining a simulation model to test the accuracy of the model in relation to the analytical model and for purposes of comparative studies. An additional research task was to review industrial robot models available in the literature in order to determine the conditions and scope of own research in the MATLAB and Simulink environment using the System Identification Toolbox (see: Guide for ..., 1992-2024b).

The research problem was therefore reduced to designing the movement of the end of the arm of the industrial robot PR-02 by obtaining a model through parametric identification, and then to conducting a simulation of the model of the robot end of the arm movement system in the MATLAB and Simulink environment, and to comparing the obtained thereby results with regard to the analytical model. In this respect, the discrepancies between the output values of the two models and, consequently, the absolute and relative errors between the ARX parametric model and the analytical model were investigated in order to determine the identification accuracy in the regression-based machine learning process.

In order to carry out the identification, a research experiment was set up consisting in preparation of numerical data, based on the input and output values of the analytical model generated within the permissible error range for the numerical values of all three spatial coordinates. The data obtained on this basis were used in parametric identification, the final effect of which was the obtained ARX model.

Then, based on the ARX model, a simulation model was built in Simulink and a series of simulation and comparative studies were carried out to select the best approach to identification, so that the finally generated model had a trajectory of the end of the PR-02 robot arm movement close to the trajectory of the analytical model. In this way, the studies confirmed that the adopted direction of search in the identification of the PR-02 robot arm movement models is correct.

Therefore, the actual object of research is the movement of the end of the PR-02 industrial robot arm, and the adopted research method is the ARX parametric identification method using regression machine learning. Within this context, the research methodology, related to the research problem considered, consists, in particular, of (see Söderström and Stoica, 1997; Tchórzewski, 2013; Zimmer and Englot, 2005; Trusz and Tserakh, 2017; Hayati, Shafieirad M, et al., 2021):

- developing a method for generating data for identification,
- developing an identification experiment,
- conducting identification in order to obtain a model of the PR-02 robot arm end motion system based on input and output data,
- developing a simulation and comparative experiment,
- designing a simulation model for comparative studies of output signals from the parametric model in relation to output signals from the simulation model,
- conducting simulation studies.

2. Results of literature review

In order to obtain reliable knowledge on the methods used in modeling to obtain industrial robot models, including the generation of their courses based on analytical, neural, parametric, etc. models, as well as the possibility of performing identification using the System Identification Toolbox in the MATLAB environment and the method of conducting simulations and comparative studies in the Simulink environment, a critical review of the literature on the subject was conducted.

The review accounted for three distinguishing features, i.e.: object, modeling environment and modeling method, to which detailed information, obtained from the available literature was assigned. The selected results of literature research obtained are synthetically presented in Table 1.

Table 1. Selected results in the field of industrial robot modeling methods

Lp.	Authors, years	Distinctive features		
		Object	Environment	Methods
1.	T. Szkodny (2004b, 2009)	MRP - industrial robot, MRE - experimental robot, PR-02 robot	computational examples	analytical model
2.	J. Tchórzewski and P. Lewandowski (2014)	PR-02 robot	MATLAB and Simulink	evolutionary-analytical model
3.	K. Tchoń, A. Mazur, et al. (2000)	Manipulator IRb-6, Manipulator of SCARA type	computational examples	analytical model
4.	A. Morecki and J. Knapczyk (1993)	Manipulator 3-RPR, Manipulator 3-SPS	computational examples	analytical model
5.	Ł. Kocurek (2013)	Manipulator Fanuc S-420F	MATLAB and Simulink, CATIA	analytical model
6.	J. J. Craig (1995)	Cincinnati Milacron 776 manipulator	computational examples	analytical model
7.	J. Tchórzewski and A. Wielgo (2021)	Humanoid Robot	MATLAB and Simulink with NNT/DLT	neuronal model
8.	K. Kozłowski, P. Dudkiewicz and W. Wróblewski (2012)	Manipulator IRp-6	computational examples	analytical model
9.	A. Labuda, J. Pomirski and A. Rak (2009)	SCARA type manipulator	Borland Delphi	analytical model with two rotating joints

Table 1, continued

10.	B. Borowik (2015)	four-degree-of-freedom robot arm	MATLAB control via MACH3 program	analytical model
12.	J. Tchórzewski and P. Domański (2018)	Robot PR-02	MATLAB and Simulink with NNT	quantum inspired neuronal model
13.	E. Dindorf (2019)	PR-02 Robot with Pneumatic Drive	new solenoid valve block	control system modernization project
14.	G. Służałek, M. Kubica and P. Będek (2012)	Educational robot arm for self-assembly STV-KSR10 by Velleman	CAD, CAE, kinematics were mapped in Solid Edge and finite element analyses were performed in Inventor	analytical model
15.	K. Niderla and G. Kłosowski (2024)	Robot KUKA iiwa 14	Python with pybullet library	Reinforcement Learning Modeling
16.	W. Szpura (2023)	Robot PR-02	MATLAB and Simulink z SIT	Machine Learning Regression Modeling
17.	V. K. Banga, K. J., Kumar and Y. Singh (2011)	Robot arm	-	analytical model improved with AE and fuzzy logic
18.	A.A. Okubanjo, O.K. Oyetola, O.O. Olaluwoye (2017)	Robot arm	MATLAB and Simulink	mathematical modeling based on Lagrange and Euler-Lagrange equations
19.	J. Tchórzewski and P. Tymoszuk (2023)	End of the PR-02 robot arm	Matlab and Simullink with NNT	neural model

Source: own elaboration (see: Szpura, 2023).

