Introduction

Control, Shape and Topology Optimization for Distributed Parameter Systems is a challenging and rich research field in the modern theory of calculus of variations, partial differential equations, differential geometry, as well as numerical analysis. Professor Jan Sokolowski is recognized as pioneering and leading researcher in this field. In celebration of his 75th birthday, we have the pleasure and honor to offer this Special Issue that brings ten relevant papers on the subject.

And so, in particular, infinite horizon optimal control problems subject to semilinear parabolic equations are considered by **Eduardo Casas and Karl Kunisch**. Existence of optimal controls is rigorously addressed. First and second order optimality conditions are derived. Finally, the infinite horizon control problem is approximated by the finite horizon control problems and error estimates are provided. Then, the paper by **Michel Delfour** presents a comprehensive overview and some new results on the Hadamard semidifferentials in the context of matrix groups, groups of diffeomorphisms, and groups of characteristic functions. Finally, shape and topological derivatives are introduced in the context of Hadamard semidifferentials. An optimal control problem in the Mayer form with an autonomous control system, a bounded control in a bounded set, endpoint equality constraints, and one pointwise state inequality is considered by **Andrei Dmitruk and Nikolai Osmolovskii**. The associated maximum principle is derived in details, making it accessible for a wide audience. The paper by **Xuan-Nam Do, Valentin Calisti and François Ganghoffer** deals with synthesis of periodic microstructures in elasticity by using the topological derivative method. Some representative classes of symmetry are considered. The resulting microstructures induce varying macroscopic features, including auxetic and anti-auxetic behaviors, and very large and small bulk-to-shear modulus ratios. Gâteaux semiderivatives are used by **Nico Goldammer, Volker Schulz and Kathrin Welker** to derive optimality conditions for shape optimization problems constrained by variational inequalities in general and contact condition in particular. The extension of Lyapunov functions with exponential weights to a family of hyperbolic functions is considered by **Martin Gugat**, leading to better stability regions. It is also shown that the proposed approach can be applied to a wide range of control problems, including the ones governed by quasilinear hyperbolic partial differential equations. Sharp-interface multiphase problems are considered by **Antoine Laurain**. The lower envelope method for modeling the evolution of multiple phases within a geometrical domain is introduced and then analyzed in detail. Finally, the resulting algorithm is used for solving an inverse conductivity problem with three phases, showing its robustness with respect to noisy data. The inverse one-phase Stefan problem is investigated in the paper by **Marta Lipnicka, Artur**

Lipnicki and Andrzej Nowakowski. The inverse problem is rewritten in the form of an optimal control problem. Sufficient approximate optimality conditions are derived in the form of a verification theorem, which induces a numerical algorithm. Then, in the subsequent paper, the theory of holomorphic functions is applied by **Pavel Plotnikov** to Bernoulli free boundary problems. The existence of solution to the associated variational problem is ensured under natural nondegeneracy condition. In the proof, the notion of convergence in the sense of Caratheodory-Hausdorff is introduced, which represents the main contribution of the paper. The paper by **Yixin Tan, Lingkang Yan and Shengfeng Zhu** deals with structural topology optimization, based on the level-set method. The forward problems are governed by linear and nonlinear elasticity systems into two spatial dimensions. The discontinuous Galerkin finite element method is used to discretize the elasticity systems as well as the transport equation governing the level-set function. Finally, a nice set of numerical experiments are presented, showing different features of the proposed approach.

Thus, this set of papers presents, indeed, an extraordinary contribution to the field that is developing very rapidly, and constitutes a valuable gift for the 75th anniversary of Jan Sokołowski.

The Guest Editors

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