The literature review, summarized in Table 1, mainly concerns research related to modeling the PR-02 robot or similar solutions, from analytical modeling, through neural modeling, to parametric modeling, hence these are mainly items related to Polish, but English-language, literature. As it turns out, although the neural and analytical models of the PR-02 robot system are relatively well developed, there are very few research results in the field of machine learning using regression or reinforcement learning methods, including the use of identification methods.

As can be seen from the review of Table 1, in addition to modeling the PR-02 robot, experimental studies were also conducted on the IRp-6 robot model, the Fanuc S-420F robot, the humanoid robot, the 3-RPR and 3-SPS manipulator, the Cincinnati Milacron 776 manipulator, the SCARA type manipulator, the MRP industrial robot manipulator, the experimental robot manipulator (MRE) and many other objects, with only the PR-02 robot having been developed for the purposes of comparative research using various methods, hence this particular robot was adopted for comparative and simulation studies (see: Szkodny, 2004b; Tchórzewski and Lewandowski, 2014; Tchórzewski and Domański, 2018).

From the point of view of the applied research environment used for modeling and simulation of industrial and humanoid robots, the most common environment was constituted by MATLAB and Simulink (see: Guide for…, 1992-2024b). From the point of view of the type of modeling used, the most common was analytical and neural modeling, and sometimes also evolutionary modeling.

However, there is no record in the literature of the subject that in the same scope as covered by the present research, a search for a PR-02 robot model was conducted by means of machine learning, including parametric modeling using the machine learning regression method (Tabor, Śmieja et al., 2022).

3. The PR-02 robot

The PR-02 robot consists of two main units, meant for manipulation and control (see Fig. 1). The manipulation unit is equipped with four joints, two providing linear movement and two providing rotary movement (Tchórzewski and Tymoszuk, 2023). The unit itself is built of mechanical modules, which allows for introducing modifications in the structure and kinematic system of the robot. This means that the workspace of this robot can be built for any possible implementation of movements.

The entire manipulation unit is controlled by the control unit that is used to implement the movements according to the given work program. The PR-02 robot control system thus provided PTP type control*. This type of control

*In point control (PTP – Point to Point) the manipulation object is moved from one

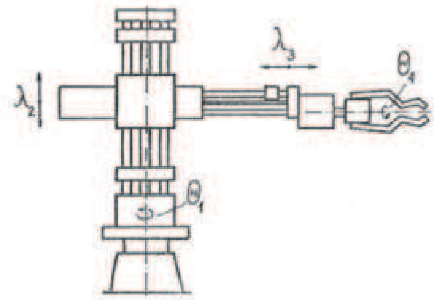

Figure 1. Industrial robot PR-02. Designations: θ_1 – angle of transformation of the rotational pair of the first system, λ_2 – magnitude of displacement of the sliding pair of the second system, λ_3 – shift of the sliding pair of the third system, θ_4 – angle of transformation of the fourth rotational pair. Sources: Dokumentacja . . . (1979); Dindorf (2019); Tchórzewski and Tymoszuk (2023)

during the movement of the robot arm from point A to point B does not take into account any supervision over the route between individual work points (Documentation ..., 1979). Based on the description of the PR-02 robot, it can be seen that the robot is designed to perform relatively simple functions, usually auxiliary, and in this version, it is used to compare new methods of modeling its movement. The pneumatic type of drive means that the lifting force of such a robot is not as big as in the case of robots with hydraulic drive. Pneumatic actuators require a much larger number of positioning points, which for obvious reasons involves the need to use additional positioning devices. The type of control itself turned out to be quite problematic. In this case, research was undertaken to improve the movement of such a robot, which was described, among others, in the work of Dindorf (2019).

4. Creating a robot model using regression machine learning

As this is shown in Table 1, various attempts have been made, as reported in the literature, to create robot models, including industrial robots and humanoid robots. When building models, it is crucial to approach the modeled system in terms of control and systems theory (see Kaczorek, Dzieliński et al., 2021; Kacprzyk, Korbicz and Kulczycki, 2020; Tchorzewski, 1990, 1992, 2013). Various modeling methods are used, with analytical models being the most common

position to the next one without controlling the course of changes in intermediate positions, the starting point and the end point only being important.

(see Szkodny, 2004b, 2009; Banda, Kaur et al., 2011; Borowik, 2015; Kocurek, Góra, 2013). In recent years, attempts have been made to accomplish modeling using various artificial intelligence methods, most often neural, evolutionary, or quantum-inspired neural modeling methods (see Tchórzewski and Tymoszuk, 2023; Tchórzewski and Wielgo, 2021; Tchórzewski and Domański, 2018; Tchórzewski and Lewandowski, 2014). One of the methods that gives great hope for obtaining well-matched models for systems is the regression machine learning method, which uses the abundance of results in the field of identification and control of technical and technical-economic systems (see Tadeusiewicz, 1993; Tchórzewski, 1990, 1992, 2013, 2021; Trusz and Tserakh, 2017; Marlęga, 2022; Tchórzewski and Marlęga, 2019).

Parametric models, obtained in process identification, are models with a strictly defined structure. Identification of such models usually begins with identification of model structure, because the number of parameters, whose values are determined at a later stage of machine learning, directly depends on structure. After the model structure is determined, the stage of determining the parameter values follows, often called parameter estimation stage, in order to emphasize that this is only one of the final stages of the entire identification process. On the other hand, the entire identification process implemented in a specific IT environment and fully automated is called regression machine learning.

Therefore, as a result of this particular kind of machine learning, not only a parametric model is obtained, but also information about the accuracy of identification, as well as about other conditions that led to a specific system model. Moreover, in machine learning it is important to obtain not only the final form of the mathematical model of the system, but also such an implementation form that it is possible to use it, e.g. as an independent block in Simulink, associated with the function supporting the model. In the experimental studies, presented in this paper, an autoregressive parametric model with an external input, the so-called ARX model (AutoRegressive with eXogenous input), was used, which is a discrete input-output model for stochastic processes of the form (see Söderström and Stoica, 1997; Tchorzewski, 2013; Zimmer and Englot, 2005; Trusz and Tserakh, 2000; Marlęga and Tchórzewski, 2022):

$$\mathrm{y}_j(t)=\frac{B\left(z^{-1}\right)}{A\left(z^{-1}\right)}\mathrm{u}_i\left(\mathrm{t}\right)+\frac{1}{A\left(z^{-1}\right)}\mathrm{e}\left(\mathrm{t}\right) \tag{1}$$

where:

A, B - polynomials containing the appropriate coefficients that are sought,

$u_i(t)$ – i-th input signal to the PR-02 robot model,

$y_j(t)$ – j-th output signal from the PR-02 robot model,

$e(t)$ – interference with the white noise properties.

5. Parametric models in system identification

The experiment on the movement of the end of the robot arm PR-02 was based on the results of the functioning of the analytical model of the robot in the MATLAB and Simulink environments (Documentation ..., 1979; Szkodny, 2004a, 2004b, 2009). For this purpose, the course of the robot arm end motion trajectory was first generated by changing the model parameters and recording the corresponding coordinates of each point of the robot arm end motion trajectory. Then, the data obtained in this way were used in parametric identification using the regression machine learning method ARX using the System Identification Toolbox in the MATLAB and Simulink environment as a computational tool.

To obtain the model of the movement regime of the end of the arm of PR-02 robot using the machine learning method, which is founded on the ARX parametric identification, the data obtained during three runs of the analytical model implemented in Simulink were used. The values of three parameters of the PR-02 robot movement were used as input data, i.e. two parameters determining the linear movement and one parameter related to rotation. In the last part of the experiment, a study model was built for the purposes of simulation and comparative studies.

5.1. Preparing the data for experiments

The data in the research experiment were generated based on the data used in this type of studies, as shown in Table 2 as the Denavit-Hartenberg parameters describing this manipulator (see Szkodny, 2004a, 2004b).

Table 2. Ranges of PR-02 robot parameter values. Designations: α_i – angle of rotation around the x_i axis; l_i - displacement along the x_i-axis; λ_i - displacement along the z axis

kinematic pair number	$\alpha_\mathbf{i}$	$\mathbf{I_i[m]}$	$\lambda_\mathbf{i}\mathbf{[m]}$	$\theta_\mathbf{i}$
1	0	0	0	-300÷0°
2	-90°	-0.110	0÷0.200	0
3	0	0	0.376÷0.676	-90°
4	0	0	0	0÷360°

Source: Szkodny (2004a, 2004b).

Therefore, in the conducted experiment, the following input quantities were assumed: the *rotation angle* of the robot of system i relative to system i-1 of the first kinematic pair θ_i, *displacement* of system i relative to system i-1 along the

z axis of the second kinematic pair λ_i and the *translation* of system i relative to system i-1 along the z axis of the third kinematic pair λ_i. The values of these three parameters were then used to force the end of the robot arm to move along the specified trajectory. In the considered kinematic chain as in Fig. 1, there are four systems connected in the series by means of kinematic pairs, hence the values of these three parameters were then used to force the end of the robot arm to move along the specified trajectory. In the tests conducted in experiments here reported, the gripper and its rotation were not used, because its position does not affect the movement of the point along the trajectory (the gripper is used to manipulate the moved goods). To determine the number of kinematic pairs used in the identification, the number of changes in the value of the first kinematic pair was selected, which involves the adoption of 301 positions. On the other hand, a simplified form of the analytical model of the PR-02 robot was used to generate the analytical model in the form:

$$\begin{aligned} x &= \lambda_2 \,\mathrm{x}\, \cos\theta_1 - \lambda_3 \,\mathrm{x}\, \sin\theta_1 \\ y &= \lambda_2 \,\mathrm{x}\, \sin\theta_1 - \lambda_3 \,\mathrm{x}\, \cos\theta_1 \\ z &= \lambda_2 \,, \end{aligned} \tag{2}$$

where:

θ_1 – theta1, rotation angle, as the angle of rotation of system i with respect to system i-1,

λ_2 – lambda2, transposition, as a shift of system i with respect to system i-1 along the z axis for the second kinematic pair λ_i,

λ_3 – lambda3, transposition, as a shift of system $i$relative to system i-1 along the axis for the third kinematic pair λ_i. for $i = 1$.

5.2. Implementation of the analytical model of the PR-02 robot movement in MATLAB

The analytical model was implemented in MATLAB in the form of an m-file (see Guide for ..., 2002-2024b). For this purpose, numerical data were first generated in the form of three tables, with input parameters needed to create the analytical model, generating data sets in the number of 301 items for each parameter, used as input quantities to the model, i.e.: $\theta_1,\ \lambda_2,\ \lambda_3$:

$$\begin{aligned} &dane1\_t1 = [-300 : 1 : 0]'; \\ &dane1\_l2 = [(0 : (0.2/300) : 0.2)]'; \\ &dane1\_l3 = [0.376 : 0.001 : 0.676]'. \end{aligned} \tag{3}$$

The data, which were generated for the purposees of machine learning using the regression method, are presented, as to their principal characteristics, in

Table 3. Then, the code was written in order to generate the values of the output parameters of the analytical model in the form of tables X, Y, Z:

$$
\begin{aligned}
&dane1\_am\_x = dane1\_l2. \bullet \cos(deg2rad(dane1\_t1)) \\
&-dane1\_l3. \bullet \sin(deg2rad(dane1\_t1)); \\
&dane1\_am\_ = dane1\_l2. \bullet \sin(deg2rad(dane1\_t1)) \\
&+dane1\_l3. \bullet \cos(deg2rad(dane1\_t1)); \\
&dane1\_am\_z = dane1\_l2.
\end{aligned} \tag{4}
$$

Table 3. Summary of the coordinate values of the end of PR-02 robot arm in the Workspace, marked X, Y and Z, respectively, which were obtained based on the input data: θ_1, λ_2, λ_3. Designations: θ_1– angle of transformation of the rotational pair of the first system, λ_2 – magnitude of displacement of the sliding pair of the second system, λ_3 – the third system pair shift, x – coordinate along X axis, y – coordinate along Y axis, z – coordinate along Z axis

θ_1	λ_2[m]	λ_3[m]	x [m]	y [m]	z [m]
-300	0.00000	0.376	-0.325626	0.188000	0.0000
-299	0.00067	0.377	-0.329408	0.183356	0.0007
-298	0.00133	0.378	-0.333128	0.178638	0.0013
...	...	...	...	...	...
-2	0.19867	0.674	0.222068	0.666656	0.1987
-1	0.19933	0.675	0.211083	0.671418	0.1993
0	0.20000	0.676	0.200000	0.676000	0.2000

Source: Own elaboration (see Tchórzewski and Lewandowski, 2014; Szpura, 2023)

Based on the values of coordinates along X, Y and Z axes, as illustrated in Table 3, the trajectory of the end of PR-02 robot arm in the workspace was generated using the plot3() function. It was assumed that 301 positions would be equivalent to the robot position movement time instances in a time unit [s]. Figure 2 shows the course of the robot arm movement generated using the plot3() function.

Then, in an identical manner, a parametric model of the end-of-arm movement of PR-02 robot was generated, obtained as a result of regression machine learning, in which, as previously mentioned, the input values were the angle of rotation and two transactional movements. In order to generate the appropriate data, a code was prepared to generate a list of 301 items for the input data of

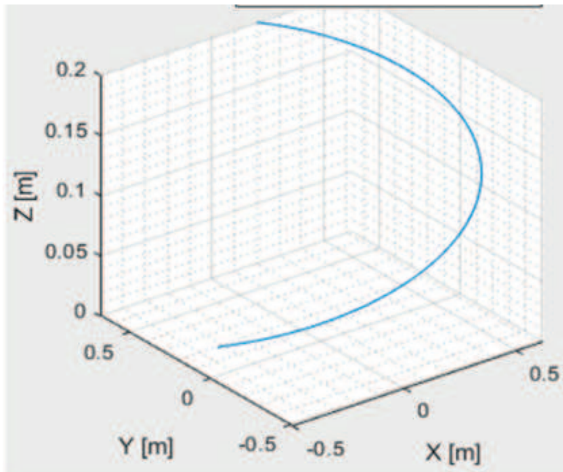

Figure 2. The trajectory of the end of PR-02 robot arm in the Workspace in the time T = 301 [s] obtained using the analytical model. Designations: X – coordinates related to the length of the workspace along X axis [m], Y – coordinates related to the width of the workspace along Y axis [m], Z – coordinates related to the height of the workspace along Z axis [m]. Source: own elaboration (see Guide for . . . , 2002-2024b; Szpura, 2023)

the analytical model: θ_1, λ_2, λ_3, in which each value has been disturbed by an appropriate error, respectively:

$$\begin{aligned} dane1\_random1\_t1 &= [-300:1:0]' + (0.01 * (randi([-30,30],301,1)));\\ dane1\_random1\_l2 &= [(0:(0.2/300):0.2)']\\ &\quad +(0.00001 * (randi([-30,30],301,1)));\\ dane1\_random1\_l3 &= [0.376:0.001:0.676]'\\ &\quad +(0.00001 * (randi([-30,30],301,1))). \end{aligned} \tag{5}$$

It was established that the data θ_1 would be generated with an error of ± 0.3 and the data λ_2 and λ_3 with an error of ± 0.0003. Then, using the code generating the output parameters of the analytical model, disturbed by the appropriate error, the following were obtained (see Table 4):

$$\begin{aligned} &dane1\_random1\_am\_x\\ &= dane1\_random1\_l2. * cos(deg2rad(dane1\_random1\_t1))\\ &-dane1\_random1\_l3. * sin(deg2rad(dane1\_random1\_t1)); \end{aligned}$$

$$
\begin{aligned}
&dane1\_random1\_am\_y \\
&= dane1\_random1\_l2. * sin(deg2rad(dane1\_random1\_t1)) \\
&+ dane1\_random1\_l3. * cos(deg2rad(dane1\_random1\_t1)); \\
&dane1\_random1\_am\_z = dane1\_random1\_l2. \qquad (6)
\end{aligned}
$$

Table 4. Summary of the coordinate values of the end of PR-02 robot arm in the X, Y, Z workspace, which were obtained based on the input data: θ_1, λ_2, λ_3. Designations as in Table 1

$\boldsymbol{\theta_1}$	$\boldsymbol{\lambda_2}$ **[m]**	$\boldsymbol{\lambda_3}$ **[m]**	**X [m]**	**Y [m]**	**Z [m]**
-300.06	-0.000030	0.37588	-0.3253397	0.188254817	-0.00003
-299.26	0.000947	0.37695	-0.3283925	0.185069058	0.00095
-297.89	0.001053	0.37777	-0.3333983	0.177642559	0.00105
...	...	...	...	...	...
-2.01	0.198417	0.67423	0.2219425	0.666855909	0.19842
-0.77	0.199293	0.67498	0.2083461	0.672240818	0.19929
-0.09	0.199850	0.67594	0.2009115	0.675625243	0.19985

Source: own elaboration using MATLAB and Simulink (see Guide for . . . , 2002-2024b, Szpura, 2023)

Based on the generated X, Y, Z values, illustrated in Table 2, using the plot3() function, the motion trajectory of the end of PR-02 robot arm in the workspace was generated in MATLAB, this trajectory having the same course as the trajectory of the analytical model, shown in Fig. 2. Then, the values of the input and output quantities were generated not only for T = 301, but also for T = 602, T = 903 and T = 1 204, and these values were then used in the System Identification Toolbox for the regression machine learning of PR-02 robot arm end motion system model using the ARX method.

5.3. Identification as a process of obtaining a model of the robot arm end motion

To identify the end-of-arm motion of PR-02 robot, the System Identification Toolbox library was used. Machine learning using the ARX regression method of the model identification was performed using the data generated for periods T = {301, 602, 903, 1 204}. The machine learning results were ARX parametric models. It turned out that estimating the model for only one period of the data set, i.e. for T = 301, generates the least accurate model, and the model generated for T = 1 204 (i.e. for four periods of arm movement) was the most accurate, which was, of course, to be expected. The comparative course of the parametric model and the analytical model of the end-of-arm motion of the PR-02 robot for T = 602 (two-cycles of movement of the arm along the specified trajectory) is presented in Fig. 3. Then, for T = 1 204, Figs. 4 through 6 present the course of the models for individual coordinates of the point on the trajectory, i.e. for X, Y and Z.

For the form (7) the following parametric models were obtained for individual coordinates:

X: y(t) = 1.338 y(t-1) + 0.3315 y(t-2) - 0.6699 y(t-3) - 8.968e-07 y(t-4)+ 0.001404u1x(t) - 0.001883 u1x(t-1) - 0.0004647 u1x(t-2) + 0.0009431 u1x(t-3)+ 0.0753 - 0.106 u2(t-1) - 0.02323 u2(t-2)+ 0.05392 u2(t-3))+ 0.2545 u3(t) - 0.3581 u3(t-1) - 0.07852 u3(t-2) + 0.1822 u3(t-3),

Y: y(t) = 1.338 y(t-1) + 0.3315 y(t-2) - 0.67 y(t-3) + 8.968e-07 y(t-4) + 0.0005305 - 0.0006845 u1(t1) - 0.0001844 u1(t-2) + 0.0003385 u1(t-3)) + 0.2732 u2(t) - 0.3645 u2(t-1) - 0.09099 u2(t-2) + 0.1823 u2(t-3))+ 0.9233 u3(t) - 1.232 u3(t-1) - 0.3075 u3(t-2) + 0.6161 u3(t-3)

Z: y(t) = 5.371e-12 y(t-1) + 8.999e-11 y(t-2) - 1.248e-10 y(t-3) + 1.607e-13 y(t-4) + 0.000341 u1(t) - 2.215e-14 u1(t-1) - 6.187e-15 u1(t-2) + 3.248e-14 u1(t-3))+ 0.08049 u2(t) + 3.728e-12 u2(t-1) - 1.181e-11 u2(zˆt-2) + 1.295e-11 u2(t-3))+ 0.272 u3(t) + 1.621e-11 u3(t-1) - 4.608e-11 u3(t-2) + 4.218e-11 u3(t-3).

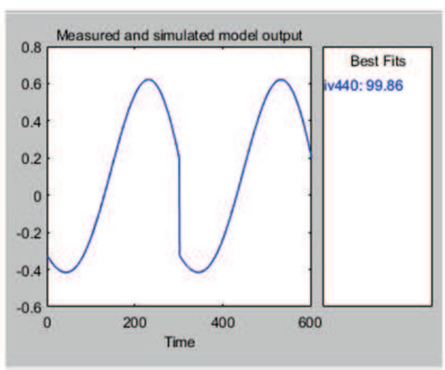

Figure 3. Comparison of the Y component of the trajectory, generated by the ARX model, with the analogous component, generated by the analytical model of the PR-02 robot arm end movement for T = 602 [s] (the degree of fit of the two models is 99.86% and so only one line is visible). Designations: Vertical axis - value of the X coordinate of the point on the trajectory [m], Time - machine learning time in [s]. Source: own study using the MATLAB and Simulink package from SIT (see: Szpura, 2023; Guide for..., 2002-2024b)

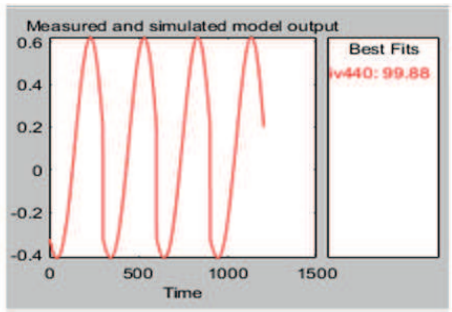

Figure 4. Comparison of the Y component of the trajectory, generated by the ARX model, with the analogous component, generated by the analytical model of the PR-02 robot arm end movement for T = 1 204 [s] (the degree of fit of the two models is 99.88% and hence only one line is visible). Designations: Vertical axis - values of the X coordinate [m], Time - machine learning time in [s]. Source: own study using MATLAB and Simulink from SIT (see Szpura, 2023; Guide for ..., 2002-2024b)

As a result of machine learning with the ARX regression method for the data generated for each of the time periods assumed, i.e. T = 301, 602, 903 and 1204, discrete MISO models were obtained in the form:

$$A(z)y(t) = B(z)u(t) + e(t), \tag{7}$$

where:

$A(z)$ - polynomial associated with $y(t)$,

$B(z)$ - polynomials associated with individual inputs $u_i(t)$,

z - operator of time shift between output and input.

In a similar way, concrete parametric models for T= 602 and T= 1 204 were obtained, and then the thus obtained models were compared with the respective analytical models. The optimal model turned out to be the model generated using data for T = 903.

6. Simulation and comparative studies

In order to conduct in-depth comparative and simulation studies, an appropriate simulation model was built in Simulink, having the structure shown in Fig. 7 (see Osowski, 2007; Tchórzewski, 2013, Tchórzewski and Marlęga, 2019). The simulation model included parametric (identification) models, obtained by the regression machine learning using the ARX method, applied with the use of the data generated for T = 903. The simulation model was built using the blocks, available in Simulink libraries with their originally implemented functions, while the block called Idmodel was used to include parametric models (see Guide for ..., 2002-2024b).

In addition, a custom block called Subsystem was designed using the Create Library option, in which the operation of basic three inputs (in ports) and three outputs (out ports) was designed. A standard data acquisition block called From Workspace was connected to this block. At the output of the block diagram, a block, here called MATLAB function, was added with the assigned plot3() function to obtain spatial histories of the PR-02 robot arm end trajectory during each of the simulation calculation runs (Fig. 8).

7. Summary

The measurements performed show, among other things, that in the case of the obtained outputs concerning coordinate x on the X axis, the error was 0.0299%, for coordinate y on the Y axis it was 0.0033%, and for coordinate z on the Z axis it was 0.0684×10^{-12}%. Such small errors indicate, among other things, that the proposed regression machine learning method ARX leads to identification of a very accurate model of the PR-02 robot arm movement. The models were obtained with an accuracy of 99.87% for coordinate x, 98.4% for coordinate y, and 100% for coordinate z. Moreover, as a result of simulation studies, it turned out, in particular, that the mean square error, MSE, between the output of the analytical model and the output of the parametric model was, respectively: 2.19% for coordinate x, 0.446% for coordinate y and 0.0109×10^{-10}% for coordinate z. Therefore, the obtained results are, in terms of error values, very low, which indicates the possibility of using the regression machine learning method in the design of the end-of-arm motion of the PR-02 robot as an alternative to the analytical model in more complex situations, when the non-analytical model is too complex or difficult to describe mathematically. Further studies should proceed with verifying the usefulness of implementing other regression machine learning methods, or possibly coupled with the reinforcement machine learning method.

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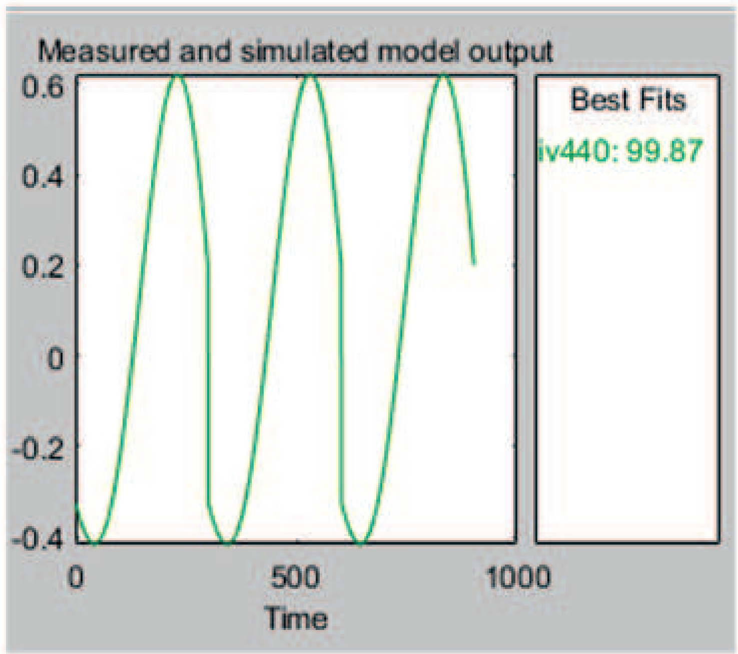

Figure 5. Comparison of the Y component of the trajectory, generated by the parametric model, with the analogous component, generated by the analytical model of the PR-02 robot arm end movement for T = 903 [s] (the degree of fit of the two models is 99.87% and so only one line is visible in the figure). Designations: Vertical axis - value of the Y coordinate of the point on the trajectory [m], Time - machine learning time in [s]. Source: own study using the MATLAB and Simulink package from SIT (see: Szpura, 2023; Guide for..., 2002-2024b)

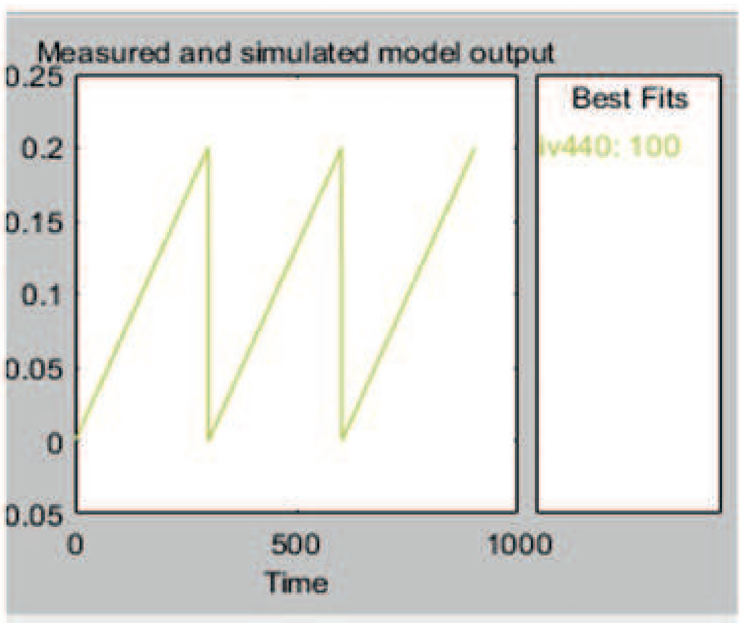

Figure 6. Comparative course of the output of the Z coordinate from the parametric model with the analogous output from the analytical model for the position of a point on the trajectory of the end of the PR-02 robot arm for T = 903 (the degree of fit of the two models is 100.00% and hence only a single line is visible in the figure). Designations: Vertical axis – Z coordinate of the point position on the trajectory [m], Time – machine learning time [s]. Source: own study using MATLAB and Simulink with SIT (see: Szpura, 2023; Guide for..., 2002-2024b)

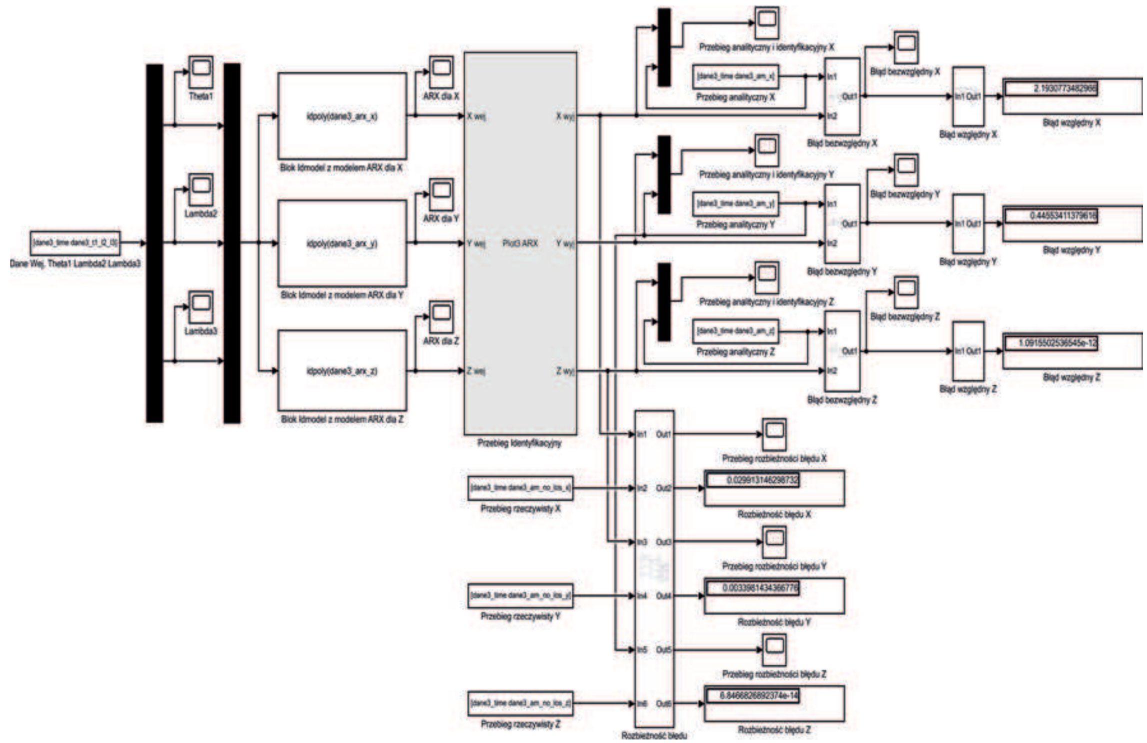

Figure 7. Block scheme of the simulation model of the end-of-arm movement of PR-02 robot. Source: own study using the MATLAB and Simulink environments (see Guide for . . . , 2002-2024b)

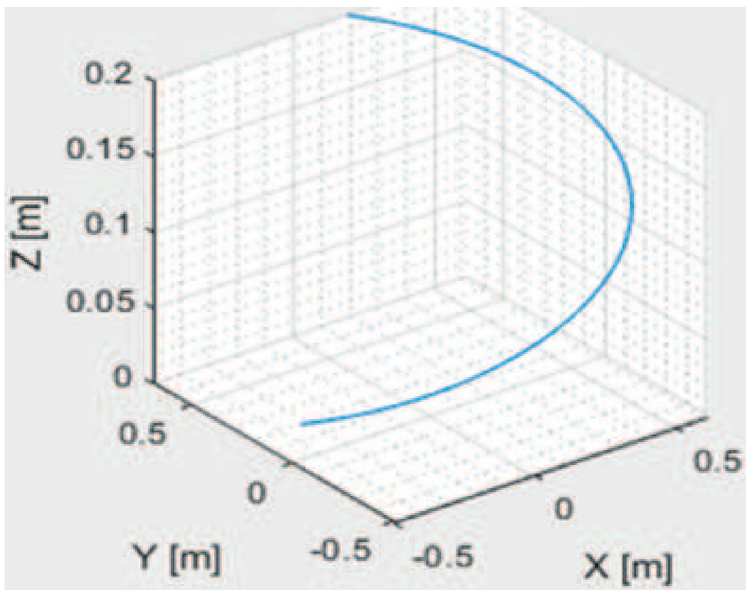

Figure 8. The trajectory of the end of PR-02 robot arm in the Workspace X, Y, Z, for the time period T = 301 [s], obtained from the simulation model. Designations: X – coordinate related to the length of the working field on the X axis [m], Y – coordinate related to the width of the working field on the Y axis [m], Z – coordinate related to the height of the working field on the Z axis [m]. Source: own elaboration using MATLAB and Simulink (see Guide . . . , 2002-2024; Szpura, 2023)